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Application of Hermite-Rodriguez Functions to Pulse Shaping Analog Filter Design

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Abstract—In this paper, we consider the design of pulse shaping filter using orthogonal Hermite-Rodriguez basis functions. The pulse shaping filter design problem has been formulated and solved as a quadratic programming problem with linear inequality constraints. Compared with the existing approaches reported in the literature, the use of Hermite-Rodriguez functions offers an effective alternative to solve the constrained filter synthesis problem. This is demonstrated through a numerical example which is concerned with the design of an equalization filter for a digital transmission channel.

Keywords—channel equalization filter, Hermite-Rodriguez, pulse shaping filter, quadratic programming.

I. Introduction

In signal processing many filter design problems can often be cast as constrained optimization problems where the constraints are defined by the specifications of the filter. These specifications can arise either from practical considerations or from the standards set by certain regulatory bodies [3], [4]. A pulse shaping filter (also known as envelope constrained filter) is a filter that process an input signal and produce an output satisfying the constraints. Pulse shaping filters are common in signal processing applications such as in TV channel equalization and pulse compression in radar and sonar systems. In these applications it is necessary for the output signal to be fitted into a prescribed waveform to obtain good system performance.

In real physical systems, almost all the signals are analog. Even in a hybrid system [5], the digitized signal, after being processed through a digital processor, needs to be converted into an analog signal through a linear interpolator. Therefore, it is of interest to directly investigate the design of filters in the continuous time domain. By doing so, we avoid the need to sample potentially very high frequency signals. This is an important factor in high speed applications where low power consumption and small integrated circuit area are required [6]. In this paper, we shall directly tackle the continuous time pulse shaping filtering problem by using continuous time Hermite Rodriguez series expansion.

Although the pulse shaping filtering problem was initially posed in the continuous time domain as a constrained optimization problem, most of the existing approches solved the discretized version of the problem using FIR structure, see e.g. [7], [9]. Although FIR filters are attractive due to their simplicity, they generally require a large number of taps. Furthermore, the number of taps needed in general is highly sensitive to the sampling rate. In [8], the continuous time envelope constrained filtering problem was directly tackled using a finite dimensional Laguerre filter structure and quadratic programming techniques. Compared with the FIR filter structure, the use of Laguerre filters does offer a robust, low order design alternative.

In this paper, we shall study the continuous time pulse shaping filtering problem by using orthogonal Hermite Rodriguez (HR) basis functions. We shall demonstrate that the use of HR functions provides a very efficient alternative in pulse shaping filtering problem. These basis functions are derived as the product of Hermite polynomials and a Gaussian function. Due to Gaussian weighting function, the HR functions have proven to be useful in many signal processing applications such as in the design of radar signals, biomedical signals, electromagnetic signals, and image processing [1], [2], [12], [13], [14].

The Hermite-Rodriquez series expansion is useful because of the following properties and advantages: (i) The HR basis functions are orthogonal, (ii) HR polynomials can easily be computed through a recurrence relation, (iii) any signal can be represented to a high degree of accuracy by using a sufficient number of terms in the expansion, and particularly important (iv) HR expansion is well suited for the design of pulse shaping filter with finite time support [1], [2].

The rest of the paper is organized as follows. Section II describes Hermite filter using orthogonal Hermite-Rodriquez basis functions. In Section III the pulse shaping problem is described and converted into a constrained quadratic programming problem. Solution method is then proposed. In Section IV a numerical example is presented to illustrate the effectiveness of the proposed approach. Section V concludes the paper.

II. Hermite-Rodriguez Filter

In this section, we shall summarize the relevant background material of Hermite Rodriquez functions. For more details, see [1] and the references therein.

The Hermite-Rodriquez filter $h(t)$ of order N is defined as

$$h(t) = \sum_{k=0}^{N-1} x_k \omega_{\lambda,k}(t) \tag{1}$$

where $\omega_{\lambda,k}(t)$ are the continuous Hermite Rodriguez functions defined as

$$\omega_{\lambda,k}(t) = \frac{1}{\sqrt{2^k k!}} H_k\big(\frac{t}{\lambda}\big) \frac{1}{\sqrt{\pi}\lambda} e^{-t^2/\lambda^2} \quad k \in [0, \infty). \tag{2}$$

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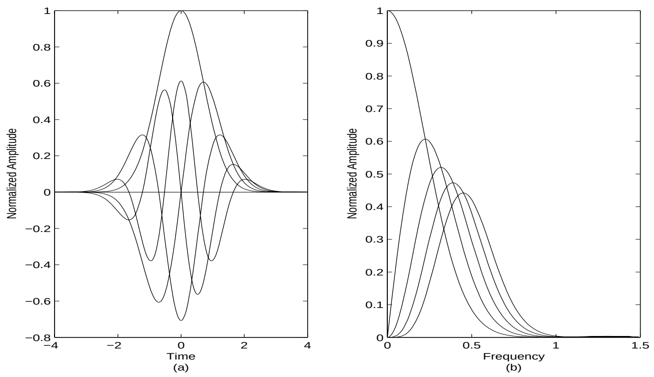

Fig. 1. (a) Hermite-Rodriguez functions order zero to four, $\lambda = 1$ and, (b) magnitude of their Fourier transform.

k is the order of the function and λ is a scaling parameter and $H_k(t)$ are the Hermite polynomials defined recursively as

$$H_k(t) = \begin{cases} 1 & k = 0 \\ 2t & k = 1 \\ 2tH_{k-1}(t) - 2(k-1)H_{k-2}(t) & k \geq 2. \end{cases} \quad (3)$$

The set of functions $\{\omega_{\lambda,k}(t)\}$ constitutes an orthogonal basis with respect to the inner product

$$< \omega_{\lambda,j}, \omega_{\lambda,k} >= \sqrt{\pi}\lambda \int_{-\infty}^{+\infty} \omega_{\lambda,j}(t)\omega_{\lambda,k}(t)e^{-t^2/\lambda^2}dt. \quad (4)$$

This expression is zero for $j \neq k$. Consequently, a signal $f(t)$ can be expanded into an HR series as

$$f(t) = \sum_{k=0}^{\infty} \alpha_{\lambda,k}\omega_{\lambda,k}(t) \quad (5)$$

where

$$\alpha_{\lambda,k} =< f(t), \omega_{\lambda,k}(t) >= \frac{1}{\sqrt{2^k k!}} \int_{-\infty}^{+\infty} f(t)H_k(\frac{t}{\lambda})dt. \quad (6)$$

Fig. 1. shows the Hermite-Rodriguez functions of order zero to four for $\lambda = 1$ and their Fourier transforms.

Note that the HR filter (1) can be written as

$$h(t) = \mathbf{x}^T\omega(t) \quad (7)$$

where

$$\begin{aligned} \mathbf{x} &= [x_0,\ x_1,\ x_2,\ \ldots\ x_{N-1}]^T \\ \omega(t) &= [\omega_{\lambda,0}(t),\ \omega_{\lambda,1}(t),\ \omega_{\lambda,2}(t),\ \ldots\ \omega_{\lambda,N-1}(t)]^T. \end{aligned}$$

III. Summary of Problem Description and Solution Method

The design objective is to design a time invariant filter h with impulse response $u(t)$ to process a given input pulse $s(t)$ which is corrupted by zero mean white noise $n(t)$, see Fig. 2(a). The noiseless output $\psi(t)$ is required to fit into a prescribed pulse shape envelope defined by the lower and upper boundaries $\varepsilon^-(t)$ and $\varepsilon^+(t)$ as shown in Fig. 2(b). The optimal filter is defined as the filter which minimizes the output power due to the input noise $n(t)$ while satisfying the pulse shape constraints. This filter design is formulated into the following constrained optimization problem

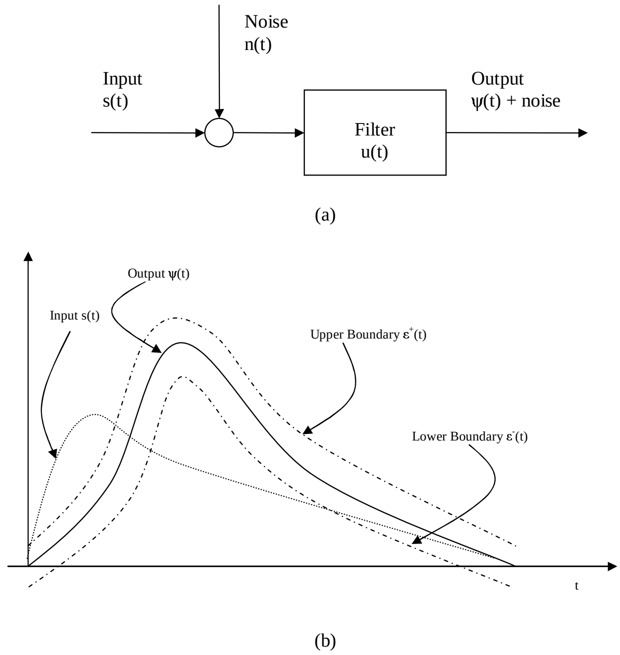

Fig. 2. Pulse shaping filter design problem. (a) Block digram. (b)Pulse shape envelope.

$$\min \int_{-\infty}^{+\infty} \Phi(\omega)|H(j\omega)|^2 d\omega \quad (8)$$

subject to

$$\varepsilon^-(t) \leq \psi(t) \leq \varepsilon^+(t)$$

where $H(j\omega)$ is the Fourier tranform of $h(t)$, $\Phi(\omega)$ is power spectral density of the noise n and

$$\psi(t) = \int_0^{\infty} s(\tau)h(t-\tau)d\tau.$$

The output $\psi(t)$ can be expressed as

$$\psi(t) = \mathbf{y}^T(t)\mathbf{x} \quad (9)$$

where

$$\begin{aligned} \mathbf{y}(t) &= [\psi_0(t),\ \psi_1(t),\ \psi_2(t),\ \ldots\ \psi_{N-1}(t)]^T \\ \psi_k(t) &= \int_0^{\infty} s(\tau)\omega_{\lambda,k}(t-\tau)d\tau \end{aligned}$$

for $k = 0,\ 1,\ 2,\ \ldots\ N-1$.

Using Parseval's theorem and (7), the minimization problem (8) can be converted into an equivalent quadratic programming (QP) problem as

$$\min \mathbf{x}^T A\mathbf{x} \quad (10)$$

subject to

$$\varepsilon^-(t) \leq \mathbf{y}^T(t)\mathbf{x} \leq \varepsilon^+(t) \quad (11)$$

where A is a positive definite matrix depending on HR functions. As can be seen that (10),(11) specify a semi-infinite quadratic programming problem where the number of

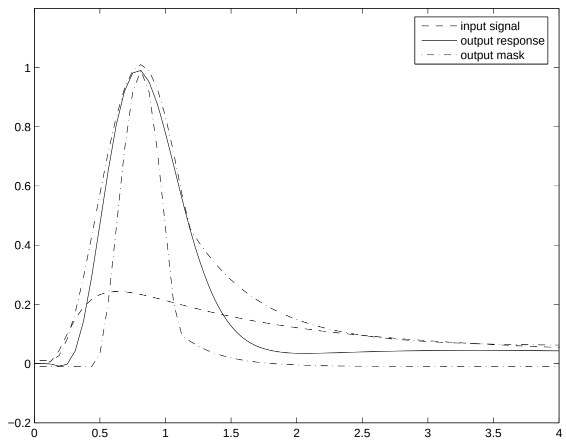

Fig. 3. Plot of the equlised time domain output signal (solid line), the input signal (dashed-line) as the impulse response of a coaxial cable, and the output mask (dash-dotted lines) as the DSX-3 standard pulse template. $N = 4$, $\lambda = 0.5$ and output noise gain = 48.154.

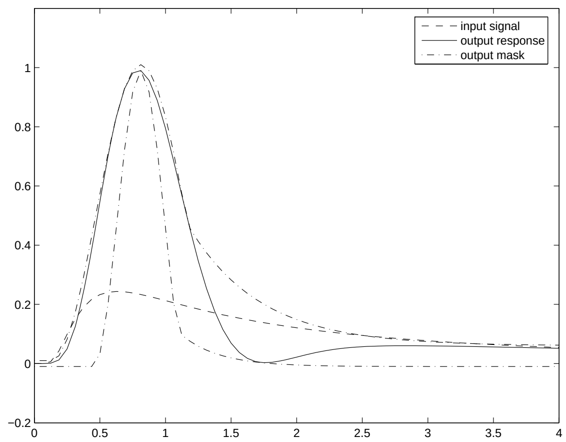

Fig. 4. Plot of the equlised time domain output signal (solid line), the input signal (dashed-line) as the impulse response of a coaxial cable, and the output mask (dash-dotted lines) as the DSX-3 standard pulse template. $N = 4$, $\lambda = 0.7$ and output noise gain = 234.5846.

variables to be optimized is finite but the number of inequality constraints is infinite (depend on t). There are at least two ways to solve the convex optimization problem. One way is by discretizing the time variable and then solved using existing software such as Matlab and its Optimization Toolbox. Another way is by directly solving as a semi-infinite quadratic programming as proposed in [10].

Note that in the design of the filter $h(t)$, if there is no time or frequency domain constraints, the coefficients x_k computed using (6) will be the optimal $\mathcal{L}_2$ space solution. However, in this design, time domain constraints are needed in order to force the output to lie within a prescribed mask. In this case, the coefficients x_k are necessarily the solution of the well-formulated constrained optimization problem (10),(11).

IV. An Illustrative Numerical Example

To illustrate the effectiveness of Hermite Rodriguez functions, a numerical example is studied in this section. It deals with the design of an equalization filter for a digital transmission channel consisting of a coaxial cable on which data is transmitted according to the DSX3 standard (see [3], [4]). The design objective is to find an equalizing filter which takes impulse response of a coaxial cable as input (see Fig. 3 dashed line) and produces an output waveform which lies within the envelope given by DSX-3 pulse template, see Fig. 3 (dash-dotted lines).

TABLE I
Output Noise Gain

Filter	Noise Gain
HR ($N = 4$, $\lambda = 0.5$)	48.154
HR ($N = 6$, $\lambda = 0.34$)	19.7941
Laguerre ($N = 6$) [11]	53.7996

In this particular equalization problem, we obtained an HR filter with $\lambda = 0.5$ using only 4 terms ($N = 4$). From Fig. 3, it is clear that the output response (solid line) lies within the lower and upper output masks for the given impulse response. Comparing the number of terms, the HR filter used less terms than the Laguerre filters reported in [8] and [11] where 14 and 6 terms were used respectively. The ability of HR filter to satisfy the output mask constraints using less terms is due to the property of Hermite Rodriguez functions where it is well suited for signals with finite time support [1], [2]. Besides that the width of HR signals can also be adjusted using the scaling factor, λ.

To study the effectiveness of the HR filter in minimizing the output noise, we compared the output noise gain of HR filter with Laguerre filter [11]. Table I shows the output noise gain for each filters. It can be seen that the HR filter is better than Laguerre filter in reducing the output noise.

To study the effect of the scaling factor, λ on the effectiveness of HR filter, we compared the output noise gain (the objective cost) for $\lambda = 0.1,\ 0.2,\ 0.3,\ \ldots,\ 1.0$. Table II shows the noise gain for each λ for $4th$ order HR filter ($N = 4$) and Fig. 4 shows the output response for $\lambda = 0.7$. We can see that the output response in Fig. 4 is closer to the boundaries that the output in Fig. 3. Therefore proper choice of λ is important to obtain the best filter. This suggests further study on how to systematically obtain optimal λ for a certain application.

In Fig. 3 and 4, although the output signals lie within the upper and lower boundaries, part of the signals at certain points are very close to the boundaries. This means that any disturbance at the input of the channel or any filter implementation errors could cause the corresponding output to violate the envelope constraint. Further research to achieve a robust design could be carried out in the future.

V. Conclusion

In this paper we have applied orthogonal Hermite Rodriguez basis functions to pulse shaping filtering problem. We have formulated the filter design problem into a quadratic programming problem which can be solved efficiently. The numerical studies presented showed that low order filter can be obtained due to property of the Hermite Rodriguez functions. Future research on the scaling factor could discover new potentials of Hermite Rodriguez functions.

TABLE II
OUTPUT NOISE GAIN

λ	**Noise Gain**
0.1	126188.89
0.2	9535.1009
0.3	5060.5244
0.4	142.5767
0.5	48.154
0.6	261.7193
0.7	234.5846
0.8	96.0914
0.9	67.0994
1.0	115.5826

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