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Fuzzy Voting in Internal Elections of Educational and Party Organizations

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Abstract—This article presents a method for elections between the members of a group that is founded by fuzzy logic. Linguistic variables are objects for decision on election cards and deduction is based on t-norms and s-norms. In this election's method election cards are questionnaire. The questionnaires are comprised of some questions with some choices. The choices are words from natural language. Presented method is accompanied by center of gravity (COG) defuzzification added up to a computer program by MATLAB. Finally the method is illustrated by solving two examples; choose a head for a research group's members and a representative for students.

Keywords—fuzzy election, fuzzy electoral card, fuzzy inference, questionnaire.

I. INTRODUCTION

COLLECTING experts into a group for any organization costs a lot. The organization undertakes the costs to achieve some aims. For two reasons, in electing a manager or leader for such groups, manner of election is important. First, who dose deserve leadership, in group's members opinion? Second, election must hinder from creating unnecessary favoritism. In fuzzy election not only the candidates are confronted by an opinion poll with great care of candidate's background but also fuzzy election cause members unable to create favoritism.

Choosing one from a group of experts is always difficult. It will be more difficult if the person wants to be the leader of the group. In traditional elections members have to vote someone and don't vote another one, absolutely rejected or absolutely accepted. The group members, who have suffrage, have to vote in favor of only one candidate. Therefore they vote someone whereas they do not entirely accept him or her and deprive others of their ballot whereas in group's opinion they are almost qualified too. A new approach against traditional one that is presented in this article enables us to resolve the problems.

II. NECESSARY POINTES

For evaluating and choosing someone, it is necessary study his or her behavior, after studying each behavior must be identified and defined explicitly. For example if it wants to reply to a question like 'How much candidate A is collaborates in the group?' The word "collaboration" may make different senses among members. In someone's opinion collaboration means compatibility with others and for another one it means, an obedience to the group's regulations. Therefore it evaluated only one subject with two meanings compatibility and obedience to group's regulations. These mistakes should be prevented before preparing questionnaire. Then it is bound to discuss on the behavior defined in the assay.

Scaling errors are other important problems and the members should pay attention to them. Scaling errors occur in different manners [1]. These manners are defined as follows:

A. Halo Effect

Halo effects maybe occur when evaluate a candidate's characteristics on the base of special behavior that is impressed with. For example a sociable candidate may be evaluated as a very qualified one whereas it is not necessarily so and vice versa that is, an unsociable behavior may change the other's opinion about his or her qualifications.

B. Contradict Error

Contradict error occur when someone evaluate candidate's behavior according to his or her own behavior. For example latecomers evaluate those who are sometimes arriving on time as punctual. This error occurs when one scaling others behavior in comparison with his or her own behavior.

C. Word Error

Some words are ambiguous; they have or express more than one possible meaning, which cause word error. For example 'not very active' means who is calm and also lazy.

D. Central Tendency Error

If candidate's behavior is evaluated in average scale frequently it will face with central tendency error. Central tendency error has two reasons: first the voters are conservative second they do not know their candidate adequately, therefore they choose middle choice. Most of the time with design even choice questionnaire can prevent central tendency error.

E. Lenient and Softness Error

Sometimes the candidate's faults are ignored by voters and being scaled with high scores. It is called lenient and softness error.

F. Severity Error

In the opposition to the lenient and the softness error, some evaluators evaluate their candidates with unusual severity and

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scale them with low scores.

In fuzzy voting election acceptance or rejection of a candidate as a leader or a group manager requires evaluation of his or her characteristics. Therefore if there is not too many candidates it is recommended that the candidates not to participate in voting. The evaluation has more validity when we have more questions in voting cards (questionnaires). Since the questionnaire has n choices therefore more choices is equal to more validity. For example the questionnaire with 6 choices is more valid than 5, 4 or 3 choices.

III. MATHEMATICS MATERIALS

Let **U** is universal set then fuzzy subset f in **U** identify by membership function μ_f. There μ_f is a map from **U** into close interval [0,1]. Therefore fuzzy subset of **U** is a set of ordered pairs:

$$f = \left\{ \left(u, \mu_f(u)\right) \middle| u \in U \right\} \tag{1}$$

There is first component u belong to f with $\mu_f(u)$ scaling.

The capacity for identifying members of sets is increased by the fuzzy subset definition. For example if a question is 'Is candidate A a good manager?' therefore it can use words from natural language rather 'yes' or 'no'. Each of this words is an equivalent to a membership function. Provided that a membership function is designed exactly in a way that will be indicate knowledge and information of who uses that word.

That need inference from membership functions then we need to use special operators for 'not', 'or', 'and'. This requires definition of complement, union and intersection of fuzzy sets until enable us to analys voters replies. Accordingly fuzzy complement, fuzzy union and fuzzy intersection are defined as follows.

A. Fuzzy Complement

Let f is a fuzzy set from universal set **U** and f identified with $\mu_f : U \to [0,1]$, C is function from [0, 1] to [0, 1] that satisfies the following conditions:

$$
\begin{aligned}
&1. C(0) = 1, C(1) = 0 \\
&2. C(a) \ge C(b) \, if \, a \le b \, for \, \forall a, b \in [0,1]
\end{aligned}
\tag{2}
$$

For example let μ_f is a membership function for fuzzy set f. It can define complement of f as follows:

$$C\left(\mu_f(x)\right) = \mu_{f^c}(x) = 1 - \mu_f(x). \tag{3}$$

B. Fuzzy Union or s-norm

Suppose μ_f and μ_g are two membership functions for fuzzy subsets f and g of **U**.

$$\mu_f : U \to [0,1], \mu_g : U \to [0,1] \tag{4}$$

Then $\mu_{f \cup g}(x) = S(\mu_f(x), \mu_g(x))$ is membership function of $f \cup g$. S is two-valued function from $[0,1] \times [0,1]$ into $[0,1]$ that satisfy the following conditions:

$$
\begin{aligned}
&1. S(1,1) = 1, S(a,0) = S(0,a) = a, \forall a \in [0,1] \\
&2. S(a,b) = S(b,a), \forall a,b \in [0,1] \\
&3. S(a,b) \le S(c,d) \, if \, a \le c, b \le d, a,b,c,d \in [0,1] \\
&4. S(a,(S(b,c))) = S(S(a,b),c), \forall a,b.c \in [0,1]
\end{aligned}
\tag{5}
$$

For example let μ_f and μ_g are membership functions for fuzzy sets f and g. It can define union of f and g as follows:

$$\mu_{f \cup g}(x) = S(\mu_f(x), \mu_g(x)) = \mu_f(x) + \mu_g(x) - \mu_f(x)\mu_g(x). \tag{6}$$

C. Fuzzy Intersection or t-norm

Suppose μ_f and μ_g are two membership functions for fuzzy subsets f and g of **U**.

$$\mu_f : U \to [0,1], \mu_g : U \to [0,1] \tag{7}$$

Then $\mu_{f \cap g}(x) = t(\mu_f(x), \mu_g(x))$ is membership function of $f \cap g$. t is two-valued function from $[0,1] \times [0,1]$ into $[0,1]$ that satisfy the following conditions:

$$
\begin{aligned}
&1. t(0,0) = 0, t(a,1) = t(0,1) = a, \forall a \in [0,1] \\
&2. t(a,b) = t(b,a), \forall a,b \in [0,1] \\
&3. t(a,b) \le t(c,d) \, if \, a \le c, b \le d, a,b,c,d \in [0,1] \\
&4. t(a,(t(b,c))) = t(t(a,b),c), \forall a,b.c \in [0,1]
\end{aligned}
\tag{8}
$$

For example let μ_f and μ_g are membership functions for fuzzy sets f and g. It can defined intersection of f and g as follows:

$$\mu_{f \cap g}(x) = t(\mu_f(x), \mu_g(x)) = \mu_f(x)\mu_g(x). \tag{9}$$

D. Linguistic Variables

Any variable that can take natural linguistic term like a quantity is a linguistic variable. Maybe professor Zadeh's example is the clearest and olden example for diction of linguistic variable. He said "Age will be a linguistic variable when it replace numerical quantities like 20, 45, 15, 60 and 70 with young, not young, very young, old, very old and so on". But why do we have intention to change numerical variable to linguistic? For an individual who wants to design a system for

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an inference and decision, most advanced model which is in him disposal, is human. They don't take any advantage of numerical variables in their communication. For example if the room is very hot, you won't ask your host to turn the control of the heater $14°$ in the clockwise. But you will ask her or him to turn down the heater. For another example suppose you want to judge about someone's management ability. You assign qualities like incapable, powerless, faint, normal, fit, powerful or excellent to management abilities and then make your decision on that base. People in their social behaviors and their decisions seldom use numeric variables and by using numeric variables they cause confusion in an organization. Considering modern computer's high speed; the replacement of linguistic variables by numerical variables will be suitable [3].

IV. MEMBERSHIP FUNCTIONS

Suppose m voter want to vote p candidates. In this election *classical ballots* are replaced by $m \times p$ *questionnaire ballots and a voter takes p questionnaire ballots. The questionnaire ballot consists of s questions and each question is followed by n choices. Each of the n choices are words from natural language like* incapable, powerless, faint, normal, fit, powerful, excellent and etc. It can substitute the linguistic terms for membership functions on interval [a, b]. These n membership functions from left to right show bad attributes toward good attributes respectively. It should observe this configuration in designing questions and its choices, that is, in all questions of the *questionnaire ballot* electing first choice always show most negative attribute to the candidate and elect n[th] choice shows most positive attribute to the candidate (see appendix).

Membership functions allocated for all n choices aren't uniform but for $i = 1,2,...,n-2$ membership functions will be bell-shaped and if $i = 0$ or $i = n-1$ they will be sigmoid.

Let
$$h = (b-a)/n + 1 \qquad (10)$$

in this case membership functions allocated for each term of choices is defined as follows:
$$\mu_i(x) = \alpha_i \exp\left(-\left((x-(a+(i+1)h))/\sigma\right)^2\right) \qquad (11)$$

Where
$$0 \le \alpha_i \le 1, \sigma = h/2\left(\sqrt{-\ln(0.5)}\right) \qquad (12)$$

Where σ cause membership functions to intersect each other with value of $1/2$. Obviously this will be right when it choose α_i's uniformly and α_i's are chosen from close interval of [0, 1]. Each α_i will be called *impact factor* for i[th] question that is defined by questionnaire designer. For example if it choose $\alpha_i = 1$ that means i[th] question has maximum impact if it

choose $\alpha_i = 0$ that means i[th] question hasn't impact and when α_i is chosen from open interval (0, 1) it shows value and weight of ith question.

First and last membership functions (See fig. 1) are defined as follows:

$$\mu_0(x) = \alpha_0 / (1 + \exp(x - (a + 3h/2))), 0 \le \alpha_0 \le 1 \qquad (13)$$

$$\mu_{n-1}(x) = \alpha_{n-1} / (1 + \exp(-(x - (n - 1/2)h))), 0 \le \alpha_{n-1} \le 1 \qquad (14)$$

V. INFERENCE AND DECISION

Consider thay have m voters and they will vote p candidates by answering s questions (follows by n choices). If that want to know evaluation and opinion of members about k[th] candidate for l[th] question we have to find the intersect of views. It will be a membership function that will be shown as follows:

$$\mu^{k_l} \qquad (15)$$
where $k = 1,2,..., p, l = 1,2,..., s$.

How can it find μ^{k_l}? If elected choice of the jth voter is i that can show that by following membership function:

$$\mu_{i_j}^{k_l} \qquad (16)$$
where $j = 1,2,...,m, i = 1,2,...,n$.

Consider that we would like to calculate intersect of views; therefore it will be right if it use a t-norm:

$$\mu^{k_l}(x) = t\left(\mu_{i_1}^{k_l}(x), \mu_{i_2}^{k_l}(x), ..., \mu_{i_m}^{k_l}(x)\right) \qquad (17)$$
where $i_j \in \{0,1,2,...,n-1\}$.

Choosing a t-norm will be important for example using minimum t-norm will not be suitable because in this case it choose only one element belongs to the set of

$$\left\{\mu_{i_1}^{k_l}(x), \mu_{i_2}^{k_l}(x), ..., \mu_{i_m}^{k_l}(x)\right\} \qquad (18)$$

That use one of them and the rest is ineffective in making a decision. Another t-norm can be used like algebraic product

$$\mu^{k_l}(x) = \mu_{i_1}^{k_l}(x) \cdot \mu_{i_2}^{k_l}(x) \cdot ... \cdot \mu_{i_m}^{k_l}(x) \qquad (19)$$

where all of them participate in our decision. The objection which can be made is that increasing number of voters may leads to production of small numbers. It will be solved if we use powerful software like MATLAB. Our software must be able to plot μ^{k_l}s because it can show us how members evaluate k[th] candidate for l[th] question. Finally votes are gotten from s questions by candidates have to be gathered, for this

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purpose we use a s-norm (union operator) and μ^k will be found as follows:

$$\mu^k(x)=S\left(\mu^{k_1}(x),\mu^{k_2}(x),...,\mu^{k_s}(x)\right) \qquad (20)$$

where $k=1,2,,...p$.

Plotting evaluation curve for each of μ^ks should be possible to show final opinion of members about each of the candidates and could be displayed for members.

Choosing s-norm isn't independent of chosen t-norm because they have to be conjugating it means they in the company of a fuzzy complement should be satisfied De Morgan laws [2].

Consider that, algebraic product is using then you have to use algebraic sum as follows:

$$s(a,b)=a+b-a.b \qquad (21)$$

Because they are conjugate. Furthermore the computer program is designed in a way that you can use your arbitrary t-norm and s-norm. For sorting our candidate regarded as sufficient, popular and acceptable that will need to use a defuzzification. Center of gravity defuzzifier is better than the others because it calculates average of random variables and then it has high justification in the visual manner [4]. The COG method is given by

$$x_k=\left(\int_a^{a+(n+1)h} x\mu^k(x)dx\right)\Big/\left(\int_a^{a+(n+1)h} \mu^k(x)dx\right),k=1,2,...,p. \qquad (22)$$

The winner of election will be calculated as follows:

$$x_w=\max\left\{x_1,x_2,...,x_p\right\} \qquad (23)$$

VI. COMPUTER PROGRAM BY MATLAB

This section presents a computer program by MATLAB based on the aforementioned theory. If each question is followed by n choices and each membership functions defined on [a, b]. In that case membership function evaluated as follows:

```
h=(b-a)/n;    %  step
o=h/(2*sqrt(-log(0.5))); % sigma
x=linspace(a,b,1000);
```

Suppose that have m voter want to vote p candidates and the questionnaire ballot consists of s questions. In this case it can evaluate μ^{k_L} as follows:

```
for k=1:p

  for l=1:s
    if k==1
    fprintf('Insert Impact facroe of %d th question',l);
```

```
    alfa=input('l:');
    end
    mu{k,l}=ones(1,1000).*alfa;  % impact factor
    for j=1:m
        fprintf...
 ('\nPlease read %d th question of %d th vote for %d th
candidate',l,j,k);
        i1=input(' i=');
        i=i1-1;
        if i>0 && i<n-1
          mu{k,l}=t_norm(mu{k,l},exp(-((x-(a+i*h))/o).^2));
        elseif i==0
          mu{k,l}=t_norm(mu{k,l},1./(1+exp(x-(a+3*h/2))));
        elseif i==n-1
          mu{k,l}=t_norm(mu{k,l},1./(1+exp(-(x-(a+(n-
1/2)*h)))));
        else
          error('Wrong number of input arguments');
        end
    end
  end
end
```

Now we can plot μ^{k_L} :

```
plot(x,mu{k,l});
```

Union of answers:

```
n=mu;
for k=1:p
  for l=1:s-1
    n{k,l+1}=s_norm(n{k,l},n{k,l+1});
  end
end
```

Now we can plot μ^k :

```
  plot(x,n{k,s});
```

Script file for evaluate t-norm:

```
 function tn=t_norm(a,b)
 tn=a.*b;
```

Script file for evaluate s-norm:

```
function sn=s_norm(a,b)
sn=a+b-a.*b;
```

Defuzzification:

```
for k=1:p
  integralnum=trapz(x,x.*n{k,s}); % Trapezoidal numerical
integration
  integralden=trapz(x,n{k,s});
```

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```
    Defuzzi(k)=integralnum/integralden;
end

for k=1:p
    fprintf('Gruop opinion about %d th candidate is %f
.\n',k,Defuzzi(k));
end
```

VII. ILLUSTRATIONS

That will employ the presented method in this section. For this purpose, it will be inspect two examples in different conditions. Voting process will be performed, traditional and fuzzy, and then the results will be compared.

A. Example1

In this example the goal is to choose a representative for elected students. This class has 27 students who are males and they know each other for less than a year. It had three candidates. First election is performed in traditional way, in which each student writes the name of candidate on his ballot. After gathering and counting the votes the following results are obtained:

First candidate polled 10 votes,

Second candidate polled 12 votes and

Third candidate polled 5 votes.

Therefore second candidate with two extra votes is the winner of our election.

Now our election will be repeated by fuzzy voting cards, questionnaire, this questionnaire is comprised of ten questions followed by six choices. Students who have suffrage (candidates don't have suffrage) answer the questions about first candidate and then we gather questionnaires. This process is repeated for second and third candidates. In this example as well as next one it use questionnaire available in appendix. That should be pointed out that the impact factor of the questions are equal then α always is 1. Closed interval [0, 10] is taken for linguistic variables (membership functions). All gathered data are fed to computer pregame presented the last section, following results are obtained:

First candidate polled 5.436466 prominences from [0, 10], second candidate polled 6.139652 prominences from [0, 10] and third candidate polled 3.972239 prominences from [0, 10].

When we review our results in two methods we find out that second candidate is winner. In traditional voting second candidate polled 44% of the vote whereas he polled 61% of prominences. It can show the curve of student's opinion about their candidates before defuzzification (See fig. 1).

By the way we can plot student's opinions for each candidate about any question exists in questionnaire. Fig. 2 shows group's opinion about first, second and third candidate on 8th question.

B. Example2

In this example the goal is to choose a head for a research group. This group has 15 members and they know each other for more than three year.
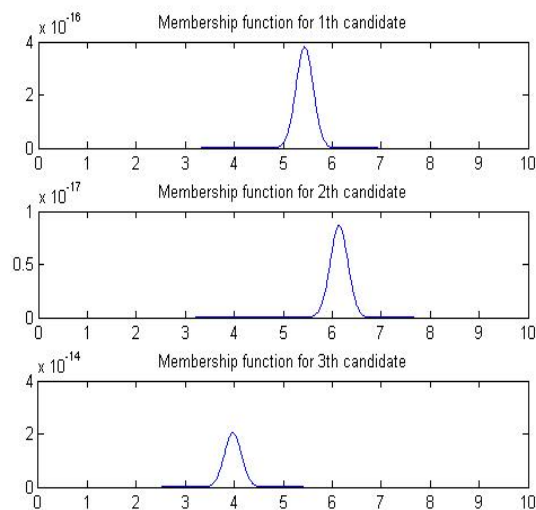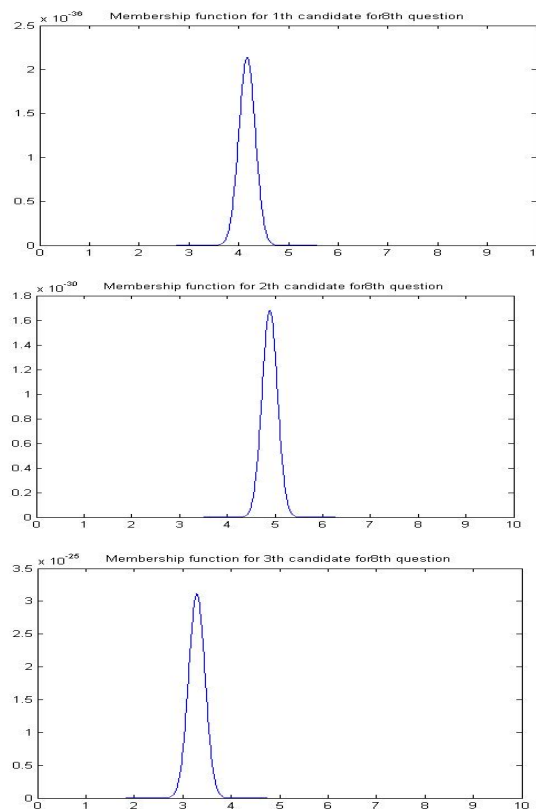

Fig. 1 Membership function

Fig. 2 Membership function

This group has five male and ten female members and we have two candidates one of them is male and the other one is female. First election is performed in traditional way, in which each student writes the name of candidate on his ballot. After gathering and counting the votes the following results are obtained:

First candidate polled 7 votes and

Second candidate polled 8 votes.

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Therefore second candidate with one extra vote is the winner of our election.
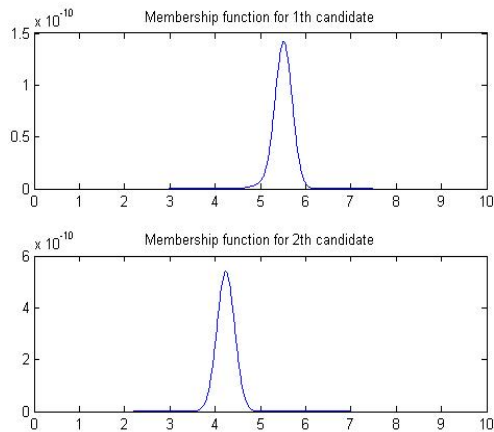

Fig. 3 Membership function

Now election will be repeated by fuzzy voting cards, questionnaire, this questionnaire is comprised of ten questions followed by six choices. Members who have suffrage (candidates have suffrage) answer to questions about first candidate and then we gather questionnaires. This process is repeated for second candidate. In this example we use the questionnaire available in appendix too. We should be pointed out that the impact factor of the questions are equal then α always is 1. Closed interval [0, 10] is taken for linguistic variables (membership functions). All gathered data are fed to computer pregame presented in the last section following results are obtained:
First candidate polled 5.500373 prominences from [0, 10] and Second candidate polled 4.236372 prominences from [0, 10].
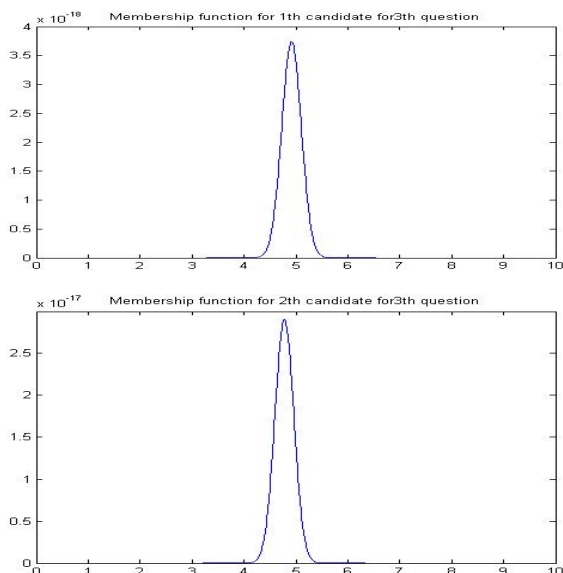

Fig. 4 Membership function

The upper results show us that the winner in fuzzy election has been changed. It isn't a surprise because in this case we have a male and a female candidate. Voters in traditional method concentrated on their friendship and probably sexuality. Whereas in fuzzy method voters answer purely to the questionnaire. It can show the curve of student opinion about their candidates before defuzzification.

By the way it can plot member's opinion for each candidate about any question exists in questionnaire. Fig. 4 shows group's opinions about first and second candidate on 3th question.

APPENDIX

In this section a questionnaire is presented for fuzzy election. Questions have six choices and choices are started from the most negative attribute to the most. This questionnaire has ten questions. Remember that it can change the number of questions as well as number of choices in questionnaire.
A fuzzy electoral card:

First: How responsible is s/he in her/ his duties?

1) ○ Never
2) ○ Seldom
3) ○ Not much
4) ○ Much
5) ○ Very much
6) ○ Completely

Second: How self-confidence is s/he?

1) ○ Never
2) ○ Seldom
3) ○ Not much
4) ○ Much
5) ○ very much
6) ○ Completely

Third: How good is s/he in his/her management?

1) ○ Not at all suitable
2) ○ Too weak to manager
3) ○ A weak manager
4) ○ A good manager
5) ○ A very good manager
6) ○ An ideal manager

Fourth: Dose s/he participate in group's meetings regularly?

1) ○ This meeting isn't important for him or her
2) ○ S/He participates very few
3) ○ S/He seldom participates
4) ○ S/He often participates often
5) ○ S/He participates too often
6) ○ S/He participates regularly

Fifth: Dose s/he participate in discussions and present useful proposals in emergencies?

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1) ○ S/He doesn't participate in discussions
2) ○ S/He participates in discussions a little and s/he never has useful proposals
3) ○ S/He participates in discussions a little and seldom has useful proposals
4) ○ S/He participates in discussions and sometimes has useful proposals
5) ○ S/He always participates in discussions has good ideas
6) ○ S/He always participates in discussions always has the most useful ideas

Sixth: Dose s/he fulfill his or her responsibilities on time and carefully?

1) ○ Never
2) ○ Seldom
3) ○ Sometimes
4) ○ Often
5) ○ Too often
6) ○ Always

Seventh: Dose s/he help members in emergencies?

1) ○ Never
2) ○ Seldom
3) ○ Sometimes
4) ○ Often
5) ○ Most of times
6) ○ Always

Eighth: How much is s/he nonchalant?

1) ○ Always
2) ○ Most of times
3) ○ Often
4) ○ Sometimes
5) ○ Seldom
6) ○ Never

Ninth: Dose s/he completed whatever s/he started?

1) ○ Never
2) ○ Seldom
3) ○ Sometimes
4) ○ Often
5) ○ Most of times
6) ○ Always

Tenth: Is s/he a hard-working person and does more than what have been expected from her or him?

1) ○ Never
2) ○ Seldom
3) ○ Sometimes
4) ○ Often
5) ○ Too often
6) ○ Always

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