

# Vibration Attenuation Using Functionally Graded Material

Saeed Asiri, Hassan Hedia, and Wael Eissa

**Abstract**—The aim of the work was to attenuate the vibration amplitude in CESNA 172 airplane wing by using Functionally Graded Material instead of uniform or composite material. Wing strength was achieved by means of stress analysis study, while wing vibration amplitudes and shapes were achieved by means of Modal and Harmonic analysis. Results were verified by applying the methodology in a simple cantilever plate to the simple model and the results were promising and the same methodology can be applied to the airplane wing model.

Aluminum models, Titanium models, and functionally graded materials of Aluminum and titanium results were compared to show a great vibration attenuation after using the FGM. Optimization in FGM gradation satisfied our objective of reducing and attenuating the vibration amplitudes to show the effect of using FGM in vibration behavior. Testing the Aluminum rich models, and comparing it with the titanium rich model was an optimization in this paper. Results have shown a significant attenuation in vibration magnitudes when using FGM instead of Titanium Plate, and Aluminium wing with FGM Spurs instead of Aluminium wings. It was also recommended that in future, changing the graphical scale to 1:10 or even 1:1 when the computers' capabilities allow.

**Keywords**—Vibration, Attenuation, FGM, ANSYS2011, FEM.

## I. INTRODUCTION

IN materials science functionally graded material (FGM) may be characterized by the variation in composition and structure gradually over volume, resulting in corresponding changes in the properties of the material. The materials can be designed for specific function and applications. Various approaches based on the bulk (particulate processing), perform processing, layer processing and melt processing are used to fabricate the functionally graded materials. The concept of FGM was first considered in Japan in 1984 during a space plane project. Where a combination of materials used would serve the purpose of a thermal barrier capable of withstanding a surface temperature of 2000K and a temperature gradient of 1000K across a 10mm section [1].

There are many areas of application for FGM. The concept is to make a composite material by varying the microstructure from one material to another material with a specific gradient. This enables the material to have the best of both materials. If it is for thermal, or corrosive resistance or malleability and toughness both strengths of the material may be used to avoid

corrosion, fatigue, fracture and stress corrosion cracking. The following picture illustrates the transition between a hard material and a soft material with eight discrete layers [2]. The transition between the two materials can usually be approximated by means of a power series. The aircraft and aerospace industry and the computer circuit industry are very interested in the possibility of materials that can withstand very high thermal gradients [3]. This is normally achieved by using a ceramic layer connected with a metallic layer. The Air Vehicles Directorate has conducted a Quasi-static bending test results of functionally graded titanium/titanium boride test specimens [4]. In Toyohashi University of Technology in Japan presented an analytical formulation and a numerical solution of the thermal stress and deformation for axisymmetrical shells of FGM subjected to thermal loading due to fluid. Numerical computations are carried out for various compositional distribution profiles in FGM [5]. The modulus of elasticity and the coefficient of linear thermal expansion vary with the power product form of radial coordinate variable where the exact solution for the axisymmetric thermoelastic problem of a uniformly heated functionally graded transversely isotropic cylindrical shell, assuming that the modulus of elasticity and the coefficient of linear thermal expansion vary with the power product form of radial coordinate variable [6]. Cho and Ha studied the volume fraction optimization for minimizing steady-state thermal stresses in Ni/Al<sub>2</sub>O<sub>3</sub> heat-resisting FGM composites [7]. A general analysis of one-dimensional steady state thermal stresses in a hollow thick cylinder made of FGM is developed by Jabbari et al. [8]. Kadoli and Ganesan have proposed a linear thermal buckling and free vibration analysis of functionally graded cylindrical shells with clamped-clamped boundary conditions, based on temperature-dependent material properties [9]. Liew et al. studied the analyses of the thermo-mechanical behavior of hollow circular cylinders of functionally graded material [10]. Ootao et al. studied a neural network to optimize the problem of material compositions for a hollow circular cylinder of functionally graded material [11].

However, the aim of the work is to study the effect of functionally graded materials on the vibration behavior by using finite elements method to optimize the results to end with a graded model that attenuate the vibration amplitudes while keeping the strength and rigidity levels high.

S. Asiri is assistant professor at the King Abdulaziz University, P.O.Box. 80204, Jeddah21589, Saudi Arabia, (e-mail: saeed@asiri.net).

H. Hedia is with the Mechanical Engineering Department, King Abdulaziz University, Saudi Arabia, (email: hassanhedia87@yahoo.com).

W. Eissa is with the Turbine Maintenance Chief Engineer, Saline Water Conversion Corporation, Saudi Arabia, (email: WEissa@swcc.gov.sa).

## II. METHODOLOGY

### A. Background

ANSYS 11.0 with work bench Finite Element package (solver) was used to analysis and to simulate 2 case studies. The analysis was done on Cantilever Plate to proof and to test our methodology to demonstrate success. The same approach was then applied to a very complicated real case study (CESNA 172 Air Plane wing Model) to simulate the problem mathematically to end up with the results. Steps of modeling both studies are presented for the simple case study and for the complicated one. It consisted of building the geometry of the model, distributing the FGM material properties along the model, meshing the model with a proper smart sized mesh types, applying the loads (force, pressure, etc.) on the model, setting the boundary conditions of the model, and finally running and solving the model. Stress analysis was performed to guarantee the system strength and rigidity before solving the vibration analysis. Modal analysis was performed to find the system vibration parameter (i.e. natural frequencies, and mode shapes). Finally, the Harmonic analysis was performed to check the frequency responses of the 2 models, in order to compare the plain material model and FGM material model.

### B. Modeling

#### 1. Cantilever Plate

According to the material properties shown in TABLE I and using ANSYS 11.0 with APDL capabilities, a vibration analysis problem has been solved for a model made of a functionally graded material as a new approach, and then a simple model had to be started with in order to get predictable results. A cantilever plate model was chosen for simplicity purposes and for results prediction. The cantilever plate consists of a 10mm x 10mm x 1mm cantilever plate subjected to an upward force in the tip while supported from the other side. The width of the model was divided into 100 parts to generate 100 identical geometry volumes but with different material properties. The volume number 1 and volume number 100 have different pure metal properties, while the volumes in between has a gradually changing in properties based on the gradation theory discussed in the methodology chapter. In the simple model, a comparison between an Aluminum plate, Titanium plate, and FGM (Aluminum – Titanium) plate was performed to check the strength, rigidity, and stability when subjected to vibration excitation. So in the Aluminum model, the FGM gradation generated a uniform material (Aluminum) distribution with Aluminum properties. In the FGM (AL- TI) case, the gradation formulas generated a data base of 100 different material properties starting by the aluminum properties and ending with the titanium properties. A linear material distribution in the gradation process took place in this case. Smart sized solid octahedral elements were chosen to analyze the model because of the narrow width of the volumes that were being meshed. Narrow width appears because the 1 mm width was divided into 100 equal parts for FGM application. The plate was supported from one end and kept

free from the other end (cantilever plate) to simulate the boundary conditions of the air plant wing model.

The plate was subjected to an upward unit force (1 N) at the tip as shown in Fig. 2. The one N force was applied to make the comparison between the different models easy and to linearize the results when applying different values of forces. Solving the models was done by writing an APDL code and it was done in 3 stages. The first one is when the model was chosen to be solved for Static Analysis (Stress Analysis). And the results were a comparison between maximum stresses and maximum deflections in each model, and this will be discussed in the coming section. The second one is when the model was chosen to be solved for Modal Analysis. And the results were a comparison between the first 20 natural frequencies in each model, and this will be discussed in the following sections.

TABLE I  
 MATERIAL PROPERTIES OF ALUMINUM AND TITANIUM

	Elastic Modules	Poison's Ratio	Density
Aluminum	72 Gpa	0.3	2600 kg/m3
Titanium	103 Gpa	0.4	4800 kg/m3

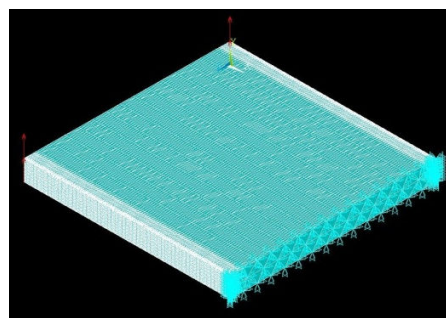


Fig. 1 Cantilever Plate

The last one is when the model was chosen to be solved for Harmonic Analysis. And the results were a comparison between the frequency response amplitude values in each model, and this will be discussed in the following sections.

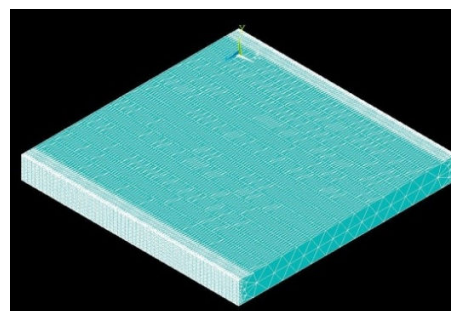


Fig. 2 Meshed Cantilever Plate

### C. Air Plane Wing

CESNA 172 Airplane was selected to study its proto type wing which is shown in the Model Figure. Stress analysis, modal analysis, and harmonic analysis were done to study the model from the strength, rigidity, and stability when subjected to a vibration excitation. In the Airplane wing model, a

comparison between a complete Aluminum wing, complete Titanium wing, Aluminum wing with FGM Spurs, and a Titanium wing with FGM was performed to check the strength, rigidity, and stability when subjected to vibration excitation. Octahedral smart sized element type was used to mesh the wing stiffeners and Spurs as shown in their figures. Tetrahedral smart sized element type was used to mesh the wing shell volumes as shown in the meshed model figure. The Airplane wing was supported from one end (Spur Areas) and kept free from the other end (cantilever plate. The wing was subjected to an upward unit force (1 N) at the tip. The one N force was applied to make the comparison between the different models easy and to linearize the results when applying different values of forces. Solving (running) the models was done by writing an APDL code and it was done in 3 stages. The first one is when the model was chosen to be solved for Static Analysis (Stress Analysis). And the results were a comparison between maximum stresses and maximum deflections in each model, and this will be discussed in the following section. The second one is when the model was chosen to be solved for Modal Analysis. And the results were a comparison between the first 4 natural frequencies in each model. The last one is when the model was chosen to be solved for Harmonic Analysis. And the results were a comparison between the time response amplitude values in each model.

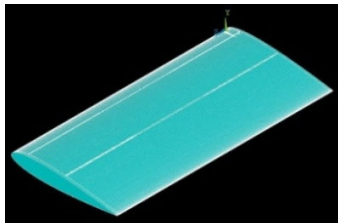


Fig. 3 CESNA 172 wing model

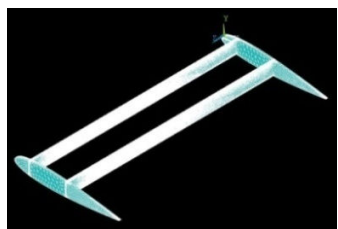


Fig. 4 Meshed Stiffeners and Spurs

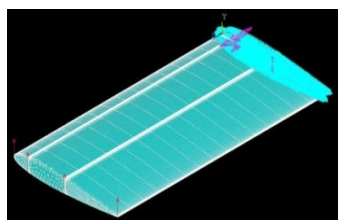


Fig. 5 Boundary Conditions

### III. THE RESULTS

#### A. Background

Results of the simple model (cantilever plate) and the airplane wing model are divided into 3 main sections. Stress analysis (Static analysis) results are to check the plate and wing strength and rigidity by means of showing the stress distribution, and the deformed when subjected to a unit force. Unit force is chosen to ease the straight forward comparison between the different models. Modal analysis results are obtained to check and compare the natural frequencies when using FGM material i.e. Aluminum- Titanium plate with a single material plate which is Aluminum or Titanium, or Aluminum with FGM spur and Titanium with FGM spur wings. Finally, representing the vibration results in the Harmonic analysis results was done to compare the amplitude of vibration in case of using the Aluminum – Titanium plate with the Aluminum or Titanium plate or Aluminum with FGM spur and Titanium with FGM spur wings. Results were presented in a frequency domain and time domain representation

#### B. Cantilever Plate Model

##### 1. Static Analysis

Stress distribution in the aluminum model is symmetrical around y- axis due to the material uniformity.

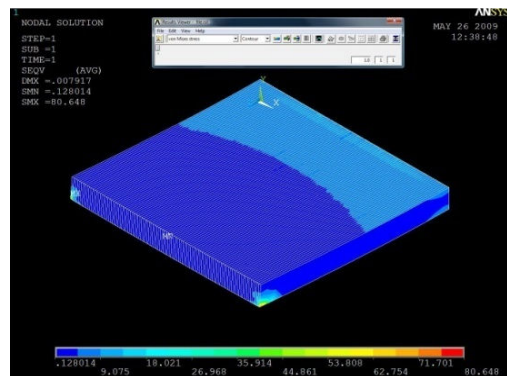


Fig. 6 Von Mises Stress Distribution of Aluminum Model

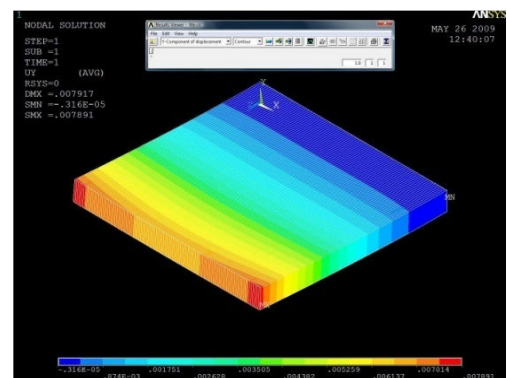


Fig. 7 Deformed Shape of Aluminum Model

It is also shown that the maximum von Mises stress is the contact stress due to the force concentration in the model corners. This is shown in Figs. 6 to 11.

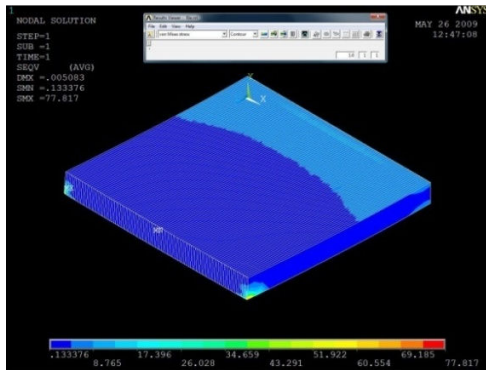


Fig. 8 Von Mises Stress Distribution of Titanium Model

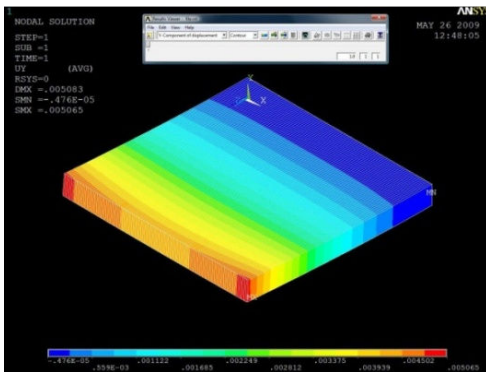


Fig. 9 Deformed Shape of Titanium Model

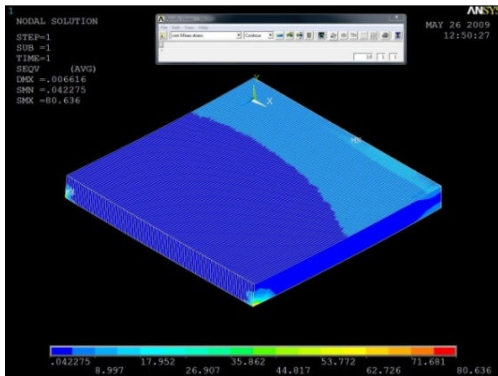


Fig. 10 Von Mises Stress Distribution of Aluminum- Titanium (FGM)

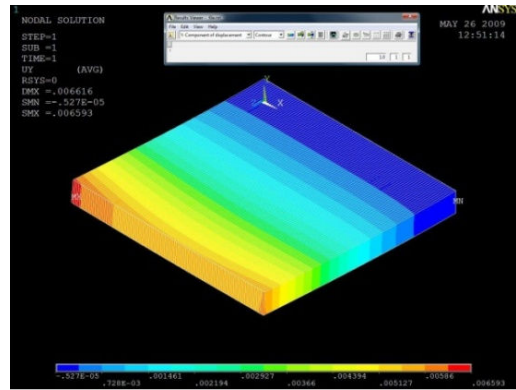


Fig. 11 Deformed Shape of Aluminum- Titanium (FGM)

Stress distribution in the aluminum model is symmetrical around y- axis due to the material uniformity.

In Aluminum- Titanium model, the materials were not uniform. The gradation started with a titanium material and gradually changed to aluminum. Due to the gradual change in material properties, von Mises stress distribution was also gradually changed around the y- axis.

## 2. Stress Analysis Results Summary

A reduction of 20% in the maximum deformation is achieved when using Aluminum- titanium plate instead of Aluminum plate while keeping the maximum von Mises stress the same as shown in Table II.

TABLE II  
 STRESS ANALYSIS COMPARISON

	Deformation (e-4) mm	Von Mises Stress (Mpa)
Aluminum	78.91	80.648
Titanium	50.65	77.817
AL- TI	65.93	80.636

A tabulated comparison is also shown in Fig. 12.

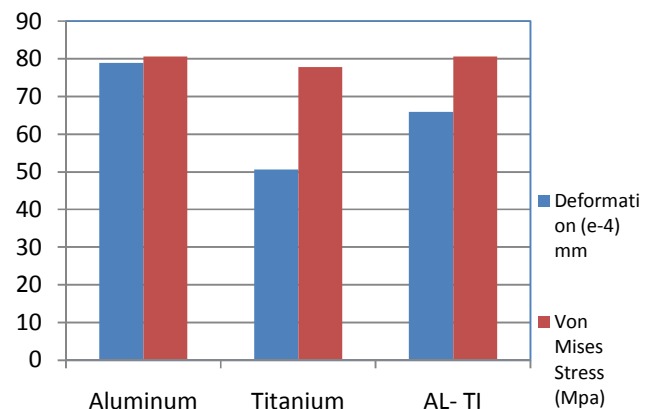


Fig. 12 Maximum von Mises Stress and Deformation Comparison  
 Modal Analysis

When comparing the first 20 natural frequencies in the Aluminum, Titanium, and Aluminum- Titanium plates as shown in Fig. 13, the FGM plate had an in between values that

were considered very logical results. Taking the average of the 20 natural frequencies has shown a reduction of 7.5 % between the Titanium plate, and the Aluminum- Titanium plate as shown in Fig. 13.

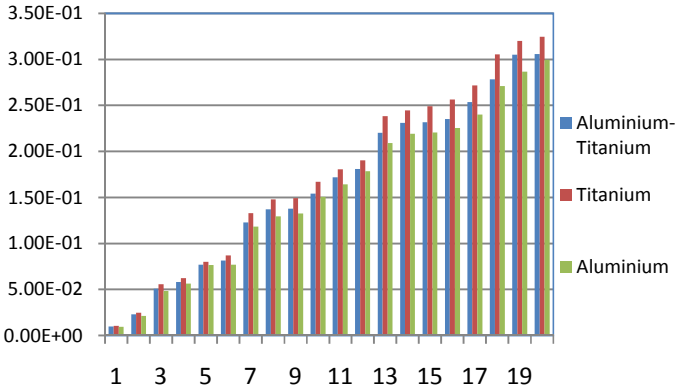


Fig. 13 First 20 natural frequencies comparison

Harmonic Analysis

Figs. 14 and 15 show the frequency responses of Aluminum model while Figs. 16 and 17 show the frequency responses of Titanium model.

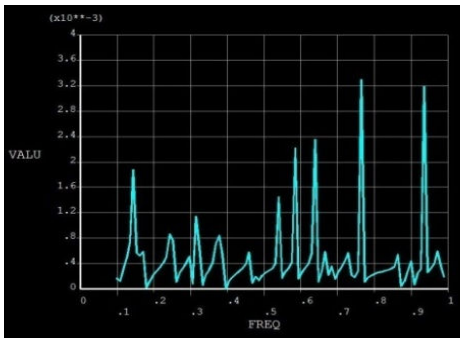


Fig. 14 Frequency Response (Linear Scale)

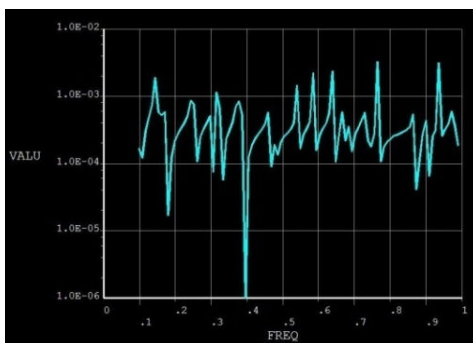


Fig. 15 Frequency Response (Log Scale)

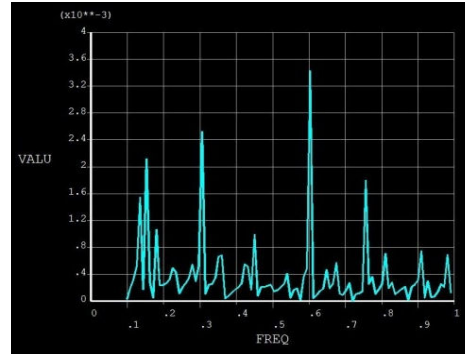


Fig. 16 Frequency Response (Linear Scale)

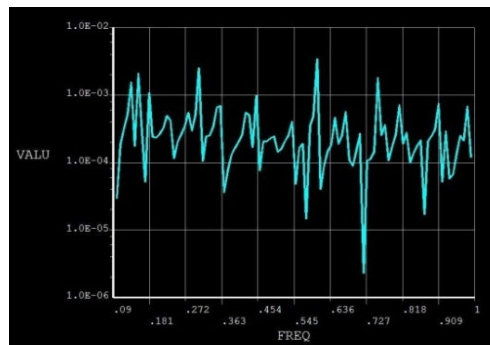


Fig. 17 Frequency Response (Log Scale)

Figs. 18 and 19 show the frequency responses of Aluminum- Titanium (FGM) model.

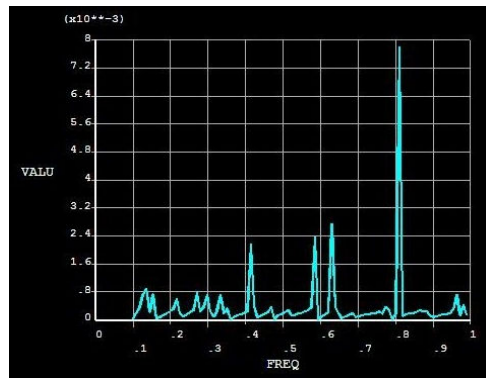


Fig. 18 Frequency Response (Linear Scale)

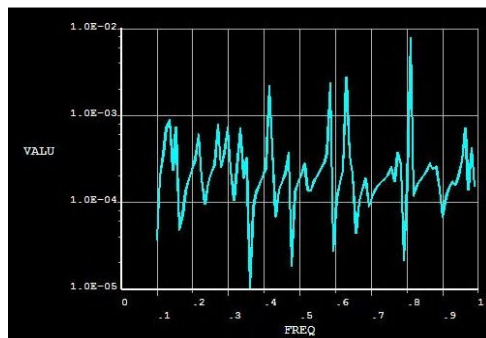


Fig. 19 Frequency Response (Log Scale)

A reduction of 60 % in the vibration amplitude is achieved when using FGM (Aluminum- Titanium) cantilever plate instead of Titanium plate.

TABLE III  
 COMPARISON BETWEEN AL, TI, AL-Ti IN AMPLITUDE AND FREQUENCY

	Amplitude	Frequency
Aluminium	3.40E-03	1.63E-01
Titanium	7.80E-03	1.57E-01
Aluminium- Titanium	3.30E-03	1.75E-01

That is considered as a significant amount of vibration attenuation when using FGM as shown in Table III and Fig. 20

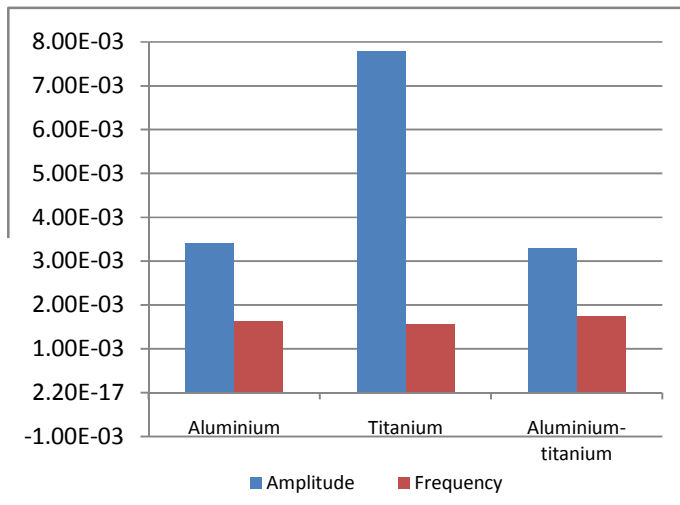


Fig. 20 Bar chart to compare Al, Ti, Al-Ti in Amplitude and Frequency

Figs. 21 and 22 show the Von Mises stress distribution of air plane wing made from Aluminum Model in case of static analysis while Figs. 23 and 24 are for the wing made from Titanium.

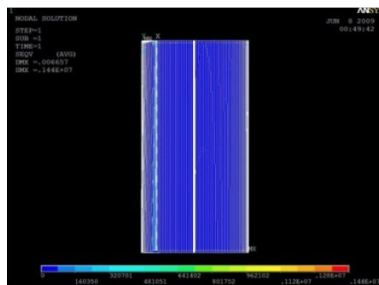


Fig. 21 Von Mises Stress Distribution

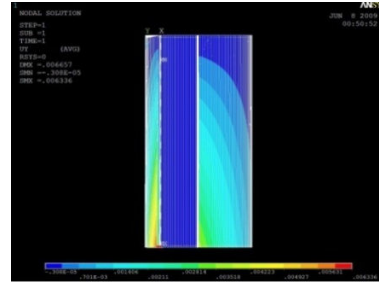


Fig. 22 Deformed Shape

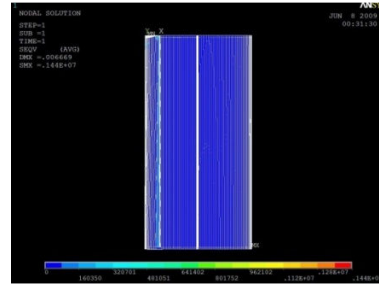


Fig. 23 Von Mises Stress Distribution

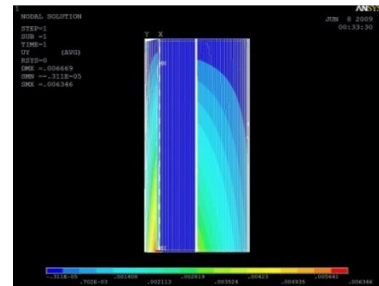


Fig. 24 Deformed Shape

Figs. 25 and 26 show the Von Mises stress distribution of air plane wing made from Aluminum Wing with FGM Spurs Model in case of static analysis while Figs. 27 and 28 are for the wing made from Titanium Wing with FGM Spurs.

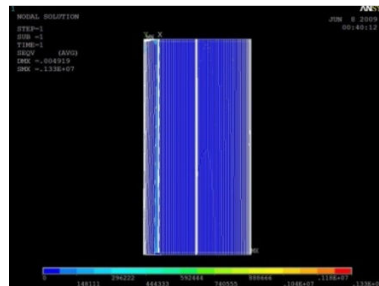


Fig. 25 Von Mises Stress Distribution

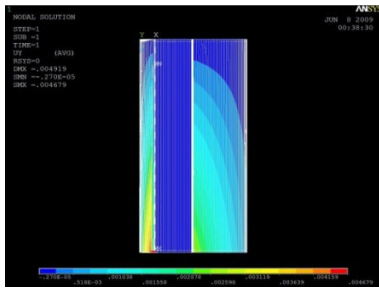


Fig. 26 Deformed Shape

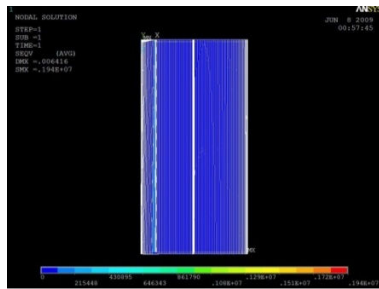


Fig. 27 Von Mises Stress Distribution

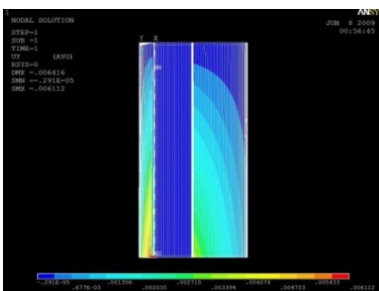


Fig. 28 Deformed Shape

The Summary of Stress Analysis Results is shown Table IV using Comparison between Al, Ti, Al with FGM spurs, and Ti with FGM Spurs in Max. Von Mises Stress and Max. Deformation.

TABLE IV  
 COMPARISON BETWEEN AL, TI, AL WITH FGM SPURS, AND TI WITH FGM MATERIAL

	Von Mises Stress	Deformation
Aluminum (Figs. 21, 22)	144 Mpa	0.6346 mm
Titanium (Figs. 23, 24)	133 Mpa	0.6336 mm
Aluminum – FGM Spurs (Figs. 25, 26)	144 Mpa	0.4679 mm
Titanium- FGM Spurs (Figs. 27, 28)	19 4 Mpa	0.6112 mm

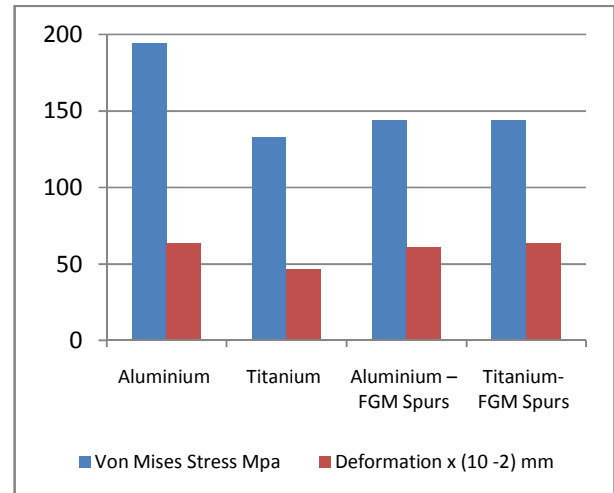


Fig. 29 Bar chart to compare Al, Ti, Al-Ti in Stress and Deformation

Fig. 2 show that a reduction of 26 % in Max. Von Mises stress is achieved when using Aluminum wing with FGM spurs instead of Aluminum wing. Also, a reduction of almost 4% in max. deformation is achieved when using Aluminum wing with FGM spurs instead of Aluminum wing.

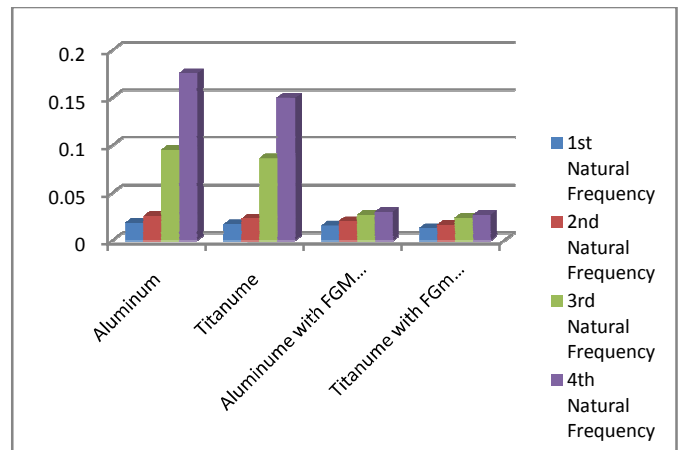


Fig. 30 Comparison between Al, Ti, Al with FGM spurs, and Ti with FGM spurs

According to Fig. 30, the 3rd and 4th natural frequencies were significantly affected when the wing spurs were made of an FGM of Aluminum and Titanium while the 1st and 2nd natural frequencies were just slightly affected. 3rd natural frequency dropped by 70 % when adding FGM spurs to the aluminum wing. 4th natural frequency was dropped by 82 % when adding FGM spurs to the aluminum wing.

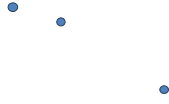


Fig. 31 Aerofoil Studied points

In the harmonic analysis of the air plane wing model, 3 points at the wing span end as shown in Fig. 31 were being studied to get a complete image about the vibration behavior of the wing before and after the functionally graded materials were used. These 3 points were called Tip Point, Middle Point, and Tale Point. The time responses representation is obtained as shown in Figs. 32-34 for Aluminum wing and Figs. 35-37 are for Titanium wing. Due to the computer limitations and the model complexity, the time responses are used instead of the frequency responses but it is highly recommended to be continued to cover the frequency response when computer capabilities allow.

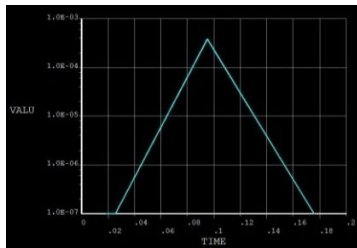


Fig. 32 Aerofoil Tip

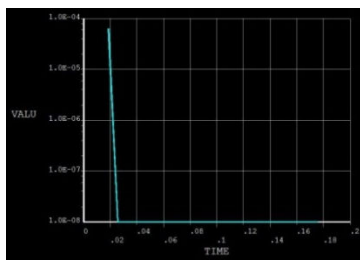


Fig. 33 Aerofoil Middle

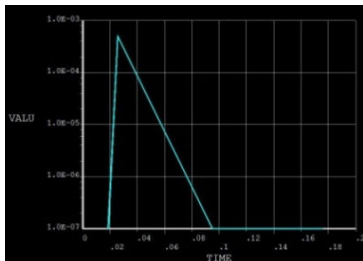


Fig. 34 Aerofoil Tale

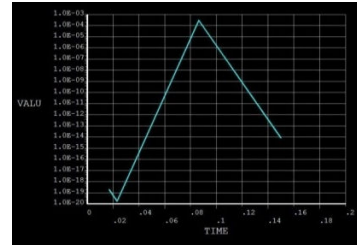


Fig. 35 Aerofoil Tip

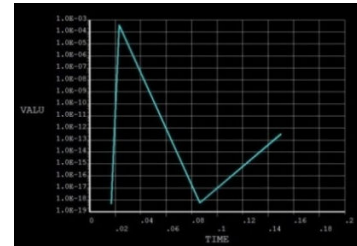


Fig. 36 Aerofoil Middle

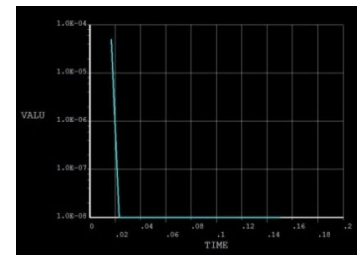


Fig. 37 Aerofoil Tale

The time responses representation is obtained as shown in Figs. 38-40 for Aluminium Wing with FGM Spurs wing and Figs. 41-43 are for Titanium Wing with FGM Spurs wing.

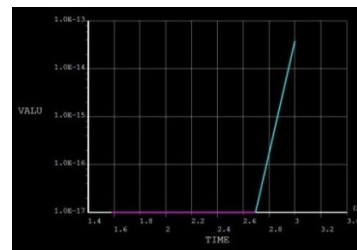


Fig. 38 Aerofoil Tip

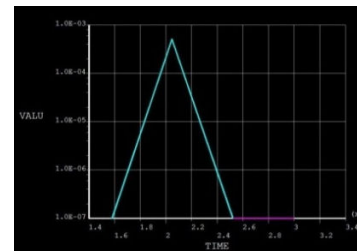


Fig. 39 Aerofoil Middle



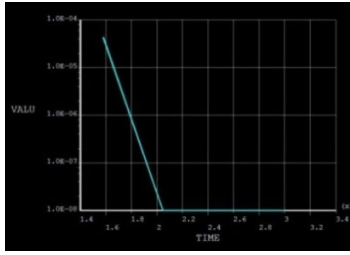


Fig. 40 Aerofoil Tale

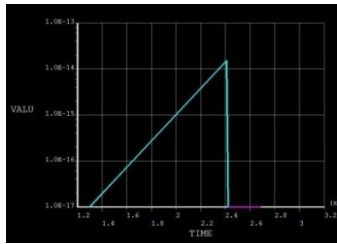


Fig. 41 Aerofoil Tip

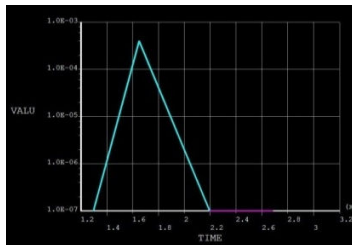


Fig. 42 Aerofoil Middle

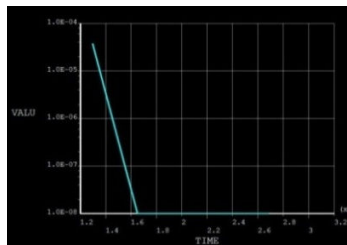


Fig. 43 Aerofoil Tale

According to the Harmonic Analysis, Table V and Fig. 44 show Vibration Amplitude Comparison Al, Ti, Al with FGM spurs, and Ti with FGM spurs- measuring at Tip point, Middle point, and Tale point.

TABLE V  
 VIBRATION AMPLITUDE COMPARISON

Tale Point	Middle Point	Tip Point	
9.00E-05	7.00E-04	6.00E-04	Aluminium
6.00E-05	7.00E-04	6.00E-14	Aluminium-FGM Spurs
7.00E-03	6.00E-04	5.00E-04	Titanium
6.00E-03	5.00E-04	2.00E-14	Titanium-FGM Spurs

Vibration amplitude calculated at the tip of the air plane wing aerofoil fully attenuated when using Aluminum wing

with FGM spurs instead of Aluminum wing. Same thing happened when using Titanium wing with FGM spurs instead of Titanium wing.

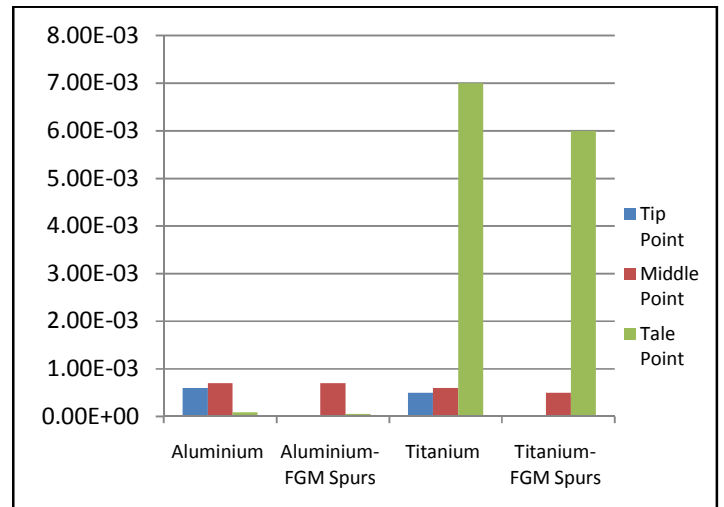


Fig. 44 Vibration Amplitude Comparison Al, Ti, Al with FGM spurs, and Ti with FGM spurs- measuring at Tip point, Middle point, and Tale point

In addition, vibration amplitude calculated at the tale point reduced by 12 % when using Titanium wing with FGM spurs instead of Titanium wing as shown in Fig. 44.

#### IV. CONCLUSIONS

The stress in Air Plane Wing Model is reduced by 26 % when using Aluminum wing with FGM Spurs. In addition, the deformation is reduced by 3.7 % when using Aluminum wing with FGM Spurs. All the natural frequencies are reduced when changing to FGM Spurs. On the other hand, Aluminum wing with FGM Spurs and Titanium wing with FGM Spurs shown a great vibration amplitude attenuation, but when taking the price into consideration, Aluminum Wing with FGM Spurs will be the right choice to attenuate the vibration amplitudes.

#### V. RECOMMENDATIONS

It's recommended to change the graphical scale to 1:10 or even 1:1 when the computers' capabilities allow. Also, it's natural extension to study the frequency harmonic analysis for the airplane wings when the computers' capabilities allow

#### REFERENCES

- [1] Noda N., "Thermal stresses in functionally graded materials", *Journal of Thermal Stresses*, 22, 477-512, 1999
- [2] Fuchiyama T., Noda N., "Analysis of thermal stress in a plate of functionally gradient material", *JSAE Review*, 16, 263-268, 1995
- [3] Obata Y., Noda N., "Steady thermal stresses in a hollow circular cylinder and hollow sphere of a functionally gradient material", *Journal of Thermal Stresses*, 17, 471-487, 1994
- [4] Reddy J.N., Chin C.D., "Thermomechanical analysis of functionally graded cylinders and plates", *Journal of Thermal Stresses*, 21, 593-626, 1998.
- [5] Takezono S., Tao K., Inamura E., Inoue M., "Thermal stress and deformation in functionally graded material, shells of revolution under

- thermal loading due to fluid", *JSME International Journal Series A*, 39, 573–581, 1996.
- [6] Ye G.R., Chen W.Q., Cai J.B., "A uniformly heated functionally graded cylindrical shell with transverse isotropy", *Mechanics Research Communications*, 28, 535–542, 2001
- [7] Cho J.R., Ha D.Y., "Volume fraction optimization for minimizing thermal stress in Ni–Al<sub>2</sub>O<sub>3</sub> functionally graded materials", *Materials Science and Engineering*, 334147–155, 2002
- [8] Jabbari M., Sohrabpour S., Eslami M.R., "Mechanical and thermal stresses in a functionally graded hollow cylinder due to radially symmetric load", *International Journal of Pressure Vessels and Piping*, 79, 493–497, 2002
- [9] Kadoli R., Ganesan N., "Buckling and free vibration analysis of functionally graded cylindrical shells subjected to a temperature-specified boundary condition", *Journal of Sound and Vibration*, 289, 450–480, 2006
- [10] Liew K.M., Kitipornchai S., Zhang X.Z., Lim C.W., "Analysis of the thermal stress behaviour of functionally graded, hollow circular cylinders", *International Journal of Solids and Structures*, 40, 2355–2380, 2003
- [11] Ootao Y., Tanigawa Y., Nakamura T., "Optimization of material composition of FGM hollow circular cylinder, under thermal loading: a neural network approach", *Composites Part B*, 30, 415–422, 1999