

# Methods of Estimating the Equilibrium Real Effective Exchange Rate (REER)

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**Abstract**—There are many debates now regarding undervalued and overvalued currencies currently traded on the world financial market. This paper contributes to these debates from a theoretical point of view. We present the three most commonly used methods of estimating the equilibrium real effective exchange rate (REER): macroeconomic balance approach, external sustainability approach and equilibrium real effective exchange rate approach in the reduced form. Moreover, we discuss key concepts of the calculation of the real exchange rate (RER) based on applied explanatory variables: nominal exchange rates, terms of trade and tradable and non-tradable goods. Last but not least, we discuss the three main driving forces behind real exchange rates movements which include terms of trade, relative productivity growth and the interest rate differential.

**Keywords**—real exchange rate, real effective exchange rate, foreign exchange, terms of trade

## I. INTRODUCTION

There are many current debates over undervalued and overvalued currencies traded on the world financial market. This paper contributes to these debates from a theoretical point of view. We present three most commonly used methods of estimation of the equilibrium real effective exchange rate (REER): macroeconomic balance approach, external sustainability approach and equilibrium real effective exchange rate approach in the reduced form.

The paper continues with the following structure. Section 2 analyzes three key approaches to the calculation of RER. In Section 3 we present key determinants of RER, while in Section 4 we discuss methods used to estimate the equilibrium REER. In Section 5 we describe the most commonly used methods of estimation of the equilibrium REER. Section 6 concludes the paper and states final remarks.

## II. APPROACHES TO THE REAL EXCHANGE RATE

There are three key concepts of the real exchange rate (RER) depending on used explanatory variables: nominal exchange rates, terms of trade and tradable and non-tradable goods.

First, the most common definition of the exchange rate is:

$$RER = \frac{E \times P^*}{P} \quad (1)$$

where  $E$  is the nominal exchange rate in form of units of foreign currency per one unit of domestic currency,  $P^*$  and  $P$  are foreign and domestic price levels respectively, measured usually by the consumer prices index or the unit labor costs index. The choice of the price level measure influences crucially the value of the RER.

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To avoid errors which might arise from different standards of computing price indices we will use the Consumer Price Index (CPI) index from the OECD database in all our calculations. Effective exchange rates are defined as a geometric weighted average of a number of bilateral exchange rates (deflated by price indices). The effective exchange rate takes into account a whole basket of currencies and therefore it can be taken as a measure of competitiveness of a country relatively to the others.

The real effective exchange rate calculations are usually constructed according to the method used in [3], which can be described by the following formula:

$$REER_i = \prod_{j \neq i} \left( \frac{E_i \times CPI_i}{E_j \times CPI_j} \right)^{w_{ij}} \quad (2)$$

where index  $i$  refers to the home country, while indices  $j$ 's indicate individual trade partners of the home country,  $E_i$  and  $E_j$  are the nominal exchange rates of the domestic and partners' currency relatively to the U.S. dollar. Exchange rates are expressed in dollars per unit of the local currency.  $w_{ij}$  denotes the weight of each currency in the basket and it reflects the share of trade with country  $j$  on the total external trade of the domestic country.

Second, in some cases the definition of real exchange rate may coincide with the definition of terms of trade (ToT) usually defined as the ratio:

$$ToT = \frac{p^{ex}}{p^{im}} \quad (3)$$

Thus price of a unit of typical good of export divided by the price of a unit of typical good of import. Ricci et al. [12] makes a brief overview of different approaches to RER and shows in which conceptions these two variables overlap. For instance, The Mundell-Fleming model assumes that home economy only produces one (composite) good that does not represent perfect substitute to the good produced by the rest of the world. In this case the real exchange rate can be expressed as:

$$RER = \frac{E \times p^{im*}}{p^{ex}} \quad (4)$$

where  $E$  is again the nominal bilateral exchange rate,  $p^{im}$  is the price of the imported good (in foreign currency) from the rest of the world and  $p^{ex}$  stands for the price of the exported domestic good (in domestic currency). In this case, the RER is obviously simply the inverse of the terms of trade. When considering several types of goods produced both domestically and abroad, however, RER and terms of trade must be treated as two distinct variables.

Yet, they are closely related. The ToT are linked to a country's current account. An increase in demand for exports of the domestic country pushes up the price of exports and hereby also the ToT and raises foreign demand for domestic currency. The increase in demand for domestic currency then leads to real appreciation of the exchange rate.

This is why terms of trade are often being used as one of the explanatory variables for real effective exchange rate movements. In practice, several approaches to approximate the prices of typical goods of export and import are used.

Chortareas, Driver [4] argue that simplified version of ToT under the form of “exports to imports ratio” is not optimal when the variable is to be used in order to explain RER movements. They claim that when nominal rigidities are present this usual measure of terms of trade, which does not distinguish between prices of goods and prices of services, may lead to identification of a correlation between real exchange rate and terms of trade which is in fact false.

Moreover, they suggest that there may be an endogeneity relation between the prices of export and real exchange rate.

To account for these problems Chen and Rogoff [4] propose to use so called commodity terms of trade (especially for countries whose exports consist mainly of commodities). These only take into account prices of exported and imported commodities, whose prices are determined on global markets.

Ricci et al. [13] use the commodity terms of trade under the form:

$$ToT = \frac{\Pi_t \left( \frac{P_t}{MUV} \right)^{X_t}}{\Pi_t \left( \frac{P_t}{MUV} \right)^{M_t}} \quad (5)$$

where commodity prices  $P_t$ 's are measured against prices of manufactured goods (represented by the manufacturing unit value index). This measure is supposed to be less troubled with endogeneity problems and should be a better indicator of real exchanges rate movements.

A third approach to the real exchange rate uses the distinction between tradable and non-tradable goods and the theory developed by [2] and [15]. This approach defines the real exchange as:

$$RER = \frac{P^{NT}}{P^T} \quad (6)$$

where  $P^T$  and  $P^{NT}$  refer to the price of the tradable good and the price of the non-tradable good in domestic currency. The dynamic version of Balassa's theory considers two economies with different paces of productivity growth. In both countries real wages are determined by marginal product of labor in the tradable sector. Perfect labor mobility is assumed between sectors but zero mobility across countries. Nominal exchange rate is supposed to adjust to the movements in productivity in the tradable sector relatively to productivity in non-tradable sector in the home and foreign country.

Let us assume that in the domestic economy relative productivity in the tradable sector to productivity in the non-tradable sector grows faster than the corresponding variable in the foreign economy. By definition, the increase in productivity is in fact an increase in marginal product of labor and is accompanied by a raise in wages. Now difference in the productivity growth is larger in the tradable sector than in the non-tradable one. At the same time, by assumption, wage increases in both sectors reflect the growth of the marginal product in the tradable sector only and “spill over” into the non-tradable sector. This will result in a differential in the

growth of wages in the non-tradable sectors between countries which is greater than their productivity growth differential. Then also prices of non-tradable goods in the domestic economy will grow faster relatively to non-tradable goods abroad then the relative productivity growth would imply. Finally, given that in each country the overall inflation is given by the weighted average of inflations in both sectors, the inflation differential between countries will be higher than their overall productivity growth differential.

Recalling formula (2), inflation does enter the calculation of real exchange rate, which is why the relative productivity growth in tradable sector with respect to non-tradable sector also constitutes an important determinant of the real exchange rate development.

### III. DETERMINANTS OF THE REAL EXCHANGE RATES

From the three approaches to real exchange rate we derived two main driving forces behind real exchange rates movements: terms of trade and relative productivity growth. There are, however, also other factors, which have at least temporary impact on the real exchange rate.

One of them is real interest rate differential. Suppose that a domestic economy offers higher interest rates relatively to other countries. Then investor will be more willing to lend money to the domestic economy than to others since it offers higher returns (under the assumption of equal expected inflation rates). A domestic country will start to accumulate foreign currency, which will result in the real exchange rate appreciation. The relation is captured by uncovered interest rate parity (UIP), which, in nominal terms can be written as:

$$\frac{E_{t+k}^e - E_t}{E_t} = \frac{1 + i_{t,t+k}}{1 + i_{t,t+k}^*} \quad (7)$$

where index  $e$  refers to expected value,  $E_{t+k}$  and  $E_t$  are of nominal exchange rates in corresponding time periods  $t$  and  $t+k$ ,  $i_{t,t+k}$  and  $i_{t,t+k}^*$  stand for domestic and foreign nominal interest rates in time  $t$  for  $k$  periods. Substituting the nominal exchange rate by the real one using the formula (1) we get:

$$\frac{\frac{Q_{t+k}^e \times P_{t+k}^e}{P_{t+k}^e} - \frac{Q_t \times P_t}{P_t}}{\frac{Q_t \times P_t}{P_t}} - 1 = \frac{1 + r_{t,t+k}}{1 + r_{t,t+k}^*} \times \frac{1 + \pi^e}{1 + \pi} \quad (8)$$

where  $r_{t,t+k}$  and  $r_{t,t+k}^*$  are real interest rates in time  $t$  for  $k$  periods. Now, when expected relative domestic and foreign

inflation term  $\frac{1 + \pi^e}{1 + \pi}$  is subtracted from both sides of the equation we get the equation in real terms. It can be easily seen that a percentage change in real exchange rate requires the real interest rate differential of  $\frac{1 + r_{t,t+k}}{1 + r_{t,t+k}^*}$ . When domestic real interest rate exceeds the foreign one, the exchange rate is expected to appreciate. Conversely, when the foreign interest rate is higher than the domestic one, the exchange rate is expected to depreciate.

Development in real interest rates may hence also help to explain real exchange rate movements. Empirical studies

estimating the impact of interest rate differentials to the RER bring miscellaneous results. Greenwood [5] found evidence for such a relationship.

Another important determinant of RER is the amount of net foreign assets (NFA) or net foreign debt of the country. Lane and Milesi-Ferretti [11] stress that globalization led to huge capital flows which allowed accumulating substantial stocks of foreign assets or liabilities. Therefore, it becomes more important to turn the attention to net investment positions of countries rather than to their gross positions when we are interested in explaining changes in the exchange rate. The amount of cumulated net foreign assets (or debt) influences the real exchange rate via intertemporal substitution. A country which is heavily indebted (in net terms) may need to depreciate its currency in order to generate a trade surplus that would service the payments of the debt in future. Montiel [11] find strong evidence for this relationship. To make the NFA comparable among countries, the value of NFA is usually divided by country's GDP or its volume of external trade, see, for instance, [7] or [14]. Government expenditures also may have an impact on real exchange rate. Government tends to spend money rather on non-tradable goods than on tradable goods. An increase in government expenditures raises the price of non-tradable goods relatively to tradable goods and hereby contributes to the real exchange rate appreciation.

Last but not least, measures of price controls and trade restrictions may be included among explanatory variables of the real exchange rate when in the countries of interest these factors significantly influence the price level. For instance, openness to trade, which can be defined as the ratio of imports to GDP or total trade to GDP, is being used to account for potential liberalization effect.

We have identified several important factors which affect real exchange rate and explained how they influence the RER. The following section provides a formal model combining the effects of all major variables affecting the RER, which allows deriving the equilibrium level of the RER.

#### IV. A FORMAL MODEL OF EQUILIBRIUM EXCHANGE RATE

In order to assess actual RER behavior we need a definition of equilibrium real exchange rate to which the actual RER could be compared. Most of empirical calculations are usually based on a simplified version of the theoretical models used in [1] and [12]. Precisely, we will use a dynamic model developed in a static framework. The Montiel's model not only distinguishes between tradable and non-tradable goods, but also further divides tradable goods into exportable and importable goods in order to incorporate the influence of terms of trade on the real exchange rate. Exportable goods are assumed not to be consumed in the home country. This implies that real exchange rate can be defined in two ways: as the real exchange rate for exportable goods:

$$RER_{exportables} = \frac{p_{NT}}{p_{ex}} \quad (9)$$

or the real exchange rate for importable goods:

$$RER_{importables} = \frac{p_{NT}}{p_{im}} \quad (10)$$

The analysis is developed for the latter. *Equilibrium* exchange rate is defined as the rate that ensures that domestic country reaches both internal balance and external balance. Internal balance refers to the situation when labor market and market for non-tradable goods clear, which requires the supply of non-tradable goods to match its demand. This is represented by the equation:

$$Y_{NT} = C_{NT} + G_{NT} = (1 - \theta) \frac{C_T^{im}}{Q} + G_{NT} \quad (11)$$

where  $Y_n$  is the total output of non-tradable goods,  $G_{NT}$  and  $C_{NT}$  are government, respectively private spending on non-tradable goods,  $Q$  stands for current real exchange rate and  $q$  stands for the share of private spending devoted to tradable (importable) goods. Supply of non-tradable output  $Y_{NT}$  is dependent on the current real exchange rate and terms of trade. When  $G_{NT}$  increases, the real exchange rate  $Q$  must appreciate in order to maintain internal equilibrium.

External position of a country depends on cumulative foreign exchange inflows and outflows. In one specific year the change in country's external position can be described by the equation:

$$\Delta NFA = TB + rNFA = Y_T^{im} + Y_T^{ex} - G_T^{im} - \theta C_T^{im} + rNFA \quad (12)$$

where  $NFA$  refers to net foreign assets and  $TB$  to trade balance. The trade balance is given by the supply of tradable (importable and exportable) goods  $Y_T^{im} + Y_T^{ex}$  minus the domestic demand for them  $G_T^{im} + C_T^{im}$ .  $r$  refers to real yield on foreign assets.

Now, external balance is assumed to be reached when the external position of a country meets steady state equilibrium, i.e. when  $\Delta NFA=0$  and the cumulative volume of net foreign assets remains constant in time.

Assuming this criterion to be met and combining both equations (11) and (12) through  $CT^{im}$  we can express the real equilibrium exchange rate as a function of the following variables:

$$RER = Q = f(Y_{NT}, Y_T^{im}, Y_T^{ex}, G_T^{im}, G_{NT}, rNFA) \quad (13)$$

where  $Y_{NT}$ ,  $Y_T^{im}$ ,  $Y_T^{ex}$ , and  $rNFA$  are endogenous in the model and  $G_T^{im}$ ,  $G_{NT}$  are considered as exogenous. We will take this equation as the basis for the choice of explanatory variables affecting the level of real equilibrium exchange rate.

Now that major driving forces behind the real exchange rates have been identified and a formal model deriving the equilibrium exchange rate formulated we need to choose appropriate method for the equilibrium real exchange rates estimation. The following section provides a brief overview of the techniques. It should be pointed out that equilibrium real exchange rates are usually estimated in form of REER rather than bilateral exchange rates.

#### V. METHODS USED TO ESTIMATE THE EQUILIBRIUM REER

Lane and Milesi-Ferretti [7] summarize the most commonly used methods of estimation of the equilibrium real effective exchange rates into three approaches: macroeconomic balance

approach, external sustainability approach and equilibrium real effective exchange rate approach in the reduced form.

First, macroeconomic balance (MB) estimation is made in three steps. The goal is to predict the current account balance (CA) for the medium term. First, a regression with historical values of CA balance as the dependent variable is performed. Fundamental macroeconomic variables such as population growth, dependency ratio, net foreign assets and economic growth are used as explanatory variables. In the second step the estimated parameters from the regression together with forecasts of the fundamental variables are used in order to predict the current account balance prevailing in the medium term. The prediction is then taken as a "current account norm". Finally, the CA balance that would occur at zero output gap is compared to this current account norm computed in previous step. The change in the exchange rate required for the current account norm to equal to the "zero output gap CA" represents the exchange rate misalignment. The macroeconomic balance approach is used to assess the medium-term real exchange rate. Therefore, several-years observations are aggregated into one observation in order to minimize fluctuations of variables.

Second, the external sustainability approach uses similar methods as the macroeconomic balance approach, but stresses the role of net foreign assets. Key element of the estimation is choosing a sustainable (or "benchmark") level of the stock of net foreign assets. The "sustainable" level of net foreign assets may differ across countries reflecting their specificities, for instance, their degree of openness to trade. After the benchmark level of NFA is set, the ratio of trade balance (or CA balance) to GDP, which is compatible with keeping this stock of NFA constant, is calculated. In the following step, trade (or CA) to GDP ratios which are expected to prevail in the medium term are estimated. The last step consists in computing the change in REER which is necessary for the actual and the sustainable trade balance to GDP ratios to equal.

Real equilibrium exchange rate approach is the third method. Sometimes it is being referred to as "reduced form" of MB approach. In fact, the long-run relationship between exchange rate and a set of fundamentals is directly estimated in a regression. Afterwards, equilibrium exchange rate is calculated using this estimate and levels of fundamentals which are supposed to prevail in the medium term. The misalignment of the REER is then calculated as a simple difference of the equilibrium and actual REER.

## VI. CONCLUSION

This paper contributes to the debates over undervalued and overvalued currencies traded on the world financial market during the global crisis from a theoretical point of view (for more details on the pending world global crisis see, for instance, [8], [9], [10], [16], [17], [18], [19] or [20]). We present three most commonly used methods of estimation of the equilibrium real REER, we discuss key concepts of the calculation of RER. Last but not least, we discuss three main driving forces behind real exchange rates movements: terms of trade, relative productivity growth and interest rate differential.

## ACKNOWLEDGMENT

Financial support from The Czech Science Foundation (project GACR 403/10/P278 - The Implications of The Global Crisis on Economic Capital Management of Financial Institutions and The Grant Agency of Charles University (project GAUK 58410/2010 - Efficiency of EU Merger Control) is gratefully acknowledged. Authors are also thankful for valuable comments to Ms. Yael Roshwalb.

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