

# Generalized Differential Quadrature Nonlinear Consolidation Analysis of Clay Layer with Time-Varied Drainage Conditions

A. Bahmanikashkouli, O.R. Bahadori Nezhad

**Abstract**—In this article, the phenomenon of nonlinear consolidation in saturated and homogeneous clay layer is studied. Considering time-varied drainage model, the excess pore water pressure in the layer depth is calculated. The Generalized Differential Quadrature (GDQ) method is used for the modeling and numerical analysis. For the purpose of analysis, first the domain of independent variables (i.e., time and clay layer depth) is discretized by the Chebyshev-Gauss-Lobatto series and then the nonlinear system of equations obtained from the GDQ method is solved by means of the Newton-Raphson approach. The obtained results indicate that the Generalized Differential Quadrature method, in addition to being simple to apply, enjoys a very high accuracy in the calculation of excess pore water pressure.

**Keywords**—Generalized Differential Quadrature method, Nonlinear consolidation, Nonlinear system of equations, Time-varied drainage

## I. INTRODUCTION

NUMERICAL models of soil consolidation have initially been developed from the noted classical theory of Terzaghi dating back to 1923 [1]. In the Terzaghi theory, the magnitude of the load and the coefficients of consolidation and impermeability have been considered constant throughout the clay layer; whereas these parameters could vary within the layer, due to the type of soil and the history of loading.

Generally, the models related to the study of clay layer consolidation phenomenon, assume constant properties for the soil throughout the layer; however, experimental findings indicate nonlinear and inhomogeneous behaviors, which cannot be explained by simple models. In 1967, Gibson *et al.* published their findings regarding the need to consider appropriate assumptions, which take into account the changing properties of soil, for the consolidation models based on the developed technical model of Terzaghi [2].

In 1969, Poskitt studied the consolidation of saturated clay layers with variable characteristics of permeability and compressibility, and presented remarkable results [3]. In 1994, Cornetto and Battaglio presented several nonlinear consolidation models for soils and also offered techniques for their analysis [4]. Then in 1996, Arnod *et al.* presented their

theory of nonlinear consolidation models of soils and the effective parameters needed for their analysis [5].

In the past years, researchers like Battaglio *et al.* have conducted studies on the subject of nonlinear consolidation models of clay which changed type from the normal to over consolidation stated; and they have associated this change of state with the critical value of excess pore water pressure of the clay layer during consolidation, which itself is highly influenced by different boundary conditions at the top and bottom of the layer [6]. Bonzani and Lancellotta studied the relations and differential equations governing the phenomenon of nonlinear consolidation, and described the parameters affecting this phenomenon [7]. Battaglio *et al.* used the Generalized Collocation Method and the parameters of London clay soil to solve the partial differential equation governing the one-dimensional nonlinear consolidation of clay layers with time-dependent drainage conditions, and presented their findings in the form of simultaneous diagrams indicating the amount of excess pore water pressure throughout the clay layer [8].

The most common numerical analysis methods for partial differential equations are the Finite Element Method and the Finite Difference Method, which require a large number of grid points in order to achieve acceptable results and the desired accuracy; and sometimes the computational cost increases due to the complexity of the problem. Since in this research, the Generalized Differential Quadrature method is used for the numerical analysis of the problem, this numerical approach is briefly described and its characteristics and limitations are pointed out.

The Differential Quadrature Method (DQM) was initially used by Bellman and Casti in 1971 as a numerical approach for solving partial differential equations [9]. In this method, the partial derivatives of the function at one point along a specific direction are expressed in terms of the linear weighted sum of function values at all the nodal points along the same direction and throughout the computational domain. This method has gained a growing popularity, since it is based on the idea of integral quadrature, and also because it produces highly-accurate results and is simple to use. The only limitation of this method is in the estimation of weight coefficients that are used for the interpolation of a function's derivatives. To improve the estimation of weight coefficients, Quan and Chang used the Lagrange's interpolating polynomials to obtain an appropriate formula for the calculation of weight coefficients of the first and second order

A. Bahmanikashkouli, M.Sc. Student of Structural Engineering, Engineering Faculty, Islamic Azad University, Bushehr, Iran. (Email: ayoub.bahmani.k@gmail.com)

O. R. Bahadori Nezhad, M.Sc. Student of Geotechnical Engineering, Civil Engineering Department, Bu-Ali Sina University, Hamedan, Iran. (Email: O.Bahadori@basu.ac.ir)

derivatives [10]. The use of this method makes it impossible to increase the number of grid points, and sometimes leads to the use of different mesh configurations when applying different boundary conditions. To remove these obstacles, Shu and Richard used the Generalized Differential Quadrature to solve fluid dynamics equations [11]. The results obtained from this method indicated the fact that this method, in spite of being simple to apply, does not have the problems associated with the DQM.

Normally, the Generalized Differential Quadrature (GDQ) method is based on the idea that the partial derivative of a function is approximated at a specific point of the problem interval as the weighted algebraic sum of function values at all discretized points of the whole region. These coefficients are obtained through the relation presented by Shu and Richard, and the coefficients of diagonal and nondiagonal members related to the matrix of weighted coefficients of different order derivatives are calculated with the help of Lagrange's interpolating functions [11].

In the year 2000, Shu presented a comprehensive introduction of the Differential Quadrature Method and described the merits and capabilities of this numerical method in analyzing the partial differential equations governing the engineering problems [12]. The new and advanced capabilities of this approach can also be found in [13].

## II. NUMERICAL METHOD OF GDQ

In this numerical method, after dividing the considered domain into a number of points termed nodes, the function derivative is calculated with respect to the independent variable at each of these nodes versus weighted algebraic sum of function values on all the points of the considered domain. In this approach, the function derivative ( $f$ ) is defined as a function of  $x$  :

$$f_x^{(n)} = \sum_{j=1}^N A_{ij}^{(n)} f(x_j) \quad n = 1, \dots, N-1. \quad (1)$$

The weight coefficients used in this method for the first-order derivative along the  $x$ -axis are obtained through the following relation:

$$A_{ij}^{(1)} = \frac{M(X_i)}{(X_i - X_j)M(X_j)} \quad i, j = 1, \dots, N_x \quad j \neq i. \quad (2)$$

In the (2):

$$M(X_i) = \prod_{\substack{j=1 \\ j \neq i}}^{N_x} (X_i - X_j). \quad (3)$$

And for the higher order derivatives:

$$A_{ij}^{(n)} = n(A_{ii}^{(n-1)} A_{ij}^{(1)} - \frac{A_{ij}^{(n-1)}}{X_i - X_j}) \quad i, j = 1, \dots, N_x \quad j \neq i, \quad (4)$$

$$A_{ii}^{(n)} = -\sum_{\substack{j=1 \\ j \neq i}}^{N_x} A_{ij}^{(n)} \quad \left\{ \begin{array}{l} i = 1, \dots, N_x \\ n = 1, 2, \dots, N_x - 1 \end{array} \right\}. \quad (5)$$

For example, by choosing five nodes, the matrix form of the first-order derivative of function  $f$  along the  $x$ -axis is obtained as:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix} = \begin{bmatrix} A_{11}^{(1)} & A_{12}^{(1)} & A_{13}^{(1)} & A_{14}^{(1)} & A_{15}^{(1)} \\ A_{21}^{(1)} & A_{22}^{(1)} & A_{23}^{(1)} & A_{24}^{(1)} & A_{25}^{(1)} \\ A_{31}^{(1)} & A_{32}^{(1)} & A_{33}^{(1)} & A_{34}^{(1)} & A_{35}^{(1)} \\ A_{41}^{(1)} & A_{42}^{(1)} & A_{43}^{(1)} & A_{44}^{(1)} & A_{45}^{(1)} \\ A_{51}^{(1)} & A_{52}^{(1)} & A_{53}^{(1)} & A_{54}^{(1)} & A_{55}^{(1)} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix}.$$

One of the important and effective issues in the convergence and accuracy of calculations in the GDQ method is the choosing of proper distance between nodal points for the mesh configuration of the considered region. The following relations indicate how this selection is made:

a) Points with equal distances:

$$X_i = \frac{i-1}{N-1} a \quad i = 1, \dots, N_x. \quad (6)$$

b) Points with unequal distances:

$$X_i = \frac{a}{2} \left\{ 1 - \cos\left(\frac{i-1}{N_x-1}\pi\right) \right\} \quad i = 1, \dots, N_x. \quad (7)$$

In the (6), and (7), " $a$ " denotes the length of the considered interval along the  $x$  direction, and  $N_x$  represents the number of nodes along the same direction. These points are known as Chebyshev-Gauss-Lobatto points. The above division was first proposed by Richards and Shu [11]. Research works demonstrate that by using the mentioned division, results with higher accuracy could be achieved. Therefore in this study, this type of division is employed.

## III. ONE-DIMENSIONAL NONLINEAR CONSOLIDATION PHENOMENON

In 2005, Battaglio *et al.* presented the partial differential equation governing the nonlinear consolidation of clay soil that leads to the change of soil type from the over consolidated state to the normally consolidated state, and evaluated a special case related to the parameters of London clay soil [8]. The governing assumptions in this study are:

1. Time and the length of the saturated clay layer are the independent variables, and the excess pore water pressure

caused by external loading is the dependent variable. The problem is analyzed in one dimension; therefore, the soil layer is confined on the sides and drainage is only achieved perpendicularly.

2. The total vertical stress  $\sigma_c$  generated by the external load is constant in time, the soil is completely saturated, and soil components and the existing pore water in the soil behave as an incompressible layer.
3. The weight of soil grains and of the existing water has been disregarded, and a constant volume has been considered for the studied soil.
4. Darcy's law governs the displacement of water in the soil layer.

In view of these assumptions, (8), expresses the relationship between void ratio ( $e$ ) and effective stress ( $\sigma'$ ) and coefficient of permeability ( $k$ ).

$$e = e_o - I_c \log\left(\frac{\sigma'}{\sigma'_o}\right) = e_o + C_k \log\left(\frac{k}{k_o}\right). \quad (8)$$

In the (8),  $I_c$  is the compressibility index,  $C_k$  is the permeability index, and  $e_o$  and  $k_o$  corresponding to the initial value of effective stress ( $\sigma'_o$ ).

Arnold *et al.* introduced the effective parameters of nonlinear consolidation as follows [5]:

$$\eta = \frac{I_c}{1+e_o}, \quad \lambda = 1 - \frac{I_c}{C_k}, \quad \mu = \frac{\sigma'_f - \sigma'_o}{\sigma'_o}. \quad (9)$$

In normal modeling of the consolidation phenomenon, the above parameters are considered constant throughout the process. However, when the change of soil type occurs, experimental findings indicate considerable variation in parameter  $\eta$ , from its highest value ( $\eta_M$ ) in normally consolidated soil to its lowest ( $\eta_m$ ) in over consolidated soil, which these changes are associated with the critical value of excess pore water pressure ( $u_c$ ).

Battaglio *et al.* considered large but discontinuous changes for the above parameters throughout the nonlinear consolidation phenomenon, and presented the following function for the simulation of experimental data [6]:

$$\eta = \eta(u - u_c) = \eta_m \frac{S_1(u - u_c)}{S_2(u - u_c)}, \quad (10)$$

where

$$S_1(u - u_c) = \exp\left[\alpha \frac{(u - u_c)}{u}\right] + \eta_M \exp\left[-\alpha \frac{(u - u_c)}{1 - u}\right], \quad (11)$$

and

$$S_2(u - u_c) = \exp\left[\alpha \frac{(u - u_c)}{u}\right] + \eta_m \exp\left[-\alpha \frac{(u - u_c)}{1 - u}\right], \quad (12)$$

in the (11), and (12),  $\alpha$  is a suitable positive parameter. The slope of the changes of  $\eta$  with respect to  $u$  is obtained by taking the derivative of (10), as follows:

$$\eta'(u; u_c, \alpha) = \alpha \eta_m (\eta_m - \eta_M) \frac{1}{S_2^2(u)} \times \exp\left[\alpha \frac{(u - u_c)(1 - 2u)}{u(1 - u)}\right] \left[ \frac{u_c}{u^2} + \frac{(1 - u_c)}{(1 - u)^2} \right]. \quad (13)$$

In their research, Battaglio *et al.* had assumed the two parameters of  $\mu$  and  $\lambda$  to be constant throughout the consolidation process [6]. In fact, in consideration of empirical results, the ratio of  $I_c/C_k$  is considered constant in many practical cases; because the permeability index ( $C_k$ ) shows many similar changes with  $I_c$  in connection with the critical value of excess pore water pressure ( $u_c$ ).

In view of the effective variables, parameters and assumptions, the mass balance equation corresponding to the nonlinear consolidation process can be presented.

$$\begin{aligned} \frac{\partial}{\partial t} \left( \frac{e}{1+e} \right) &= \frac{\partial}{\partial x} \left( \frac{k}{k'} \frac{\partial u}{\partial x} \right), \\ \frac{e - e_o}{1 + e_o} &= -\eta \log_{10} \varepsilon, \\ \frac{k}{k_o} &= \varepsilon^{\lambda - 1}, \\ \varepsilon &= 1 + \mu(1 - u). \end{aligned} \quad (14)$$

Where  $k' = k_o \ln 10 / \mu \eta_M$ . The first relation expresses the mass balance equation with regards to Darcy's law. The second and third relations express (8), with new parameters. Since the relationship between the normal effective stress and excess pore water pressure is expressed by  $\sigma'_f - \sigma' = \sigma_c \cdot u$  and  $\varepsilon$  represents the ratio  $\sigma'/\sigma'_o$ , the fourth relation can be derived.

Considering the equations presented in (14), the differential equation related to excess pore water pressure in the nonlinear consolidation process is expressed as follows:

$$\frac{\partial \varepsilon}{\partial t} = -\mu \frac{\partial u}{\partial t}. \quad (15)$$

Relation (15), is an immediate outcome of the equations presented in (14), and regarding the nonlinear consolidation phenomenon in saturated clay layers in which the soil type changes, Battaglio *et al.* have presented the following nonlinear partial differential [8]:

$$\frac{\partial u}{\partial t} = \frac{\mu \eta_M}{\mu \eta(u) - \eta'(u) \varepsilon(u; \mu) \ln(\varepsilon(u; \mu))} \times \left[ h(u) \frac{\partial^2 u}{\partial x^2} + p(u) \left( \frac{\partial u}{\partial x} \right)^2 \right]. \quad (16)$$

The values of  $\eta(u)$  and  $\eta'(u)$  have already been given in the (10), and (13), respectively. In (16),  $h(u)$  and  $p(u)$  are obtained through the relations below:

$$\begin{aligned} h(u) &= (1 + e_o) [1 - \eta(u) \log_{10} \varepsilon]^2 \varepsilon^\lambda, \\ p(u) &= (1 - \lambda) \frac{\mu}{\varepsilon} h(u), \end{aligned} \quad (17)$$

by considering the constant value of  $\eta = \eta_M$  in a special case, Battaglio *et al.* expressed (16), as a dimensionless relation for the nonlinear consolidation of saturated clay layers [6], [8]:

$$\frac{\partial u}{\partial t} = \left[ h(u) \frac{\partial^2 u}{\partial x^2} + p(u) \left( \frac{\partial u}{\partial x} \right)^2 \right], \quad (18)$$

in the (18),  $h(u)$  and  $p(u)$  have already been defined by (17),  $u$  denotes the excess pore water pressure,  $t$  is the time and  $x$  is the vertical distance in the soil layer.

In this phenomenon, the dependent variable of excess pore water pressure ( $u$ ) is associated with the independent variables of time ( $t$ ) and vertical coordinate in the soil layer ( $x$ ). The analysis of (18), is performed in one dimension; meaning that the soil is assumed to be confined from the sides and drainage is achieved only in the vertical direction. All the assumptions of the Terzaghi consolidation phenomenon govern this phenomenon as well, and parameters  $\mu$  and  $\lambda$  have been considered constant during the consolidation process. In this study, a case of boundary conditions, which represents the manner of clay layer drainage at different times, is evaluated. The amount of excess pore water pressure at different times is calculated by the Generalized Differential Quadrature (GDQ) method, and presented as simultaneous diagrams.

In solving partial differential equations with time derivatives through the GDQ numerical method, by discretizing the domains of the other independent variables (such as length) and obtaining the matrices of weight coefficients and inserting them into the partial differential equation (PDE) that governs the problem, this partial equation can be converted into an ordinary differential equation (ODE) system, as follows:

$$\frac{d\{u\}}{dt} = [A]\{u\} + [B]^2 \{u^2\}, \quad (19)$$

in this relation, by discretizing the independent variable of length into  $N$  nodes, matrices  $A$  and  $B$  with the  $N \times N$  dimension and matrix  $u$  with the  $N \times 1$  dimension will be obtained, and the weight coefficient matrices  $A$  and  $B$  will be formed by relations associated with the second-order and first-order derivatives, respectively.

The obtained system of ordinary differential equations can be analyzed by ordinary differential equation solution methods such as the Rung-Kutta and Finite Difference Methods. However in this research, to solve the governing differential equation, the domains of both independent variables, i.e., length ( $x$ ) and time ( $t$ ), are discretized by means of points with unequal distances into  $N$  and  $M$  nodes, respectively, and the matrices of weight coefficients are calculated with regards to the number of nodes and the domain lengths of independent variables, and are substituted into the governing differential equation. Thus, a  $N \times M$  grid is formed to obtain the final solutions.

To analyze the partial differential equation governing the nonlinear consolidation process by the Generalized Differential Quadrature method, after discretizing the domain of independent variables into nodes with unequal distances from one another, the following procedure is implemented:

$$\frac{\partial u}{\partial t} = \left[ h(u) \frac{\partial^2 u}{\partial x^2} + p(u) \left( \frac{\partial u}{\partial x} \right)^2 \right] \quad (20)$$

$$\begin{aligned} \frac{\partial u}{\partial t} &= h(u) \left( \sum_{k=1}^N A_{ik}^{(2)} u_{kj} \right) + p(u) \left( \sum_{k=1}^N A_{ik}^{(1)} \right)^2 (u_{ki})^2 \\ \sum_{k=1}^M A_{jk}^{(1)} u_{kj} - h(u) \left( \sum_{k=1}^N A_{ik}^{(2)} u_{kj} \right) - p(u) \left( \sum_{k=1}^N A_{ik}^{(1)} \right)^2 (u_{ki})^2 &= 0 \end{aligned} \quad (21)$$

$$i = 1, 2, \dots, N \quad j = 1, 2, \dots, M,$$

in the (21),  $A_{ik}^{(1)}$  are the first-order derivative weight coefficients,  $A_{ik}^{(2)}$  are the second-order derivative weight coefficients of  $u$  with respect to the length axis, and  $A_{jk}^{(1)}$  are first-order derivative weight coefficients of  $u$  with respect to the time axis.

The proper substitution of boundary conditions is very important in obtaining an accurate final solution. In view of the initial and boundary conditions associated with the manner of drainage of saturated clay layers in the nonlinear consolidation phenomenon, and considering the nonlinearity of the governing equation, the following nonlinear system of equations is obtained:

$$[K]\{u\} = \{f\}. \quad (22)$$

By solving this nonlinear system of equations through the Newton-Raphson numerical approach, the value of the unknown dependent variable ( $u$ ) at each node of the problem's grid system is obtained.

#### IV. NUMERICAL PROBLEM

In this section, an example of the drainage conditions corresponding to the relations and equations presented in the previous chapter is reviewed and analyzed. This problem has been chosen due to its attractiveness and applicability in geotechnical engineering. First, the problem is described and then, the governing boundary and initial conditions of the problem relevant to the partial differential equation are discussed. It should be mentioned that these conditions are associated with the drainage conditions of the saturated clay layer, and they vary with time. With the help of unequal distance points, or the Chebyshev-Gauss-Lobatto series, the domains of independent variables (i.e., length ( $x$ ) and time ( $t$ )) are discretized. Then, considering the number and value of nodes within the range of independent variables, the matrices of weight coefficients corresponding to the governing equations are established using the relations cited in the previous chapters, and by combining and rearranging them, a system of nonlinear equations is obtained and solved through the Newton-Raphson numerical method. Following the numerical analysis of the problem and obtaining the final solution, simultaneous time history diagrams that show the amount of excess pore water pressure ( $u$ ) at various points of the saturated clay layer with time-varied drainage conditions are presented and discussed. For the nonlinear consolidation example, the amount of excess pore water pressure ( $u$ ) at various distances of the saturated layer and at different times is presented in a diagram; and in a separate diagram, the excess pore water pressure ( $u$ ) in the middle of the layer is compared to the same parameter values at the boundaries of the clay layer; because in most of the problems related to clay layer consolidation process, in the middle of the layer values can provide good approximations. Finally, the obtained numerical results are compared to the results presented by Battaglio *et al.* [8].

The saturated clay layer has been confined from the top and bottom by sand layers, and the ground water level is at the surface of the ground, and Pore water pressure increases linearly with soil depth. By creating a water well to the lower part of the clay layer, the flow of water and the boundary conditions governing this phenomenon can be altered with time. The initial water level in the upper sand layer remains fixed; but in the lower sand layer, pore water pressure will decrease relative to water flow in the well. The initial and Dirichlet boundary conditions relevant to this problem [8] are:

$$u(0, x) = 1 \quad \forall x \in [0, 1], \quad (23)$$

$$\begin{aligned} u(t, 0) &= 0 \quad \forall t > 0, \\ u(t, 1) &= 0.1(1 + e^{-ct}) \quad \forall t > 0. \end{aligned} \quad (24)$$

The existence of parameter  $t$  in the (24), changes these relations throughout the consolidation process. By considering the numerical values of  $c = 7$  in the (24), and analyzing the

governing equation by means of the Generalized Collocation Method with regards to the boundary and initial conditions, Battaglio *et al.* presented the simultaneous diagrams of excess pore water pressure associated with this problem [8]. In this research, with regards to the mentioned initial condition and boundary conditions, the numerical method of Generalized Differential Quadrature (GDQ) has been used to obtain the concurrent curves of excess pore water pressure at various points of the clay layer, according to Fig. 1.

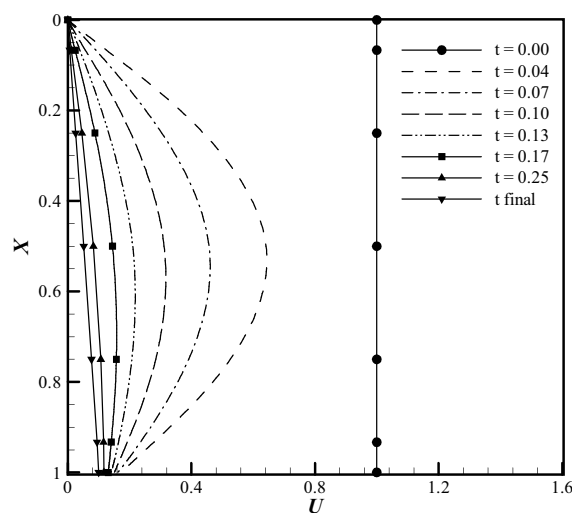


Fig. 1 Simultaneous curves of excess pore water pressure of clay layer

The concurrent curves of Fig. 1 indicate that due to the excess pore water pressure remaining constant at the top boundary of the clay layer, the excess pore water pressure decreases at the bottom boundary and various point of the clay layer, shortly after the start of the consolidation process, but it is not totally eliminated.

For the comparison of analysis results and the degree of convergence of the GDQ, TABLE I has been provided.

TABLE I  
 U VALUES OBTAINED FROM THE GDQ METHOD IN THE MIDDLE OF THE LAYER,  
 AT  $t = t_{final}$ , FOR DIFFERENT NUMBERS OF NODES.

$N$ :	5	6	7	8
$U$ :	0.053	0.052	0.052	0.052

TABLE I shows that by using the GDQ method, it is possible to estimate the excess pore water pressure up to three decimal places. It is also observed that through this method, relatively identical values have been obtained in the middle of the layer, at  $t = t_{final}$ , for different numbers of nodes, which is due to the use of the Chebyshev-Gauss-Lobatto series. Applying the Generalized Collocation Method, Battaglio *et al.* obtained a value of 0.05 for  $U$  [8], which has a good agreement with the values obtained through the GDQ method for various node quantities.

In the following diagrams, the amount of excess pore water

pressure at various distances from the surface of the saturated clay layer has been shown during the consolidation process; and the nonlinear changes of excess pore water pressure are clearly visible.

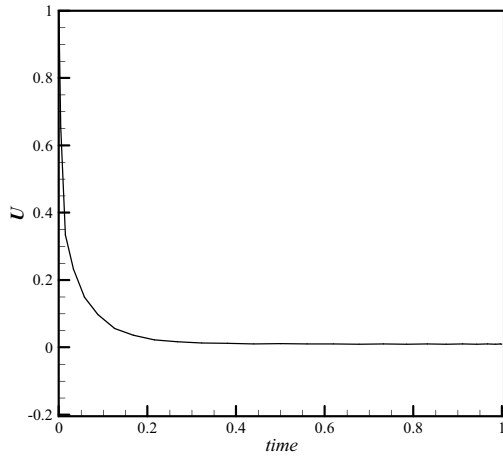


Fig. 2 Changes of excess pore water pressure in clay layer at  $X = 0.1$

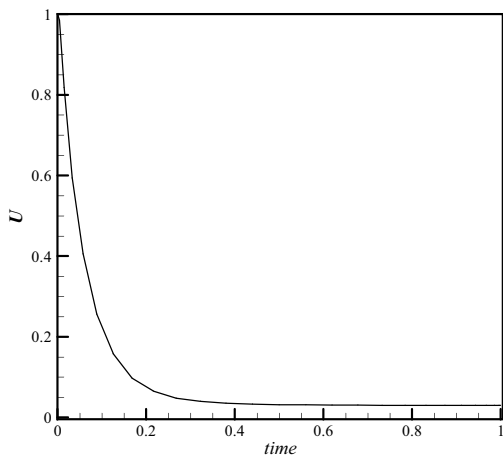


Fig. 3 Changes of excess pore water pressure in clay layer at  $X = 0.3$

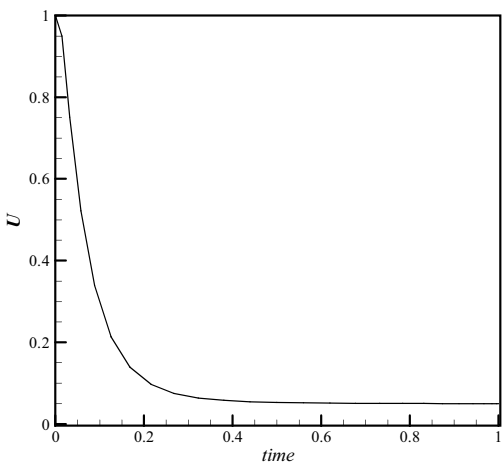


Fig. 4 Changes of excess pore water pressure in clay layer at  $X = 0.5$

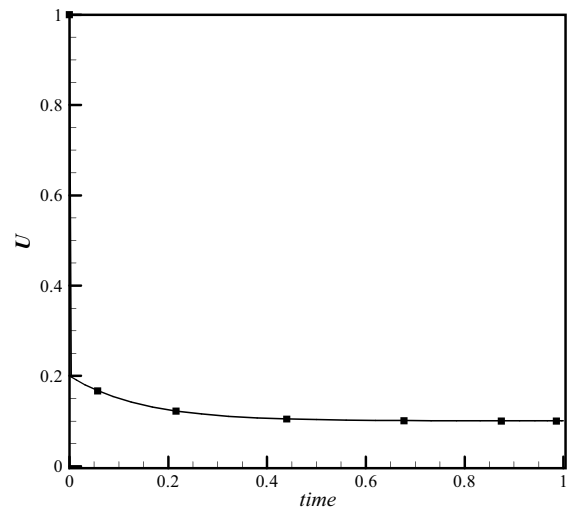


Fig. 5 Changes of excess pore water pressure at the bottom boundary of clay layer

Fig. 6 shows the changes and comparison of pore water pressure in the middle and at the bottom boundary of clay layer within the time interval. As is obvious in this example, changes are very little at the bottom boundary and initially after the start of the consolidation process, large changes occur in the middle of the layer, but as time passes, these changes diminish; and there are minor changes of excess pore water pressure in the second half of the considered time interval, while these changes are quite substantial in the first half of the said interval. Based on the presented figures, contrary to the Terzaghi consolidation phenomenon, in the nonlinear consolidation process, the pore water pressure produced throughout the saturated clay layer as a result of external loading is never eliminated, even when the consolidation process stops.

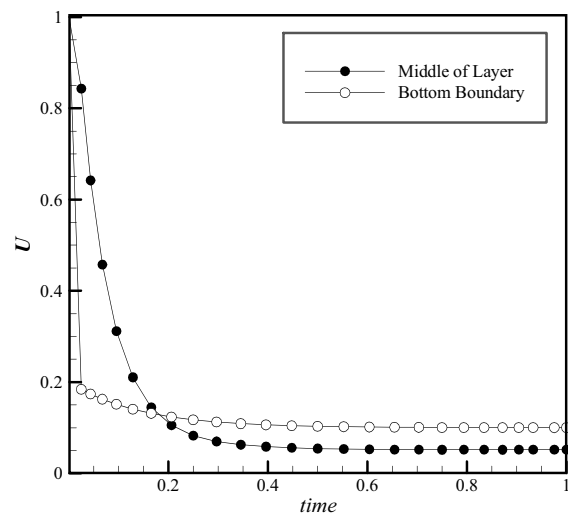


Fig. 6 Changes of excess pore water pressure in the middle and at the bottom boundary of clay layer

## V. CONCLUSION

In this article, the nonlinear consolidation process in a saturated clay layer subjected to time-varied drainage conditions has been studied, and numerically analyzed. Since the analytical solution of the nonlinear partial differential equation governing this phenomenon is time-consuming, the relatively new numerical method of Generalized Differential Quadrature (GDQ) has been used to obtain the amount of excess pore water pressure at various points of the saturated clay layer at different times. The high accuracy and simple computational procedure of this approach in obtaining the solution to the presented problem was verified in this paper. An important numerical example in the area of geotechnical engineering was reviewed; the partial differential equation governing this problem was presented; and its relevant initial and boundary conditions was described. The Chebyshev-Gauss-Lobatto series were used to discretize the domains of independent variables, namely, length and time, which based on the previous experiences, resulted in improved accuracy and reduced time in the convergence of solutions. The findings indicate that there is very good agreement between the values of excess pore water pressure in the middle of the clay layer obtained from the GDQ and Generalized Collocation Method, Battaglio *et al.* [8], with the difference that the GDQ method has yielded more accurate values of excess pore water pressure, compared to the Generalized Collocation Method. Also in this approach, the answers for different numbers of nodes are almost identical. The diagrams and results presented in this research indicate a major difference between the linear and nonlinear consolidation phenomena in saturated clay layers under time-dependent drainage conditions. In view of the special physics of this problem, the excess pore water pressure created in the saturated clay layer under external loading conditions diminishes with time, but never goes away completely.

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