World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:3, No:11, 2009

Application of Generalized NAUT B-Spline Curve on Circular Domain to Generate Circle Involute

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Abstract—In the present paper, we use generalized B-Spline curve in trigonometric form on circular domain, to capture the transcendental nature of circle involute curve and uncertainty characteristic of design. The required involute curve get generated within the given tolerance limit and is useful in gear design.

Keywords—Bézier,Circle Involute, NAUT B-Spline, Spur Gear.

I. Introduction

In recent years, several new spline curves and surface schemes have been proposed for geometric modeling in CAGD.

For the higher order curve, uniform hyperbolic polynomial B-Splines were presented by Lü et al. [11] over the space spanned by $\{1, t, ., ., ., t^{k-3,} cosh(t), sinh(t)\}$using an integral approach. Q. Chang and Wang [14] used this integral approach to present a class of Bézier like curve over trigonometric space. Wang et al.[10] presented a new kind of splines called non uniform algebraic trigonometric (NUAT) B-Spline and showed that NUAT B-Spline curves shares most of the properties of B-Spline curves.

In [1] a hyperbolic class of B-Spline curve was presented in generalized way by an integral approach by taking parameter values α and m respectively as scale and shape parameters, generated over space

$\Omega_k(t) = span\{1, t, t^{k-3}, \sinh(mt), \cosh(mt)\}$

where $k \geq 3, m \in \mathbb{R} - \{0\}$ and $\alpha \in \mathbb{R}^+ - \{0\}$ and it has shown that the basis behaves like conventional B-Spline curve in the limiting case of definition and defines hyperbolic class of B-Spline curve in generalized form. Further this curve shows most of the properties of B-Spline curve over polynomial space. The parameter values are used as weights in rational form of B-Spline curve, and give more accurate result compared to rational form of B-Spline curve, in case of generation of transcendental curve.

For α and $m = 1$, if we replace $sinh(t)$ by $sin(t)$, we get non uniform algebraic trigonometric basis, see Wang et al.[10].

But in practical way for curves and surfaces representation fixed parameter values generate some problems. First, fixed value constraints bring up conflicts at later design stages, specifying determined parameter values implicitly adds rigid constraints on geometry. If an interval instead of a fixed value is assigned to a parameter so that any real value within the interval is valid, the degree of freedom of geometric entities is increased. Second, the requirement of fixed parameter values makes the development of conceptual design tools difficult. In fact, at this stage, actual values of parameter may not be known. The problem of parameter partial integrity can be solved if a parameter is specified in a range. Sederberg and Farouki [15] introduced the interval Bézier curve that can transfer a complete description of approximation errors along with the curve to applications in other system. Shen and Patrikalakis [16] presented numerical and geometric properties of interval B-Splines. Tuohy et al. [18] presented an interesting paper on interval B-Spline curve and surfaces, respectively adding a new dimension to robustness in solid modeling system and reverse engineering. Further in [2] we discussed an interval form of B-Spline curve and obtained an envelope of interval B-Spline curve as well as presented a method to find vector interval based on de- Boor algorithm. We extended the idea of rectangular representation and gave least square approximation to fit scattered data using B-Spline curve on circular domain [3] using disk arithmetic ([9], [12]).

S.Baron [17] presented a gear geometric design by B-Spline curve fitting and sweep surface modeling. But Baron used conventional B-Spline in algebraic form for curve fitting of involute profile used for gear geometric design. The iterative algorithm mentioned is quite lengthy and time consuming. However, this problem lies in the transcendental nature of circle involute curve. To do away with it, Fumitaka et al. [7] developed an approximation algorithm for circle involute curve in terms of polynomial functions. The circle involute curve is approximated using Chebeyshev approximated formula [7], which enables to represent the involute in term of polynomial as Bézier curve.

But as Bézier curve has global support property, it is always better to use B-Spline curve having local support property. Also B-Spline curve has less oscillation, strong convex hull property etc. compare to Bézier curve [6]. So in the present paper, we use generalized B-Spline curve in trigonometric form on circular domain, covering the idea of [1], [3], [7], [10], [13] and [17], to capture the transcendental nature of circle involute curve and uncertainty characteristic of design. The required involute curve gets generated within the given tolerance limit.

The paper is organized as; section 2 consists of definition of NAUT B-Spline basis functions in generalized way with the definition of curve on circular domain, in this section we also mention the differential geometry of circle involute curve for the use of reader. In section 3 we give the method to

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approximate circle involute curve using generalized NAUT B-Spline curve on circular domain by transforming the B-Spline parameter range according to the length of the involute curve. Section 4 consists of software implementation and in section 5 we presents an illustrative example with an application of generated involute curve in spur gear, and comparison of our method with previous method.

II. BASIC DEFINITIONS AND DIFFERENTIAL GEOMETRY OF CIRCLE INVOLUTE CURVE

For the advantage of reader we present the definition of basis functions from [1] in generalized form as special case in trigonometric form. Then we define a NAUT B-Spline curve in generalized way on circular domain and the basic differential geometry and terminology of involute profile.

A. Definition of Generalized NAUT B-Spline Basis Functions

Let T be a given set of knot vectors with $t_i < t_{i+1}$ and α be considered as a shifting parameter which can be assumed constant for a particular case. We first give a set of initial functions over $\Omega_2[T]$:

$$N_{i,2}(t) = \begin{cases} \frac{\sin(m(t-\alpha t_i))}{\sin(m(\alpha t_{i+1}-\alpha t_i))}, & \alpha t_i < t \leq \alpha t_{i+1} \\ \frac{\sin(m(\alpha t_{i+2}-t)}{\sin(m(\alpha t_{i+2}-\alpha t_{i+1}))}, & \alpha t_{i+1} < t \leq \alpha t_{i+2} \\ 0, & otherwise \end{cases} \tag{1}$$

where $m \in \mathbb{R} - \{0\}$is considered as shape parameter and $\alpha \in (0, 1)$ with $\alpha m < \pi$.

The generalized NAUT B-Spline basis function for order k>3 in the space can be defined recursively as:

$$N_{i,k}(t) = \int_{-\infty}^{t} (\delta_{i,k-1}N_{i,k-1}(s) - \delta_{i+1,k-1}(s)N_{i+1,k-1})\, ds \tag{2}$$

for $k \geq 3$.

where $\delta_{i,k}^{-1}$is the area bounded by $N_{i,k}(t)$ and the parameter axis, given by

$$\delta_{i,k} = \left(\int_{-\infty}^{\infty} N_{i,k}(t)dt\right)^{-1}. \tag{3}$$

with$\int_{-\infty}^{\infty} \delta_{i,k}N_{i,k}(t)dt = 1$.

In case of multiple knot sequence [10], set $\delta_{i,k}N_{i,k} = 0$ when $N_{i,k} = 0$, however in order to ensure that $N_{i,k}$ have partition of unity when $N_{i,k} = 0$, consider

$$\int_{-\infty}^{t} \delta_{i,k}N_{i,k}(t)dt = \begin{cases} 1, & t \geq t_{i+k} \\ 0, & t < t_{i+k} \end{cases} \tag{4}$$

The derivatives are given by

$$\left.\begin{array}{l} N'_{i,k}(t) = \delta_{i,k-1}N_{i,k-1}(t) - \delta_{i+1,k-1}N_{i+1,k-1}(t) \\ N''_{i,k}(t) = \delta_{i,k-1}N'_{i,k-1}(t) - \delta_{i+1,k-1}N'_{i+1,k-1}(t) \end{array}\right\} \tag{5}$$

B. Representation of NAUT B-Spline Curve on Circular Domain

To define the parameter in an interval, we [3] used the interval on circular domain, i.e., in a disk format, presented by Lin et al. [12], and gave a method to fit data using B-Spline curve on circular domain [3]. On this basis generalized NAUT B-Spline curve on circular domain is defined as

$$(P)_{\varepsilon,\theta_f}\, t = \sum_{i=0}^{n} (P_i)\, N_{i,k}(t) \tag{6}$$

where,
$(P_i) = (C_{x_i}, C_{y_i}) + \varepsilon_i(\cos\theta_f, \sin\theta_f)$ for $0 \leq \theta_f \leq 2\pi$.
and
$N_{i,k}(t)$ is given by (1) and (2), (C_{x_i}, C_{y_i}) are the given polygon points, with allowable error ε_i.

Thus interval NAUT B-Spline curve on circular domain is a set of all NAUT B-Spline curves $P(t)$ defined by control points $P_i \in (P_i)$ for $i = 0, 1, - - -, n$.The required curve is one of these curves. It has two components as mentioned in [3], written as

$$(P)t = \left(\overline{C}\right) t + (\cos\theta_f, \sin\theta_f)(E)t \tag{7}$$

where $\left(\overline{C}\right) t$, is centered curve and error component is given by $(E)t$. It can be viewed as a NAUT B-Spline curve with given error tolerance as shown in figure 1.

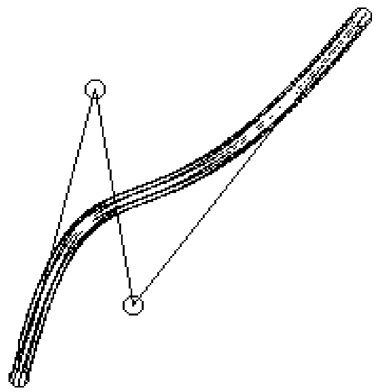

Fig. 1. Some cubic NAUT B-Spline curves of the family on Circular domain.

C. Differential Geometry of Circle Involute Curve

A circle involute curve represents the underlying geometry behind a gear tooth.

The involute of a circle is defined as the curve which is generated by the end point of chord which is kept taut while being unwound from a circle.

The equation of circle of radius r_b centered at origin is given by

$$X = r_b\cos(\theta), Y = r_b\sin(\theta).for 0 \leq \theta \leq 2\pi. \tag{8}$$

The mathematical equation for involute curve ([8],[5]) is given by

$$\left.\begin{array}{l} X = r_b(\sin(\theta) - \theta\cos(\theta)) \\ Y = r_b(\cos(\theta) + \theta\sin(\theta)) \end{array}\right\} \quad (9)$$

To generate all tooth of the profile, we consider the equation

$$\left.\begin{array}{l} X_n = X\cos(n\theta_c) - Y\sin(n\theta_c) \\ Y_n = X\sin(n\theta_c) + Y\cos(n\theta_c) \end{array}\right\} \quad (10)$$

where θ_c is the angle of rotation corresponding to circular pitch of the gear.

The arc length, unit tangent vector and radius of curvature for involute curve([7],[8]) is derived as

$$\left.\begin{array}{l} S(\theta) = \frac{r_b\theta^2}{2} \\ T(\theta) = (\cos(\theta), \sin(\theta)) \\ R(\theta) = r_b\theta \end{array}\right\} \quad (11)$$

The radius of curvature of involute varies directly as varies θ. The radius of curvature is zero or the curvature is infinite at the base circle, where the tracing point of the chord leaves the base circle. Thus curve near the base circle is very difficult to produce and should be avoided whenever possible [7].

The involute curve is used in design of gear tooth profile. The ending point of the involute curve along the tooth profile is determined by computing the intersection point between the circle involute curve and the addendum circle of radius r_a .

Thus

$$r_a^2 = r_b^2(1 + \theta^2). \quad (12)$$

From which the parametric value of θ_a of the intersection is derived to be

$$\theta_a = \frac{\sqrt{r_a^2 - r_b^2}}{r_b} \quad (13)$$

Therefore tooth profile is obtained by equation (9) varying parameter θ from $0 \leq \theta \leq \theta_a$. The arc length of gear tooth profile is given by

$$S(\theta) = \frac{r_b\theta_a^2}{2} \quad (14)$$

The circle involute curve of gear tooth under construction is defined by selecting module M, the number of gear teeth Z, and pressure angle Φ[8]. The pitch circle radius r, base circle radius r_b and addendum circle radius r_a are related by

$$\left.\begin{array}{l} r = \frac{MZ}{2} \\ r_b = r\cos(\Phi) \\ r_a = r + M \end{array}\right\} \quad (15)$$

III. NAUT B-SPLINE APPROXIMATION OF INVOLUTE CURVE

To generate circle involute curve we are using generalized NAUT B-Spline curve with open knot vectors given by $T = \{\alpha t_i\}_0^{n+k}$ as

$$\left.\begin{array}{ll} T_i = 0, & 0 \leq i \leq k \\ T_i = (i-k)\alpha & k+1 \leq i \leq n+1 \\ T_i = (n-k+2)\alpha & n+1 \leq i \leq n+k \end{array}\right\} \quad (16)$$

Basis functions are given by equations (1) and (2), for $n+1$ as number of polygon points and $k-1$ as degree of NAUT B-Spline curve.

To generate involute profile using theses knot vectors we transform the involute parameter range $0 \leq \theta \leq \theta_a$ to $0 \leq t \leq t_m\alpha$ by transformation

$$t = \left(\frac{\theta}{\theta_a}\right) t_m\alpha \quad (17)$$

where $t_m = (n-k+2)$ and $\alpha = \frac{\theta_a}{t_m}$,as an example for $n = 8$ and $k = 4$ open knot vector is given by set $\{\ 0\ 0\ 0\ 0\ \alpha\ 2\alpha\ 3\alpha\ 4\alpha\ 5\alpha\ 6\alpha\ 6\alpha\ 6\alpha\ 6\alpha\ \}$ and $\alpha = \frac{\theta_a}{6}$, so that the NAUT B-Spline parameter 't' will varies from 0 to θ_a .

To avoid high curvature span of curve near base circle we change the involute parameter range by using hundredth of arc length of tooth profile, as calculating $\theta_s = \sqrt{\frac{s_a}{100r_b}}$ and varying parameter from $\theta_s \leq \theta \leq \theta_a$.

Now the first step of fitting procedure, using generalized NAUT B-Spline on circular domain concerns with the selection or detection of $n+1$ data points $(D_c)_j$ for $j = 0, 1, ---, n$.Than we parameterize the data points $(D)_i = (D_c, D_\varepsilon)_i$ with parameter t_i as $t_0 = \theta_s$ and $t_i = \left(\frac{i}{n}\right)\theta_a$, for $1 \leq i \leq n$ for equally spaced data points or using chord length approximation for unevenly spaced data points [4]as

$$t_0 = \theta_s, t_i = \frac{\sum_{k=0}^{i} \left|(D_c)_{k+1} - (D_c)_k\right|}{\sum_{k=0}^{m-1} \left|(D_c)_{k+1} - (D_c)k\right|} t_m\alpha \quad (18)$$

for $i = 1, 2, ---, n$.Now data point $(D_c)_i$ is on the required centered NAUT B-Spline curve corresponding to parameter t_i, so we can write the following system of equation

$$(D_c)_i = \sum_{i=0}^{n} (P_{c_i}) N_{i,k}(t_j) \quad (19)$$

for $j = 0, 1, ---, n$. The $(n+1)$ by $(n+1)$ system of equation can be easily solved to get unknown control points P_{c_i}Thus we find the center coordinate point for each control disk using least square approximation and assume the radius for each control disk is same as that of selection error of corresponding data point.

Now using (6) we generate the family of NAUT B-Spline curve for the given involute points, on circular domain. The best fit involute curve out of theses generated family of involute is obtained by finding average deviation of each generated curve with theoretical involute. We select the best fit curve within the given tolerance limit having minimum deviation from given involute, using the following

$$E = \frac{\sqrt{\sum_{j=0}^{N} \left(P_{\varepsilon,\theta_f}(t_j) - I(\theta_j)\right)^2}}{N} \quad (20)$$

where $\theta_j = \frac{t_j\theta_a}{t_m\alpha}, 0 \leq \theta_f \leq 2\pi, 0 \leq \varepsilon \leq \frac{\sum_i \varepsilon_i}{n}$ and N is the total number of points used to find deviation. Now by varying ε and θ_f we find the best fit involute curve for the minimum average deviation 'E'.

Now the first and second derivatives of generated involute curve is given by

$$\left.\begin{aligned} P'(t) &= \sum_{i=0}^{n-1} (P_i)\, N'_{i,k}(t) \\ P''(t) &= \sum_{i=0}^{n-1} (P_i)\, N''_{i,k}(t) \end{aligned}\right\} \quad (21)$$

where $N'_{i,k}(t)$ and $N"_{i,k}(t)$ are given by (5). First and second derivative of theoretical involute is given by

$$\left.\begin{aligned} &X'(\theta) = r_b\theta\sin\theta, Y'(\theta) = r_b\theta\cos\theta \\ &X''(\theta) = r_b(\cos\theta - \theta\sin\theta), Y''(\theta) = r_b(\sin\theta + \theta\cos\theta) \end{aligned}\right\} \quad (22)$$

Using these derivatives values, we compare the tangents and curvature values of generated involute curve with the true involute curve at corresponding $\theta_j = \frac{t_j\theta_a}{t_m\alpha}$ and obtained the result with reasonable fidelity [13].

IV. SOFTWARE IMPLEMENTATION

Input detected points on invloute profile, with possible error of detection.

Input addendum and base circle radius of involute.

System responds with required open knot vectors using equation (16) and (17) corresponding to number of points for cubic NAUT B-Spline curve.

System find center of each control disc using system of equation (19) required to fit centered NAUT B-Spline for the input points, following equations (18) as per the points equally spaced or unequally spaced.

Using these centered control disk points, and with radius of control disk same as that of input error mentioned for points, we get family of NAUT B-Spline curve for involute profile.

Input the required error tolerance and select NAUT B-Spline curve out of generated family with minimum average deviation from theoretical involute using (20).

If this deviation is within given tolerance we get the required profile otherwise system increases the number of input points and repeats the process.

The program made in C++ can generate involute profile for any number of points using cubic NAUT B-Spline curve on circular domain.

V. APPLICATION TO SPUR GEAR MODELING

Ones we get the required involute profile within the given error tolerance, input module 'M' and number of teeth 'Z' of the gear.

The circular pitch is given by $P = \pi M$ ([8],[5]). Find the mirror image of involute profile with rotation corresponding to half of circular pitch. One tooth profile of gear gets generated. Now to get all teeth as per the number, rotate this tooth using equation (10) on the base circle with rotation corresponding to circular pitch.

Input it into CAD system to model gear.

As an illustrative application, an invloute profile for the following data[1] of gear has been modeled by the method described above.

- The number of teeth $Z = 20$
- The module $M = 1/16$
- Pressure angle $= 14.5\ ^o$
- The Pitch Diameter $(D) = Z * M = 20/16 = 1.25$"
- The Pitch Radius$(r) = D/2 = .625$"
- The Base Circle Diameter $(D_b) = D * COS(\Phi) = 1.210$"
- The Base Circle Radius $(r_b) = Db/2 = 0.605$"
- The Addendum $(a) = 1/16 = 0.0625$
- The Dedendum $(d) = 1.157/P = 1.157/16 = 0.0723$" (rounding off at 0.0001")
- Outside Diameter $(D_O) = D + 2 * a = 1.375$"
- Addendum Radius $(r_a) = r + M = 0.6875$"
- Root Diameter$(D_r) = D - 2 * b = 1.1054$"
- Root Radius $(r_r) = r - d = 0.625" - 0.0723" = 0.5527$

Calculated Results

$\theta_a = 0.5397$" by eqn(10).

For n=8,K=4 we get t_m=6 and α =0.08995" ,

For n=11, k=4 we get t_m=9 and α=0.059966".

We detect the equally spaced points on true involute profile with error of detection 0.0001 and m=1. We summarized the result in following table, showing the variation of generated profile with the true involute profile. A comparison of mentioned method with the Baron method [17] is also given in the table.

TABLE I
SUMMARY OF NORMALIZED DEVIATION COMPUTED AS DIFFERENCE BETWEEN GENERATED CURVE AND TRUE INVOLUTE CURVE.

	Number of polygon points(n+1) and degree (k-1)	**Average Deviation (mm)**	**Maximum Deviation(mm)**
S.Baron's Method	$n = 8, k = 4$	2.1x10^{-3}	11x10^{-3}
	$n = 11, k = 4$	0.57x10^{-3}	2.9x10^{-3}
Our Method	$n = 8, k = 4$	$3.99x10^{-5}$	9.46x10^{-5}
	$n = 11, k = 4$	2.50x10^{-5}	7.08x10^{-5}

Figure 2 and 3 shows the generated involute profile for $n = 8, k = 4$ and $n = 11, k = 4$ respectively. In figure 3 theoretical profile get overlapped by generated profile as error is negligible. Figure 4 shows the tangent plots and deviation of slop of tangent with tangent slop of theoretical involute. Figure 5 shows curvature plots for $n = 8$ and $n = 11$ with $k = 4$. It is noticed that the tangent and curvature plot for $n = 11$ matches with the tangent and curvature plot for theoretical involute profile and the result can be improved for large n. Figure 6 shows the generated spur gear as an application.

[1]Source of data: From already designed gear www.cartertools.com /The Involute Curve, Drafting a Gear in CAD and Applications (April 27th , 2007)

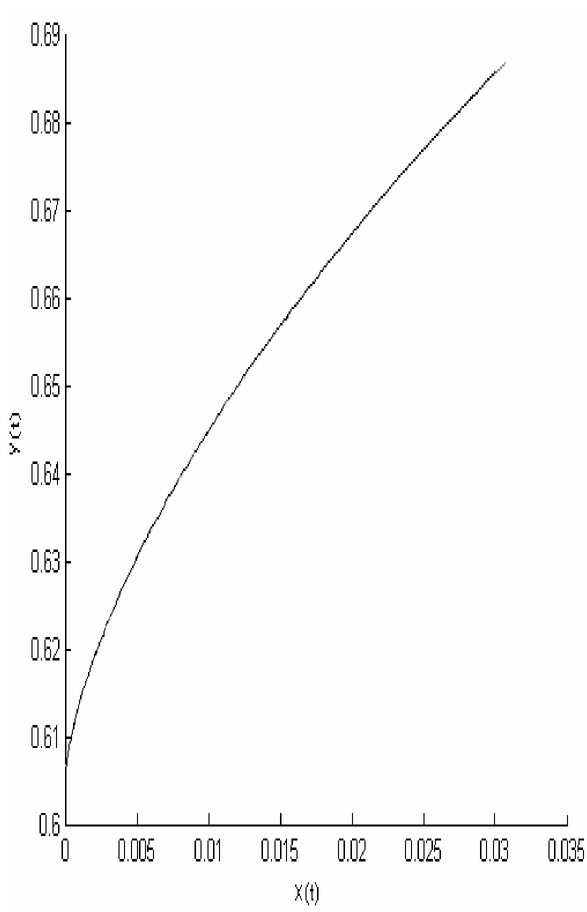

Fig. 2. Generated and true Involute profile for n+1= 9, k=4.

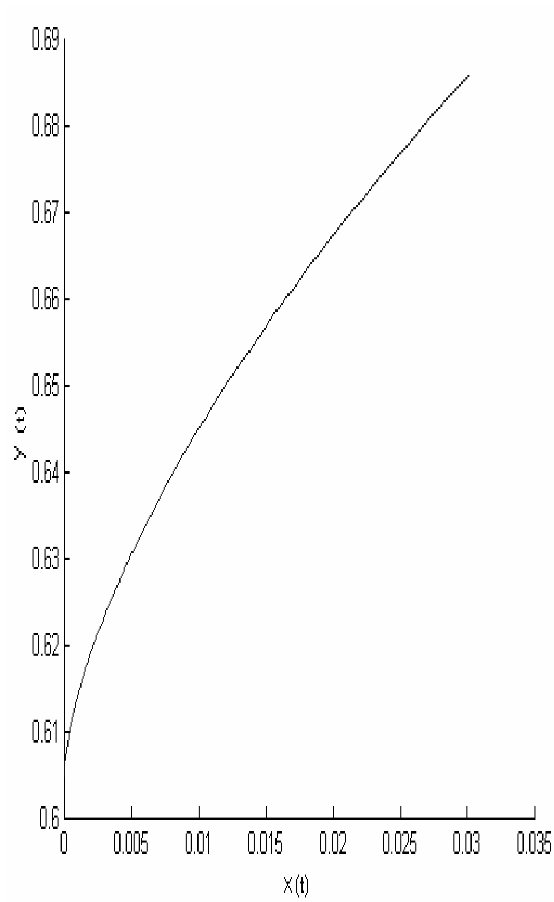

Fig. 3. Generated and true Involute profile for n+1= 12, k=4.

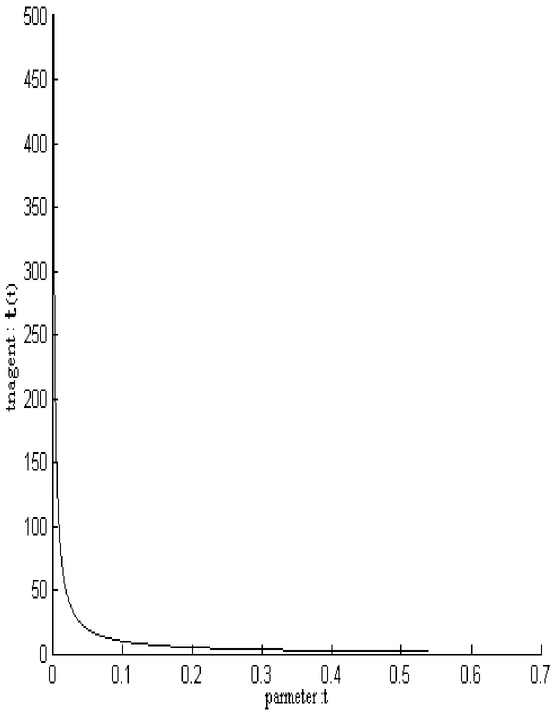

fig 4(a):Tangent plot for n=8,k=4.

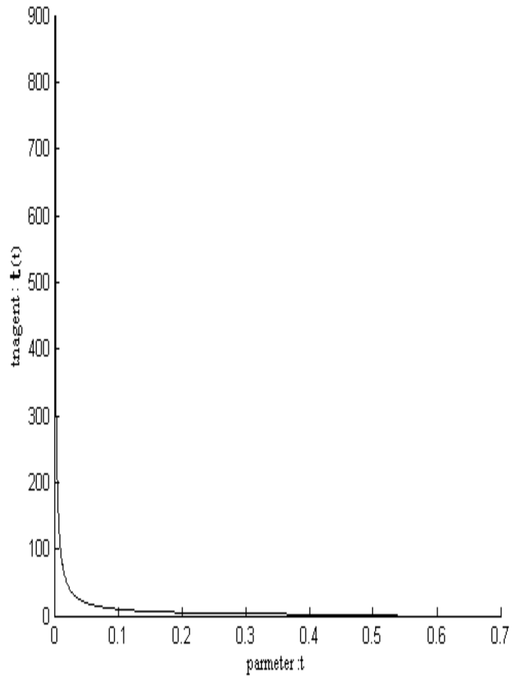

fig 4(b) :Tangent plot for n=11,k=4.

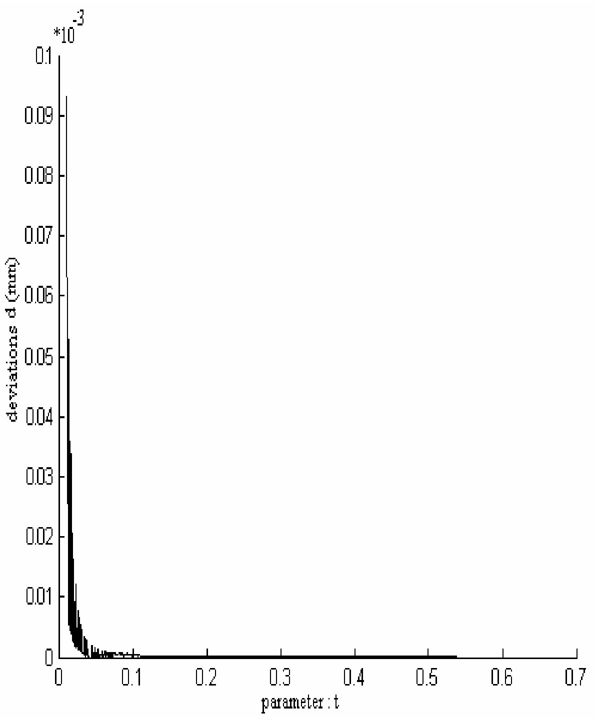

fig 4(c):Plot of deviations d (mm) between the slopes unit tangent vector of generated involute and unit tangent vector of theoretical involute for n+1=9 and k=4, with average deviation of slopes = 0.00007859.

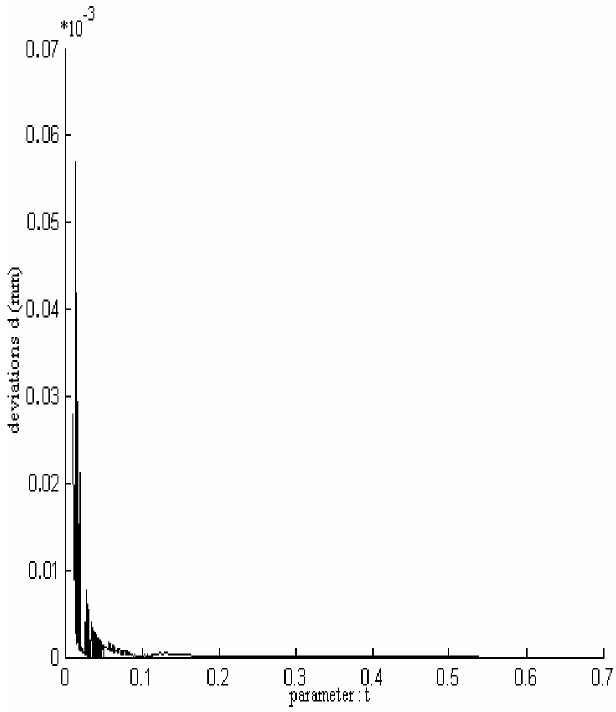

fig 4(d) Plot of deviations d (mm) between the slopes unit tangent vector of generated involute and unit tangent vector of theoretical involute for n+1=11 and k=4, with average deviation of slopes = 0.000049042

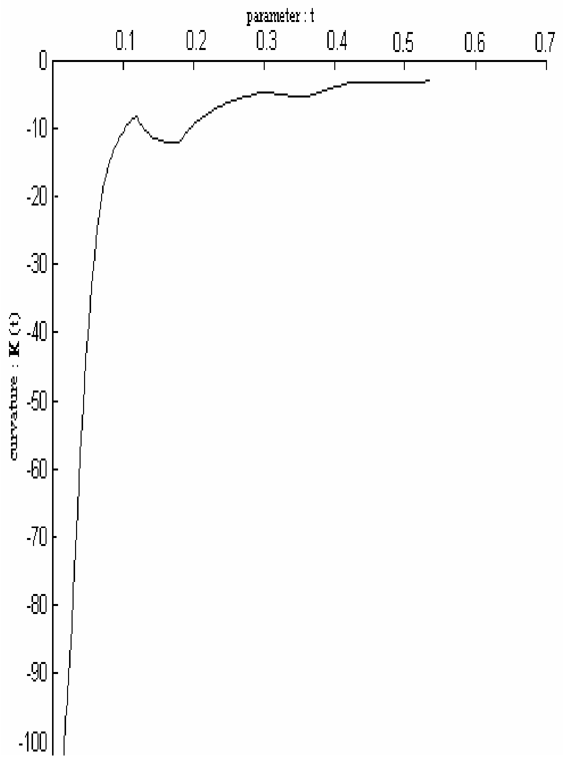

fig 5(a) :Curvature plot for n=8, k=4 of generated involute.

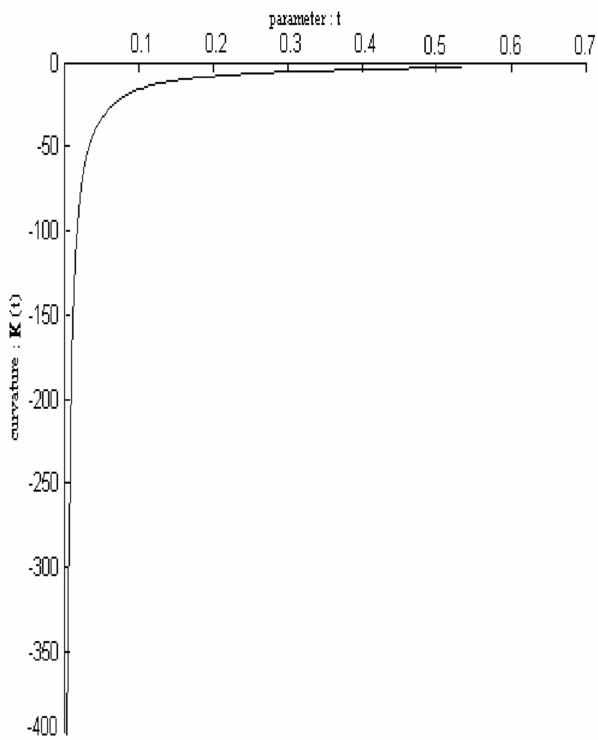

fig 5(b):Curvature plot for n=11 ,k=4 of generated involute

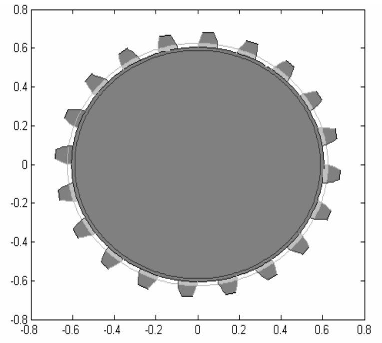

Fig 6: Generated Gear, two dimensional view.

VI. Conclusion

Thus, a generalized B-Spline curve in trigonometric form on circular domain is used to generate circle involute curve within the given tolerance limit also a spur gear is generated as an application. The error is much improved vis-à-vis S.Baron's method and other methods mentioned in the introduction. The error analysis has confirmed the validity of generated involute. The technique can be applied to various gear typologies

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