Periodicity for a Food Chain Model with Functional Responses on Time Scales

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Abstract—With the help of coincidence degree theory, sufficient conditions for existence of periodic solutions for a food chain model with functional responses on time scales are established.

Keywords—time scales, food chain model, coincidence degree, periodic solutions.

I. INTRODUCTION

Recently, the continuation theorem of coincidence degree theory has been widely applied to the existence problems of periodic solutions in differential equations and difference equations, such as [1],[2]. However, the research methods and results are similar. Is there a unified way to investigate these problems. The theory of calculus on time scales, which was initiated by Stefan Hilger in [3], well solved these problems and unified the differential and difference analysis. In [4],[5], Bohner and Fan systematically studied the existence of periodic solutions of dynamic equations on time scales of predator–prey type and competition type.

In [6], a food chain model with mixed selection of functional responses was constructed,

$$\begin{cases} \dot{X}(\tau) = rX\left(1 - \frac{X}{K}\right) - b_1 XY, X(0) > 0, \\ \dot{Y}(\tau) = -d_1 Y + c_1 XY - \frac{b_2 YZ}{a_1+Y}, Y(0) > 0, \\ \dot{Z}(\tau) = -d_2 Z + \frac{c_2 YZ}{a_1+Y}, Z(0) > 0. \end{cases} \quad (1)$$

Here the positive constants b_1, d_1, c_1, b_2, a_1, d_2 and c_2 respectively denote the predation rate of the predator, the death rate of the predator, the conversion rate, the maximal growth rate of the predator, the half saturation constant, the death rate of the super predator and the conversion factor.

For simplicity, such transformations were made in [6]: $x = \frac{X}{K}$, $y = \frac{Y}{K}$, $z = \frac{Z}{K}$ and $t = r\tau$, and system (1) is equivalent to

$$\begin{cases} \dot{x}(t) = x(1 - x) - bxy, x(0) > 0, \\ \dot{y}(t) = -dy + cxy - \frac{pyz}{1+ay}, y(0) > 0, \\ \dot{z}(t) = -mz + \frac{qyz}{1+ay}, z(0) > 0, \end{cases} \quad (2)$$

where $b = \frac{b_1 K}{r}$, $d = \frac{d_1}{r}$, $c = \frac{c_1 K}{r}$, $a = \frac{K}{a_1}$, $p = \frac{b_2 K}{a_1 r}$, $m = \frac{d_2}{r}$, $q = \frac{c_2 K}{a_1 r}$. Taking into account the periodicity of environment, it is realistic to assume that the parameters in (2) are periodic functions of period ω. Then we have the non-autonomous system

$$\begin{cases} \dot{x}(t) = x(t)(1 - x(t)) - b(t)x(t)y(t), x(0) > 0, \\ \dot{y}(t) = -d(t)y(t) + c(t)x(t)y(t) - \frac{p(t)y(t)z(t)}{1+a(t)y(t)}, y(0) > 0, \\ \dot{z}(t) = -m(t)z(t) + \frac{q(t)y(t)z(t)}{1+a(t)y(t)}, z(0) > 0, \end{cases} \quad (3)$$

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where $b(t)$, $d(t)$, $c(t)$, $a(t)$, $p(t)$, $m(t)$, $q(t)$ are all positive ω−periodic functions. By using the method in [7], we can get the discrete analogy of the previous system

$$\begin{cases} x(k+1) = x(k) \exp\{1 - x(k) - b(k)y(k)\}, \\ y(k+1) = y(k) \exp\left\{-d(k) + c(k)x(k) - \frac{p(k)z(k)}{1+a(k)y(k)}\right\}, \\ z(k+1) = z(k) \exp\left\{-m(k) + \frac{q(k)y(k)}{1+a(k)y(k)}\right\}. \end{cases} \quad (4)$$

So, we mainly consider the following system on time scales

$$\begin{cases} x^\Delta(t) = 1 - e^{x(t)} - b(t)e^{y(t)}, \\ y^\Delta(t) = -d(t) + c(t)e^{x(t)} - \frac{p(t)e^{z(t)}}{1+a(t)e^{y(t)}}, \\ z^\Delta(t) = -m(t) + \frac{q(t)e^{y(t)}}{1+a(t)e^{y(t)}}, \end{cases} \quad (5)$$

where all the coefficients are rd-continuous positive ω−periodic functions on time scales $\mathbb{T}$. System (5) can be reduced to (3) and (4) when $\mathbb{T}$ is $\mathbb{R}$ or $\mathbb{Z}$ respectively.

In this paper, we mainly explore the periodic solutions of (5) by the continuation theorem in coincidence degree theory. The paper is organized as follows. In the next section, we present some preliminary results about the calculus on time scales and the continuation theorem. In Section 3, the sufficient conditions for the existence of periodic solutions for (5) are obtained.

II. PRELIMINARIES

For the convenience of reading, we first present some basic definitions and lemmas about time scales and the continuation theorem of the coincidence degree theory, more details can be found in [8],[9].

Definition 1. A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of real numbers $\mathbb{R}$.

Throughout this paper, we assume that the time scale $\mathbb{T}$ is unbounded above and below, such as $\mathbb{R}$, $\mathbb{Z}$, and $\bigcup_{k \in \mathbb{Z}}[2k, 2k+1]$. The following definitions and lemmas about time scales are from [8].

Definition 2. The forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$, the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$, and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+ = [0, +\infty)$ are defined, respectively, by $\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}, \rho(t) := \sup\{s \in \mathbb{T} : s < t\}$, and $\mu(t) = \sigma(t) - t$ for $t \in \mathbb{T}$. If $\sigma(t) = t$, then t is called right–dense(otherwise: right–scattered), and if $\rho(t) = t$, then t is called left–dense(otherwise: left–scattered).

Definition 3. Assume $f : \mathbb{T} \to \mathbb{R}$ is a function and let $t \in \mathbb{T}$. Then we define $f^\Delta(t)$ to be the number (provided it exists)

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with the property that given any $\varepsilon > 0$, there is a neighborhood U of t such that $\left| f(\sigma(t)) - f(s) - f^\Delta(t)(\sigma(t) - s) \right| \leq \varepsilon |\sigma(t) - s|$, for all $s \in U$.

In this case, $f^\Delta(t)$ is called the delta (or Hilger) derivative of f at t. Moreover, f is said to be delta or Hilger differentiable on $\mathbb{T}$ if $f^\Delta(t)$ exists for all $t \in \mathbb{T}$. Obviously, if $\mathbb{T} = \mathbb{R}$, then $f^\Delta(t) = f'(t)$; if $\mathbb{T} = \mathbb{Z}$, then $f^\Delta(t) = f(t+1) - f(t) = \Delta f(t)$.

Definition 4. A function $F : \mathbb{T} \to \mathbb{R}$ is called an antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^\Delta(t) = f(t)$ for all $t \in \mathbb{T}$. Then we define

$$\int_r^s f(t)\Delta t = F(s) - F(r) \qquad \text{for } r, s \in \mathbb{T}.$$

Definition 5. A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist(finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd}(\mathbb{T})$.

Theorem 6. *Every rd-continuous function has an antiderivative.*

Theorem 7. *If $a, b \in \mathbb{T}$, α, $\beta \in \mathbb{R}$ and $f, g \in C_{rd}(\mathbb{T})$, then*
(a) $\int_a^b [\alpha f(t) + \beta g(t)]\Delta t = \alpha \int_a^b f(t)\Delta t + \beta \int_a^b g(t)\Delta t;$
(b) *if $f(t) \geq 0$ for all $a \leq t < b$, then $\int_a^b f(t)\Delta t \geq 0$;*
(c) *if $|f(t)| \leq g(t)$ on $[a, b) := \{t \in \mathbb{T} : a \leq t < b\}$, then* $\left| \int_a^b f(t)\Delta t \right| \leq \int_a^b g(t)\Delta t.$

Theorem 8. *Let $t_1, t_2 \in I_\omega$ and $t \in \mathbb{T}$. If $g : \mathbb{T} \to \mathbb{R}$ is $\omega-$periodic, then $g(t) \leq g(t_1) + \int_k^{k+\omega} |g^\Delta(s)|\Delta s$ and $g(t) \geq g(t_2) - \int_k^{k+\omega} |g^\Delta(s)|\Delta s.$*

For simplicity, we use the following notations throughout the paper. Let $\mathbb{T}$ be $\omega-$periodic, that is $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$,

$$k = \min\{\mathbb{R}^+ \cap \mathbb{T}\}, I_\omega = [k, k + \omega] \cap \mathbb{T},$$

$$\bar{g} = \frac{1}{\omega} \int_{I_\omega} g(s)\Delta s = \frac{1}{\omega} \int_k^{k+\omega} g(s)\Delta s,$$

where $g \in C_{rd}(\mathbb{T})$ is an $\omega-$periodic real function, i.e., $g(t + \omega) = g(t)$ for all $t \in \mathbb{T}$.

Now, we introduce some concepts and a useful result in [9].

Let X, Z be normed vector spaces, $L : \text{Dom}L \subset X \to Z$ be a linear mapping, $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\text{Ker}L = \text{codim}\text{Im}L < +\infty$ and $\text{Im}L$ is closed in Z. If L is a Fredholm mapping of index zero and there exist continuous projections $P : X \to X$ and $Q : Z \to Z$ such that $\text{Im}P = \text{Ker}L$, $\text{Im}L = \text{Ker}Q = \text{Im}(I - Q)$, then it follows that $L|\text{Dom}L \cap \text{Ker}P : (I - P)X \to \text{Im}L$ is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called $L-$compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N : \bar{\Omega} \to X$ is compact. Since $\text{Im}Q$ is isomorphic to $\text{Ker}L$, there exists an isomorphism $J : \text{Im}Q \to \text{Ker}L$.

Theorem 9. *(Continuation Theorem) Let L be a Fredholm mapping of index zero and N be $L-$compact on $\bar{\Omega}$. Suppose*
(a) for each $\lambda \in (0,1)$, every solution u of $Lu = \lambda Nu$ is such that $u \notin \partial\Omega$;
(b) $QNu \neq 0$ for each $u \in \partial\Omega \cap \text{Ker}L$ and the Brouwer degree $\deg\{JQN, \Omega \cap \text{Ker}L, 0\} \neq 0$.
Then the operator equation $Lu = Nu$ has at least one solution lying in $\text{Dom}L \cap \bar{\Omega}$.

III. EXISTENCE OF PERIODIC SOLUTIONS

In this section, we will prove the theorem related to system (5).

Theorem 10. *If $a(t), b(t), c(t)$, $d(t)$, $p(t)$ and $q(t)$ are all positive rd–continuous $\omega-$periodic functions on time scales $\mathbb{T}$, and the following assumptions hold,*
(H_1) $1 - b^M e^{M_3} > 0$,
(H_2) $\left(1 - \frac{b^M}{b^L} e^{\omega \bar{c} e^{2\omega}}\right) c^L > e^{2\omega} d^M$,
where $M_3 = \ln\frac{1}{b^L} + \omega\bar{c}e^{2\omega}$, then (5) has at least one $\omega-$periodic solution.

Proof: Let $X = Z = \{(x, y, z)^T \in C(\mathbb{T}, \mathbb{R}^3) : x(t + \omega) = x(t), y(t + \omega) = y(t), z(t + \omega) = z(t), \forall t \in \mathbb{T}\}$, $\| (x, y, z)^T \| = \max_{t \in I_\omega} |x(t)| + \max_{t \in I_\omega} |y(t)| + \max_{t \in I_\omega} |z(t)|, (x, y, z)^T \in X$ or Z. Then X and Z are both Banach spaces when they are endowed with the above norm $\| \cdot \|$. Let

$$N \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = \begin{bmatrix} 1 - e^{x(t)} - b(t)e^{y(t)} \\ -d(t) + c(t)e^{x(t)} - \frac{p(t)e^{z(t)}}{1+a(t)e^{y(t)}} \\ -m(t) + \frac{q(t)e^{y(t)}}{1+a(t)e^{y(t)}} \end{bmatrix},$$

$$L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x^\Delta \\ y^\Delta \\ z^\Delta \end{bmatrix}, \qquad P \begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix}.$$

Then $\text{Ker}L = \mathscr{L}_c^\omega$, $\text{Im}L = \mathscr{L}_0^\omega$, and $\dim\text{Ker}L = 3 = \text{codim}\,\text{Im}L$. Since $\text{Im}L$ is closed in Z, then L is a Fredholm mapping of index zero. It is easy to find that P and Q are continuous projections such that $\text{Im}P = \text{Ker}L$ and $\text{Im}L = \text{Ker}Q = \text{Im}(I - Q)$. Furthermore, the generalized inverse (to L) $K_P : \text{Im}L \to \text{Ker}P \cap \text{Dom}L$ exists and is given by

$$K_P \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \int_k^t N_1(s)\Delta s - \frac{1}{\omega} \int_k^{k+\omega} \int_k^t N_1(s)\Delta s\Delta t \\ \int_k^t N_2(s)\Delta s - \frac{1}{\omega} \int_k^{k+\omega} \int_k^t N_2(s)\Delta s\Delta \\ \int_k^t N_3(s)\Delta s - \frac{1}{\omega} \int_k^{k+\omega} \int_k^t N_3(s)\Delta s\Delta t \end{bmatrix}.$$

Thus

$$QN \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{\omega} \int_k^{k+\omega} \left(1 - e^{x(t)} - b(t)e^{y(t)}\right)\Delta t \\ \frac{1}{\omega} \int_k^{k+\omega} \left(-d(t) + c(t)e^{x(t)} - \frac{p(t)e^{z(t)}}{1+a(t)e^{y(t)}}\right)\Delta t \\ \frac{1}{\omega} \int_k^{k+\omega} \left(-m(t) + \frac{q(t)e^{y(t)}}{1+a(t)e^{y(t)}}\right)\Delta t \end{bmatrix},$$

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$$K_P(I-Q)N\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} \int_k^t N_1(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t N_1(s)\Delta s\Delta t - (t-k) \\ -\frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t)\bar{N}_1 \\ \int_k^t N_2(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t N_2(s)\Delta s\Delta t - (t-k) \\ -\frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t)\bar{N}_2 \\ \int_k^t N_3(s)\Delta s - \frac{1}{\omega}\int_k^{k+\omega}\int_k^t N_3(s)\Delta s\Delta t - (t-k) \\ -\frac{1}{\omega}\int_k^{k+\omega}(t-k)\Delta t)\bar{N}_3 \end{bmatrix}.$$

Obviously, QN and $K_P(I-Q)N$ are continuous. According to Arzela–Ascoli theorem, it is easy to show that $K_P(I-Q)N(\bar{\Omega})$ is compact for any open bounded set $\Omega \subset X$ and $QN(\bar{\Omega})$ is bounded. Thus, N is $L-$compact on $\bar{\Omega}$.

Now, we are to build up the suitable open bounded subset Ω for the application of the continuation theorem. For the operator equation $Lu = \lambda Nu$, where $\lambda \in (0,1)$, we have

$$\begin{cases} x^\Delta(t) = \lambda\left(1 - e^{x(t)} - b(t)e^{y(t)}\right), \\ y^\Delta(t) = \lambda\left(-d(t) + c(t)e^{x(t)} - \frac{p(t)e^{z(t)}}{1+a(t)e^{y(t)}}\right), \\ z^\Delta(t) = \lambda\left(-m(t) + \frac{q(t)e^{y(t)}}{1+a(t)e^{y(t)}}\right). \end{cases} \quad (6)$$

Assume that $(x,y,z)^T \in X$ is a solution of system (6) for a certain $\lambda \in (0,1)$. Integrating (6) on both sides from k to $k+\omega$, we get

$$\begin{cases} \omega = \int_k^{k+\omega} e^{x(t)}\Delta t + \int_k^{k+\omega} b(t)e^{y(t)}\Delta t, \\ \int_k^{k+\omega} c(t)e^{x(t)}\Delta t = \int_k^{k+\omega} d(t)\Delta t + \int_k^{k+\omega}\frac{p(t)e^{z(t)}}{1+a(t)e^{y(t)}}\Delta t, \\ \int_k^{k+\omega} m(t)\Delta t = \int_k^{k+\omega}\frac{q(t)e^{y(t)}}{1+a(t)e^{y(t)}}\Delta t. \end{cases} \quad (7)$$

From (6) and (7), we have

$$\int_k^{k+\omega}|x^\Delta|\Delta t < \omega + \int_k^{k+\omega} e^{x(t)}\Delta t + \int_k^{k+\omega} b(t)e^{y(t)}\Delta t = 2\omega,$$

$$\int_k^{k+\omega}|z^\Delta|\Delta t < \int_k^{k+\omega} m(t)\Delta t + \int_k^{k+\omega}\frac{q(t)e^{y(t)}}{1+a(t)e^{y(t)}}\Delta t = 2\bar{m}\omega.$$

Since $(x(t),y(t),z(t))^T \in X$, there exist $\xi_i, \eta_i \in I_\omega, i = 1,2,3$, such that

$$x(\xi_1) = \min_{t\in I_\omega}\{x(t)\}, y(\xi_2) = \min_{t\in I_\omega}\{y(t)\}, z(\xi_3) = \min_{t\in I_\omega}\{z(t)\}, \quad (8)$$

$$x(\eta_1) = \max_{t\in I_\omega}\{x(t)\}, y(\eta_2) = \max_{t\in I_\omega}\{y(t)\}, z(\eta_3) = \max_{t\in I_\omega}\{z(t)\}. \quad (9)$$

From the first equation of (7), we get

$$1 > e^{x(\xi_1)} + b(\xi_1)e^{y(\xi_1)},$$

then

$$e^{x(\xi_1)} < 1,$$

and

$$x(\xi_1) < 0,$$

thus

$$x(t) \le x(\xi_1) + \int_k^{k+\omega}|x(t)|\,\Delta t \le 2\omega := M_1.$$

As a result, the following inequalities hold

$$\int_k^{k+\omega}|y^\Delta|\Delta t < \int_k^{k+\omega} c(t)e^{x(t)}\Delta t < \omega\bar{c}e^{2\omega}.$$

By the first equation of (7), we have

$$e^{y(\xi_2)}b^L < 1,$$

and

$$y(\xi_2) < \ln\frac{1}{b^L}.$$

So,

$$y(t) \le y(\xi_2) + \int_k^{k+\omega}|y^\Delta(t)|\Delta t < \ln\frac{1}{b^L} + \omega\bar{c}e^{2\omega} := M_3.$$

According to the third equation of (7),

$$m^L < q^M e^{y(\eta_2)},$$

and

$$y(\eta_2) > \ln\frac{m^L}{q^M},$$

thus,

$$y(t) \ge y(\eta_2) - \int_k^{k+\omega}|y^\Delta(t)|\Delta t > \ln\frac{m^L}{q^M} - \omega\bar{c}e^{2\omega} := M_4.$$

By the first equation of (7),

$$x^{x(\eta_1)} > 1 - b^M e^{M_3},$$

and

$$x(\eta_1) > \ln(1 - b^M e^{M_3}),$$

thus, from the assumption (H_1), we have the following estimation

$$x(t) \ge x(\eta_1) - \int_k^{k+\omega}|x^\Delta(t)|\Delta t > \ln(1-b^M e^{M_3}) - 2\omega := M_2.$$

From the second equation of (7),

$$p^L e^{z(\xi_3)} < (1 + a^M e^{M_3})c^M e^{M_1}$$

and

$$p^M e^{z(\eta_3)} > c^L e^{x(\xi_1)} - d^M > c^L e^{M_2} - d^M,$$

so, according to (H_2), we can get

$$z(\xi_3) < \ln\frac{(1+a^M e^{M_3})c^M e^{M_1}}{p^L},$$

and

$$z(\eta_3) > \ln\frac{c^L e^{M_2} - d^M}{p^M}.$$

Thus,

$$\begin{aligned} z(t) &\le z(\xi_3) + \int_k^{k+\omega}|z^\Delta(t)|\Delta t \\ &< \ln\frac{(1+a^M e^{M_3})c^M e^{M_1}}{p^L} + 2\omega\bar{m} := M_5, \end{aligned}$$

$$\begin{aligned} z(t) &\ge z(\eta_3) - \int_k^{k+\omega}|z^\Delta(t)|\Delta t \\ &> \ln\frac{c^L e^{M_2} - d^M}{p^M} - 2\omega\bar{m} := M_6. \end{aligned}$$

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From above, we have

$$\max_{t\in[k,k+\omega]}|x(t)| \leq \max\{|M_1|,|M_2|\} := R_1,$$

$$\max_{t\in[k,k+\omega]}|y(t)| \leq \max\{|M_3|,|M_4|\} := R_2,$$

$$\max_{t\in[k,k+\omega]}|z(t)| \leq \max\{|M_5|,|M_6|\} := R_3.$$

Clearly, R_1 R_2 and R_3 are independent of λ. Let $M = R_1 + R_2 + R_3 + R_0$, where R_0 is taken sufficiently large such that for the following algebraic equations

$$\begin{cases} 1 - e^x - \omega\bar{b}e^y = 0, \\ \omega\bar{c}e^x - \omega\bar{d} - \int_k^{k+\omega}\frac{p(t)e^z}{1+a(t)e^y}\Delta t = 0, \\ \omega\bar{m} - \int_k^{k+\omega}\frac{q(t)e^y}{1+a(t)e^y}\Delta t = 0, \end{cases} \qquad (10)$$

every solution $(x^*, y^*, z^*)^T$ of (10) satisfies $\| (x^*, y^*, z^*)^T \| < M$. Now, we define $\Omega = \{(x_1(t), x_2(t), x_3(t))^T \in X, \| (x_1(t), x_2(t), x_3(t))^T \| < M\}$. Then it is clear that Ω verifies the requirement (a) of Theorem 9. If $(x_1(t), x_2(t), x_3(t))^T \in \partial\Omega \cap \mathrm{Ker}L = \partial\Omega \cap \mathbb{R}^2$, then $(x_1(t), x_2(t), x_3(t))^T$ is a constant vector in $\mathbb{R}^2$ with $\| (x_1(t), x_2(t), x_3(t))^T \| = |x_1| + |x_2| + |x_3| = M$. The following inequality holds all along

$$QN\begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Moreover, define

$$\phi(x,y,z,\mu) = \begin{bmatrix} -e^x - \omega\bar{b}e^y \\ -\int_k^{k+\omega}\frac{p(t)e^z}{1+a(t)e^y}\Delta t \\ -\int_k^{k+\omega}\frac{q(t)e^y}{1+a(t)e^y}\Delta t \end{bmatrix} + \mu\begin{bmatrix} 1 \\ \omega\bar{c}e^x - \omega\bar{d} \\ \omega\bar{m} \end{bmatrix},$$

where $\mu \in [0,1]$ is a parameter. If $(x,y,z)^T \in \partial\Omega \cap \mathrm{Ker}L$, then $\phi(x,y,z,\mu) \neq 0$. In addition, we can easily see that the algebraic equation $\phi(x,y,z,0) = 0$ has a unique solution in $\mathbb{R}^3$. Thus the invariance of homotopy produces

$$\begin{aligned} \deg(JQN, \Omega \cap \mathrm{Ker}L, 0) &= \deg(QN, \Omega \cap \mathrm{Ker}L, 0) \\ &= \deg(\phi(x,y,z,1), \Omega \cap \mathrm{Ker}L, 0) \\ &= \deg(\phi(x,y,z,0), \Omega \cap \mathrm{Ker}L, 0) \\ &= 1. \end{aligned}$$

By now, we have verified that Ω fulfills all requirements of Theorem 9, therefore, system (5) has at least one $\omega-$periodic solution in $\mathrm{Dom}L \cap \bar{\Omega}$. The proof is complete. ∎

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