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# An Ising-based Model for the Spread of Infection

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***Abstract*—**A zero-field ferromagnetic Ising model is utilized to simulate the propagation of infection in a population that assumes a square lattice structure. The rate of infection increases with temperature. The disease spreads faster among individuals with low ***J*** values. Such effect, however, diminishes at higher temperatures.

***Keywords*—**Epidemiology, Ising model, lattice models

## I. Introduction

ONE of the simplest models in epidemiology separates the population into two discrete states: ***S*** for susceptible and ***I*** for infected or infective [1]. In the SI model, the time rate of change in number of susceptible and infected individuals varies is given by the following equations:

$$\frac{dS}{dt} = -\beta SI \tag{1}$$

and

$$\frac{dI}{dt} = \beta SI \ . \tag{2}$$

where $\boldsymbol{\beta}$ is the infection rate. To consider a closed system, we consider a constant population size. Thus at any given time, the sum of susceptibles and infectives is equal to some constant, say ***N***. That is, ***S(t) + I(t) = N***. In which case, the above equations yield the following solutions:

$$S(t) = \frac{N}{1+e^{\beta(t-t_c)}} \tag{3}$$

and

$$I(t) = \frac{N}{1+e^{-\beta(t-t_c)}} \ . \tag{4}$$

Eq. 3 and 4 are logistic or S-curves. The point of inflection occurs at the critical time $\boldsymbol{t_c}$.

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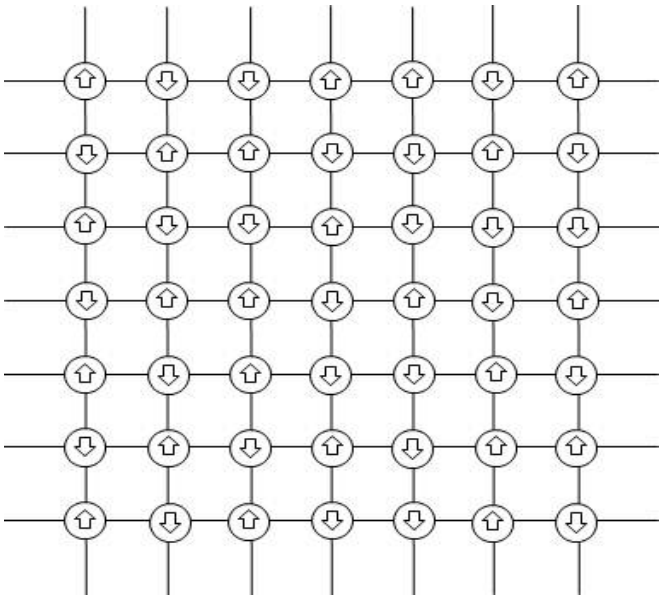

Fig. 1 A square lattice containing suceptible *spin down* and infected *spin down* individuals

In this work, we adapt the Ising model [2] framework to study dynamics of disease spread. Similar to [3], we confine our investigation to a square lattice and impose periodic boundary conditions. We observe the infection rate, $\boldsymbol{\beta}$, varies with parameters such as temperature ***T*** and neighbor interactions.

## II. The Model

Consider a closed community described by the square lattice in Fig. 1. Each site is occupied by one individual which can either be susceptible (spin down, $\boldsymbol{\sigma = -1}$) or infective (spin up, $\boldsymbol{\sigma = +1}$). The edges of the lattice are connected, thus forming a torus or a donut. Associate with each lattice configuration is a Hamiltonian which takes the form

$$H = -J \sum_{i,j} \sum_{x,y} \sigma_{ij} \sigma_{xy} \ , \tag{5}$$

Where $(\boldsymbol{x,y}) \in \{(i+1, j), (i-1, j), (i, j+1), (i, j-1)\}$ describes the Von Neumann neighborhood and ***J*** is the interaction parameter whiich described the (coupling) strength between spins.

Initially, the lattice is filled with susceptibles (all spin down). A random site is then selected and the corresponding spin flipped (from down to up). This is the first infective. For succeeding iterations, a randomly chosen site changes state if the change in energy, $\boldsymbol{\Delta H = H_{new} - H_{previous}}$, is less than or equal to zero. If $\boldsymbol{\Delta H > 0}$, the flip is accepted according to the probability

$$p = e^{-\Delta H / T} \ , \tag{6}$$

where $T$ is the scaled temperature. Flipping is allowed only in one direction (up → down, $S \rightarrow I$). Whenever a site is infected, it remains that way until the end of the simulation. Data presented in the next section correspond to the mean of 10 independent trials for a population size $N$=**100** (10 × 10 lattice).

## III. Results And Discussion

Simulation results reveal a logistic type of behavior, which is characteristic of the standard SI model. Fig. 2 shows the propagation of infection when $T$=**2.20** and $J$=**1.00**. At the onset, the spread of the disease approximates an exponential growth. But as time progresses, the growth slows down and saturates at the value of $N$. Since there is no recovery (flipping is one way), the entire community eventually becomes infected. The point of inflection, $t_c$, corresponds to the time when 50% of the population is already affected by the disease.

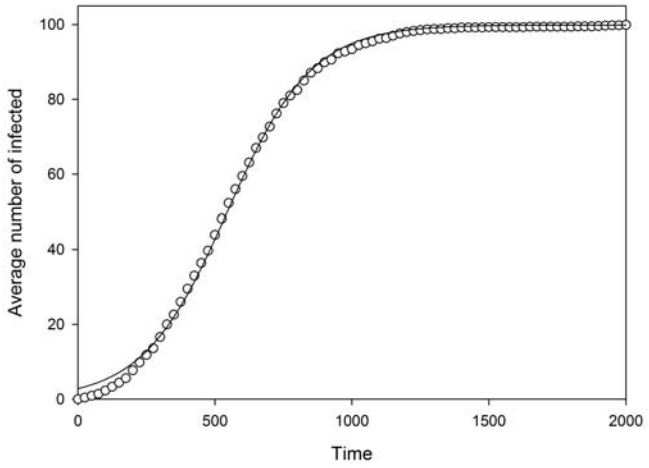


Fig. 2 Infection curve associated with $T$=**2.2** and $J$=**1.0**

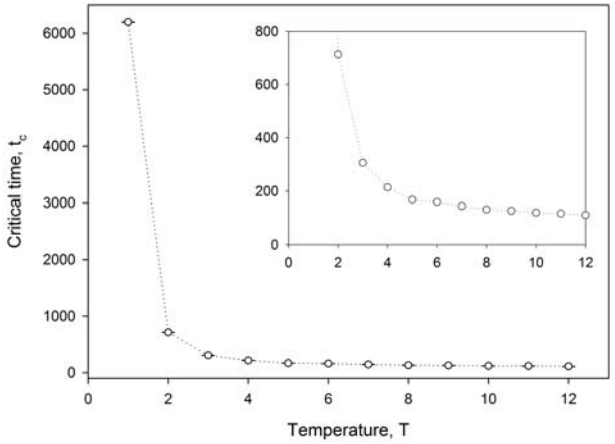


Fig. 3 Critical time, $t_c$, as a function of temperature, $T$

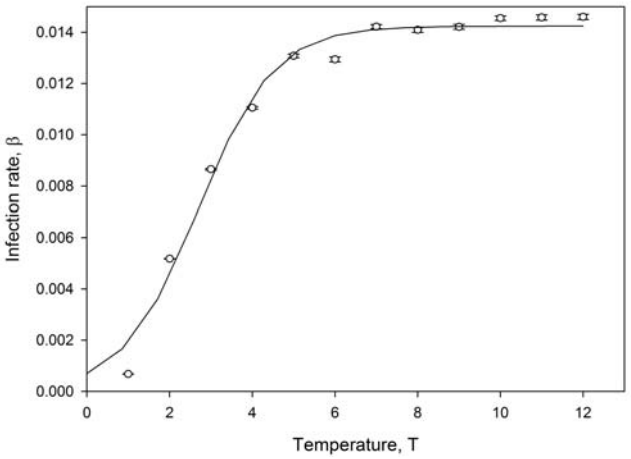


Fig. 4 Temperature dependence of the infection rate

Estimates of the critical time, $t_c$, and infection rate, $\beta$, are obtained by using Eqn. 4 as a fitting function. Fig. 3 reveal a sudden drop in $t_c$ associated with a slight rise in temperature (from 6000 to <1000 when $T$ was changed from 1.0 to 2.0). But as $T$ is increased further, the variation in critical times becomes minimal. The point of inflection occurs at an earlier time at larger $T$ values. Thus, the disease spreads more rapidly at higher temperatures. Calculated values of the rates, $\beta$, are plotted in Fig. 4. As $T$ increases, $\beta$ approaches a constant value. In our Ising-based SI model, the concept of temperature may be related to the level of aggression of a particular virus or associated to external parameters like ambient temperature and humidity. The concept of temperature may also be perceived with a broader scope to include the effects of cultural and socio-economic risk factors.

Next, we present the effect of varying interaction parameter, $J$, on the infection curves. In Fig. 5(a), saturation (100% infective) is achieved fastest when $J$=**0.25**. The rate of spread of infection decreases with increasing $J$. This parameter may be interpreted as the inverse of the contact time. Lower $J$ values can mean prolonged exposure to the agent or contagion. The effect of the parameter $J$ on the spread of the disease, however, vanishes at higher temperatures. Fig. 5(b) shows overlapping plots, independent of the $J$ value.

## IV. Summary And Conclusion

The dynamics of the spread of infection on a two-dimensional square lattice with periodic boundary conditions was analyzed using an Ising-based SI model. Beginning from a single infective, the disease was able to propagate faster at higher temperatures. Increasing the value of the interaction parameter, $J$, slowed down the infection spread. These effects, however, became less evident as the temperature is increased further.

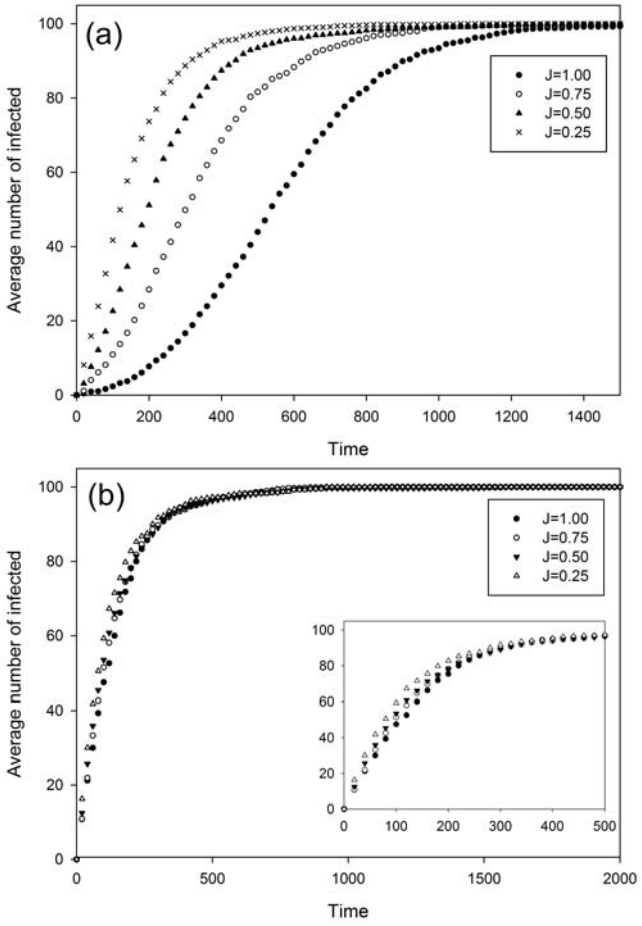


Fig. 5 Effect of varying $J$: (a) $T$=**2.2** and (b) $T$=**10.0**

## REFERENCES

[1] HW Hethcote, "Three basic epidemiological models", In: Levin, Hallan, Gross (Eds.), *Applied mathematical ecology. Biomathematics*, vol. 18, New York: Springer-Verlag, 1989, pp. 119—144.

[2] BA Cipra, "An introduction to the Ising model", *The American Mathematical Monthly*, vol. 94, no. 10, pp. 937--959, 1987.

[3] LF Lopez and E Massad, "Time-dependent discrete, Ising-like model for SIS epidemic systems", In *Proceedings of European Conference on Mathematical and Theoretical Biology*, 2011.