A Comparison between Hybrid and Experimental Extended Polars for the Numerical Prediction of Vertical-Axis Wind Turbine Performance using Blade Element-Momentum Algorithm

Gabriele Bedon, Marco Raciti Castelli, Emesto Benini

Abstract—A dynamic stall-corrected Blade Element-Momentum algorithm based on a hybrid polar is validated through the comparison with Sandia experimental measurements on a 5 m diameter wind turbine of Troposkien shape. Different dynamic stall models are evaluated. The numerical predictions obtained using the extended aerodynamic coefficients provided by both Sheidll and Klimas and Raciti Castelli et al. are compared to experimental data, determining the potential of the hybrid database for the numerical prediction of vertical-axis wind turbine performances.

Keywords—Darrieus wind turbine, Blade Element-Momentum Theory, extended airfoil database, hybrid database, Sandia 5-m wind turbine.

I. INTRODUCTION

The aerodynamic design of the vertical-axis wind turbine (VAWT) is still object of debate. The complexity of the unsteady air-blade interaction during a full revolution of the machine has induced several authors to propose different methods in order to predict rotor performances. Two main approaches are generally adopted, leading to different aerodynamic prediction tools:

- Computational Fluid Dynamics (CFD) codes, which offer a very deep understanding of the air dynamics inside the rotor. On the other hand, a great computational effort is requested, due to the long time needed to perform transient simulations.
- Blade Element - Momentum (BE-M) Theory based algorithms, which are very fast to compute a reliable global performance value, but are dependent on the availability of a complete aerodynamic database of the adopted rotor blade section.

Both the approaches have been highly developed and improved. Carrigan et al. [1] introduced a fully automated process for optimizing the airfoil cross-section of a VAWT, maximizing the torque while enforcing some typical wind turbine design constraints, such as tip speed ratio, solidity and blade profile. The design system required to maximize rotor torque incorporated both rapid geometry generation and automated hybrid mesh generation tools with viscous, unsteady CFD simulation software.

Sabarifard et al. [2] performed both experimental and CFD analyses of small scale Darrieus-type straight-bladed VAWTs, describing the effect of some design parameters, including blade number, airfoil type and turbine solidity on rotor performance.

Ferreira et al. [3] [4] presented a systematic CFD analysis of a two-dimensional single blade rotor configuration, investigating the effect of dynamic stall on VAWT performance and analyzing the influence of the turbulence model in the simulation of the vortical structures spread from the rotor blade.

Raciti Castelli et al. [5] [6] proposed a new CFD based methodology in order to obtain an estimation of both turbine aerodynamic performance and local blade angles of attack, allowing the investigation of any blade profile shape and providing a deep understanding of the flow mechanisms inside the rotor.

Performance prediction tools based on the Blade Element - Momentum (BE-M) Theory have been considered by several authors, thanks to their lower computational requirements [7] [8] [9] [10] [11] [12]. All the BE-M algorithms rely on the knowledge of the machine operative conditions, as well as the aerodynamic characteristics of the blade profile, as discussed by Bak et al. [13].

The most adopted aerodynamic database was provided by Sheidll and Klimas [14], who performed a series of wind tunnel tests in order to evaluate the lift and drag coefficients over 180° angles of attack for NACA -0009, -0012 and -0015 airfoils and for some Reynolds numbers typical of VAWT blades during operation. These results were successively extended by numerical methods to NACA-0018, -0021 and -0025 and also to a wider range of Reynolds numbers.

The availability of such extended databases is one of the main drawbacks of this method: since aeronautical databases (extending up to a limited range of angles of attack) are easily available in literature, several authors investigated the feasibility of their extension by means of numerical methods.

Bak et al. [13] analyzed several methods (based on Inverse Momentum Theory, Actuator Disc Theory, numerical optimization and quasi-3D CFD computations) for the
derivation of airfoil aerodynamic characteristics, to be used in wind turbine applications.

Raciti Castelli et al. [15] suggested a hybrid database computation methodology, allowing the numerical prediction of the aerodynamic coefficients of a symmetric NACA airfoil characterized by a thickness comprised from 0.09 to 0.25 c. The estimation was based on a combination of the results of the Xfoil numerical code and on the averaged Sandia coefficients (from: [14]), mixed up in order to minimize any possible source of error. This database was validated by Bedon et al. [16] with respect to the experimental results obtained by Sheldahl [17] for a 2-m Darrieus wind turbine, adopting the BE-M code developed by Raciti Castelli et al. [18].

In the present work, the hybrid database proposed by Raciti Castelli et al. [15] is adopted in order to predict the performances of the Sandia 5-meter Darrieus wind turbine studied by Sheldahl and Klimas [19], using a Double Disk Multiple-Streamtube algorithm developed by Raciti Castelli et al. [18]. This code includes a dynamic stall-correction algorithm based on the works of Gormont [22], Strickland et al. [23] and Berg [24].

II. BE-M THEORY

Several BE-M algorithms are available in literature. Some of them include specific corrections to keep both the expansion of the streamtubes and rotor blade dynamic stall into account [25]. As a first operational step, a structured mesh of the rotor - for a given number of vertical and azimuthal subdivisions - is created, as shown in Fig. 1.

\[
F_x = \frac{Nc}{8\pi r} \left( \frac{W}{U_\infty} \right)^2 \left( C_N - C_T \frac{\cos(\theta)}{\sin(\theta) \sin(\beta)} \right) 
\]

being \( c \) the blade chord, \( W \) the relative velocity in the cross-sectional plane of the considered blade element, \( C_T \) and \( C_N \) respectively the blade element tangential and normal aerodynamic coefficients, in formulas:

\[
C_T = C_L \sin(\alpha) - C_D \cos(\alpha) \]

\[
C_N = C_L \cos(\alpha) + C_D \sin(\alpha)
\]

with \( C_L \) and \( C_D \) respectively the lift and drag aerodynamic coefficients and \( \alpha \) the angle of attack between each blade element chord and the relative wind velocity.

Defining the axial induction factor as:

\[
a = 1 - \frac{V_{blade,i}}{U_\infty}
\]

where \( V_{blade,i} \) is the flow velocity at each blade section (i can be up or down, as shown in Fig. 2), the streamwise momentum equation can be written as:

\[
a = F_x^* + a^2
\]
forming the basis for an iterative solution.
Once the streamwise momentum equation is solved, the
torque for each streamtube can be computed by means of the
following equation:

\[ T_s = \frac{1}{2} \rho c \frac{\Delta h}{\sin(\beta)} W^2 \]  

(8)

where \( \rho \) is the air density.
Finally, the average power produced by the rotor can be
computed as:

\[ P = \omega \frac{N_a}{N_\theta} \sum_{i=1}^{N_a} \sum_{j=1}^{N_r} T_s \]  

(9)

being \( \omega \) the angular speed of the rotor, \( N_\theta \) and \( N_r \) respectively
the number of horizontal and vertical mesh subdivisions.
The rotor power coefficient can be estimated by the formula:

\[ C_p = \frac{P}{0.5 \rho T_\infty^3 S} \]  

(10)

where \( S \) is the swept-area of the turbine.
Two different dynamic stall models, based on Gornost
works [22], are implemented: Gornost-Strickland [23] and
Gornost-Berg [24]. Furthermore, the finite aspect-ratio
correction from Vienna and Corrigan studies [26] is adopted.

III. CASE STUDY AND METHODOLOGY
The 5-m Darrieus wind turbine tested by Sheldahl and
Klimas [19] is adopted as a case study for the present
computations. The rotor is 5.1 m high, with a maximum radius
of 2.5 m. The blade shape is straight-circular-straight (SCS);
this is considered a good approximation of the Troposken
architecture [27] but cheaper to manufacture. The main
geometrical characteristics of the turbine are reported in Table
I, while a picture of the described rotor installation in the
Sandia test site is shown in Fig. 3.

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<th>Table I: Main geometrical characteristics of the considered rotor (from: [19])</th>
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The main performance parameter considered in the
proposed computations is the Power Coefficient \( C_p \), as defined
in (10). The numerical results are presented with respect to the
Tip Speed Ratio, defined as:

\[ \lambda = \frac{\omega R}{U_\infty} \]  

(11)

where \( R \) is the maximum rotor radius.
Experimental measurements were conducted considering
five rotational speeds: 125, 137.5, 150, 162.5 and 175 rpm. A
wind speed ranging from 4 to 27 m/s was considered (the open
field tests were conducted in the Sandia foreground site; in the
present work, the same conditions are considered as inputs for
the BE-M algorithm). For the propose computations, the wind
is considered uniform over the turbine height since it was not
possible to collect information about the wind-shear at the test
site, while the effect of spokes is neglected.
As a first step, both Gornost-Strickland [23] and
Gornost-Berg [24] dynamic stall corrections are adopted in
combination with the Sandia coefficients from Sheldahl [14],
in order to understand which model is best describing the
rotor performance. The best model is successively adopted in
combination with the hybrid database provided by Raciti
Castelli et al. [15], in order to compare the performance
predictions computed on the basis of the two candidate
polars. The validation of the proposed hybrid database was
already observed by Raciti Castelli et al. [15] for a turbine
characterized by a height greater than 5 m and by Bedon et
al. [16] for a 2-m high rotor. The aim of the present work is
to extend the validity of the hybrid database also to a medium
size VAWT architecture.

IV. RESULTS AND DISCUSSION
A. Dynamic stall model comparison
Figs. from 4 to 8 show a comparison between measured and
computed rotor performances using both Gornost-Strickland
and Gornost-Berg models, for angular rotor velocities ranging
from 125 to 175 rpm and adopting the hybrid aerodynamic
database [14].
Differently from the conclusions drawn by Bedon et al. [16]
for the 2-m Darrieus turbine, both the Gornost-Strickland
and the Gornost-Berg models provide a good estimation of the
turbine performances. The reliability of the first model
was already assessed for small size VAWTs [16], whereas the
reliability of the second one was proved for bigger turbines
[15]; being a medium size turbine considered in this case,
both are reliable. Nevertheless, since the Gornost-Berg model
is providing a better estimation of the turbine performances
during the blade stall (for tip speed ratios lower than the
Fig. 4: Rotor performance prediction considering Gorman-Strickland (GS) and Gorman-Berg (GB) models and experimental results for a rotational speed of 125 rpm

Fig. 5: Rotor performance prediction considering Gorman-Strickland (GS) and Gorman-Berg (GB) models and experimental results for a rotational speed of 137.5 rpm

Fig. 6: Rotor performance prediction considering Gorman-Strickland (GS) and Gorman-Berg (GB) models and experimental results for a rotational speed of 150 rpm

Fig. 7: Rotor performance prediction considering Gorman-Strickland (GS) and Gorman-Berg (GB) models and experimental results for a rotational speed of 162.5 rpm

Fig. 8: Rotor performance prediction considering Gorman-Strickland (GS) and Gorman-Berg (GB) models and experimental results for a rotational speed of 175 rpm

Fig. 9: Rotor performance prediction considering Gorman-Berg model, using both the Sandia database (Sandia BE-M) and the hybrid database (Hybrid BE-M), as well as experimental results for a rotational speed of 125 rpm

one corresponding to the peak power coefficient), this will be considered for the successive computations.

B. Hybrid database results

Figs. from 9 to 13 show a comparison between measured and computed rotor performances using both Gorman-Strickland and Gorman-Berg models, for angular rotor velocities ranging from 125 to 175 rpm and adopting both the hybrid aerodynamic database [15] and the one provided by Sandia [14].

It can be observed that, for low rotational speeds (up to 150 rpm), the hybrid database is providing better results than the Sandia database. For higher rotational speeds, the hybrid database tends to overestimate rotor performance up to the peak power coefficient, while the performances for higher values of the tip speed ratio result underpredicted. This tendency is also followed by the computations based on the Sandia polars; these limitations can be therefore related to the adopted BE-M algorithm.

V. CONCLUSIONS

The 5-m Darrieus wind turbine manufactured and tested by Sandia has been considered as a case study for the
Fig. 10: Rotor performance prediction considering Gormont-Berg model, using both the Sandia database (Sandia BE-M) and the hybrid database (Hybrid BE-M), as well as experimental results for a rotational speed of 137.5 rpm

Fig. 11: Rotor performance prediction considering Gormont-Berg model, using both the Sandia database (Sandia BE-M) and the hybrid database (Hybrid BE-M), as well as experimental results for a rotational speed of 150 rpm

Fig. 12: Rotor performance prediction considering Gormont-Berg model, using both the Sandia database (Sandia BE-M) and the hybrid database (Hybrid BE-M), as well as experimental results for a rotational speed of 162.5 rpm

Fig. 13: Rotor performance prediction considering Gormont-Berg model, using both the Sandia database (Sandia BE-M) and the hybrid database (Hybrid BE-M), as well as experimental results for a rotational speed of 175 rpm

Evaluation of a hybrid database to be used in small-scale VAWT performance prediction codes. Different dynamic stall models have been evaluated using the Sandia aerodynamic coefficients from [14]. Both the Gormont-Strickland and Gormont-Berg models have provided reliable results for the considered rotational speeds.

The Gormont-Berg model has been chosen to be used in combination with the hybrid database from [15]. The hybrid database has provided good results for rotational speeds lower than 150 rpm. For higher rotational speed, the tip speed ratio of maximum power coefficient has moved towards higher wind speeds. This behavior has been observed using both the Sandia and the hybrid database, leading to the conclusion that the described effect is to be related to the BE-M code.

Further numerical simulations should be performed by considering several blade and rotor architectures, in order to establish the reliability of the hybrid database even for different NACA profiles and rotor blade geometries.

NOMENCLATURE

- $a$ [-] Axial induction factor
- $c$ [m] Airfoil chord
- $C_D$ [-] Airfoil drag coefficient
- $C_L$ [-] Airfoil lift coefficient
- $C_N$ [-] Blade element normal coefficient
- $C_P$ [-] Rotor power coefficient
- $C_T$ [-] Blade element tangential coefficient
- $F_n$ [N] Blade force normal to chord line
- $F_t$ [N] Blade force tangential to chord line
- $F_s$ [N] Resultant streamwise force
- $F_x$ [N] Streamwise force exerted by the blade element as it passes through the streamtube
- $\Delta h$ [m] Streamtube height
- $H$ [m] Rotor total height
- $N$ [-] Number of blades
- $N_h$ [-] Number of horizontal mesh subdivisions
- $N_v$ [-] Number of vertical mesh subdivisions
- $P$ [W] Power produced by the turbine
- $r$ [m] Rotor radius relative to a blade element
- $R$ [m] Wind turbine maximum radius
- $T_e$ [N.m] Blade element torque for each streamtube
- $S$ [m$^2$] Rotor swept area
- $U_\infty$ [m/s] Unhindered wind speed
- $V_{bla,rel}$ [m/s] Flow velocity at a blade section, i can be up or down
- $W$ [m/s] Relative velocity at a blade element cross-sectional plane
α [rad] Blade relative angle of attack (between airfoil chord line and relative wind velocity)

β [rad] Blade element inclination with respect to the horizontal plane

λ [-] Tip speed ratio

ρ [kg/m³] Air density (assumed 1 [kg/m³])

θ [°] Blade azimuthal coordinate

ω [rad/s] Rotor angular velocity

REFERENCES


