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Transient Combined Conduction and Radiation in a Two-Dimensional Participating Cylinder in Presence of Heat Generation

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Abstract**—**Simultaneous transient conduction and radiation heat transfer with heat generation is investigated. Analysis is carried out for both steady and unsteady situations. two-dimensional gray cylindrical enclosure with an absorbing, emitting, and isotropically scattering medium is considered. Enclosure boundaries are assumed at specified temperatures. The heat generation rate is considered uniform and constant throughout the medium. The lattice Boltzmann method (LBM) was used to solve the energy equation of a transient conduction-radiation heat transfer problem. The control volume finite element method (CVFEM) was used to compute the radiative information. To study the compatibility of the LBM for the energy equation and the CVFEM for the radiative transfer equation, transient conduction and radiation heat transfer problems in 2-D cylindrical geometries were considered. In order to establish the suitability of the LBM, the energy equation of the present problem was also solved using the the finite difference method (FDM) of the computational fluid dynamics. The CVFEM used in the radiative heat transfer was employed to compute the radiative information required for the solution of the energy equation using the LBM or the FDM (of the CFD). To study the compatibility and suitability of the LBM for the solution of energy equation and the CVFEM for the radiative information, results were analyzed for the effects of various parameters such as the boundary emissivity. The results of the LBM-CVFEM combination were found to be in excellent agreement with the FDM-CVFEM combination. The number of iterations and the steady state temperature in both of the combinations were found comparable. Results are found for situations with and without heat generation. Heat generation is found to have significant bearing on temperature distribution.

Keywords**—**heat generation, cylindrical coordinates; RTE; transient; coupled conduction radiation; heat transfer; CVFEM; LBM

I. Introduction

COUPLED transient conduction and radiation in a participating medium has numerous engineering applications in a variety of areas such as cylindrical metal-hydrogen reactor, thermal control by ceramics and low density refractory material, heat exchangers, manufacturing of glass and the window exposed to aerodynamic heating, heat pipes, gas turbine combustors, jet engines, the design of combustion chambers, rocket propulsion systems, glass manufactures, energy conservation and fibrous insulation. In this paper, we consider two dimensional transient conduction and radiation in a gray, absorbing, emitting, and isotropically scattering finite solid cylinder. The lattice Boltzmann method (LBM) has been increasingly applied in a variety of fluid mechanics [1–10], in simple and complex medium. Its application to heat transfer problems has been encouraging. Due to the direct discretization and the computational simplicity, ability and efficiency, the lattice Boltzmann method is considered the best alternative to traditional conventional computational fluid dynamics (CFD) solvers which basically solve the macroscopic transport equations of fluid flow, mass and heat transfer by directly discretizing them. The obtained partial differential equations are solved by finite difference methods (FDM), finite volume methods (FVM), etc. LBM is a mesoscopic approach inheriting many of the advantages of molecular dynamics and kinetic theories without using use complicated kinetic equations. The LBM include simple calculation procedure, efficient implementation for a parallel architecture, simplicity of boundary condition's implementation, easy and robust handling of complex geometries, and others [1–5]. In addition, the LBM is second-order accurate in time and space, which is sufficient for most engineering applications and, makes LBM competitive for complex medium. The LBM was found to provide accurate results and compatibilities of the LBM for solution of energy equation.Recently, numerous methods have been developed to solve the radiative transfer equation (RTE) in multidimensional cylindrical configurations. They include the spherical harmonic method [11], the zone method [12], the differential approximation method [13], the Galerkin finite element method [14], the discrete exchange factor method [15], the exact integral equation solutions [16, 17], the discontinuous Galerkin finite element method [18] and the finite element method [19]. The discrete ordinates method (DOM), originated by Carlson and Lathrop [20], has been widely used to solve radiative transfer problems in cylindrical configurations. Fiveland [21] and Jamaluddin and Smith [22] applied the DOM to combined heat transfer in axisymmetric geometries. Li et al. [23] studied the effect of albedo on the radiation in two-dimenional (2D) cylindrical geometry. The treatment of the radiative transfer in cylindrical enclosure by the DOM was provided by Jamaluddin and Smith [24]. A problem of unsteady cooling by conduction and radiation for a finite cylindrical medium when exposed to a rarefied cold environment was examined by Beak et al. [25] using the S4 approximation DOM. Many works have studied radiation

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problem in cylindrical enclosures using the finite volume method (FVM) [26-27]. The control volume finite element method (CVFEM) has been demonstrated to be successful in the solution of 2D [28,29] and 3D [30] rectangular enclosures, as well as for the unstructured mesh [31,32,33], and also for the solution of combined-mode heat transfer in participating media [34,35]. CVFEM is particularly a very promising approach for the solution of radiative transfer problems in cylindrical geometries. Ben Salah et al. [36] have proved its accuracy and its computational efficiency in the case of axisymmetric enclosures.The aim of this work is to combine the CVFEM which predict the axisymmetric radiative heat transfer in the cylindrical problem with the LBM and the FDM used for the corresponding energy equation. To that end, a benchmark problem dealing with transient conduction radiation heat transfer in a 2-D axisymmetric cylindrical enclosure is considered. The results obtained from the LBM-CVFEM and the FDM-CVFEM combinations are compared for the effects of various parameters, such as the effects of the emissivity and the presence of heat generation. The remainder of this article is divided into three sections. First we present the mathematical formulation and the relevant equations of the CVFEM to calculate radiative information required for the energy equation in the 2-D axisymmetric cylindrical. The mathematical and numerical formulations for the LBM approach are well illustrated next. Finally, the effectiveness of the obtained results associated to the parametric study is examined in last session.

II. Numerical Method

Equation governing unsteady heat transfer in a finite axisymmetric cylindrical medium, is as follow

$$\rho c_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial r^2} + \frac{k}{r}\frac{\partial T}{\partial r} + k\frac{\partial^2 T}{\partial z^2} - \nabla\overrightarrow{q_R} + q_g \quad (1)$$

Where ρ is the density, c_p is the specific heat, k is the thermal conductivity and q_g is the rate of heat generation. $\vec{q}_R$ represents the radiative heat flux which given by:

$$\vec{q}_R = \int_{4\pi} I\vec{\Omega}d\Omega \quad (2)$$

Where I is the radiative intensity which can be obtained by solving the Radiative Transfer Equation (RTE).

For an absorbing, emitting and scattering grey medium the RTE can be written as

$$\vec{\nabla}.(I(s,\vec{\Omega}).\vec{\Omega}) = -(k_a + k_d)I(s,\vec{\Omega}) + k_a I_b(s) + \frac{k_d}{4\pi}\int_{\Omega'=4\pi} I(s,\overrightarrow{\Omega'})P(\overrightarrow{\Omega'} \to \vec{\Omega})d\Omega' \quad (3)$$

where $I(s,\vec{\Omega})$ is the radiative intensity, which is a function of position s and direction $\vec{\Omega}$; k_a and k_d are absorption and scattering coefficients, respectively; $I_b(s)$ is the blackbody radiative intensity at the temperature of the medium; and $P(\overrightarrow{\Omega'} - \vec{\Omega})$ is the scattering phase function from the incoming $\overrightarrow{\Omega'}$ direction to the outgoing direction $\vec{\Omega}$. The term on the left-hand side represents the gradient of the intensity in the direction $\vec{\Omega}$. The three terms on the right-hand side represent the changes in intensity due to absorption and out-scattering, emission, and in-scattering, respectively. The radiative boundary condition for Eq. (3), when the wall bounding the physical domain is assumed grey and emits and reflects diffusely, can be expressed as

$$I_w(\vec{\Omega}) = \frac{\varepsilon_w \sigma T_w^4}{\pi} + \frac{1-\varepsilon_w}{\pi}\int_{\overrightarrow{\Omega'}.\vec{n}_w<0} I_w(\overrightarrow{\Omega'})\left|\overrightarrow{\Omega'}.\vec{n}_w\right| d\Omega' \text{ if } \overrightarrow{\Omega'}.\vec{n}_w > 0 \quad (4)$$

Where $\vec{n}_w$ is the unit normal vector on the wall and ε_w represents the wall emissivity.

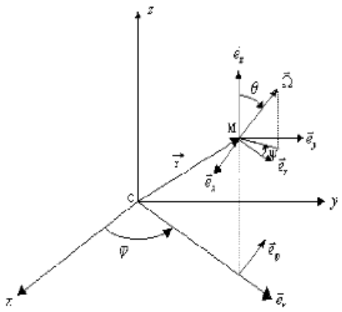

Fig. 1 Cartesian and cylindrical coordinates

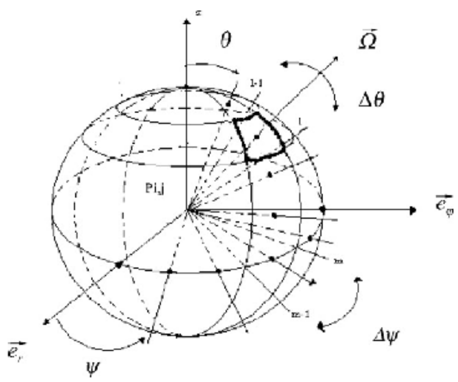

Fig. 2 Angular discretization

III. Control Volume Finite Element Method (CVFEM) Formulation

The control volume finite element method has been demonstrated to be successful in the solution of conductive radiative transfer in 2-D rectangular enclosures, two-dimensional participating cylinder [39-42]. In [39-42],

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CHAABANE *et al.* have studied this benchmark problem of coupled conductive radiative heat transfer and they found that the CVFEM is accurate and efficient. So , the CVFEM is used to discretize the RTE.

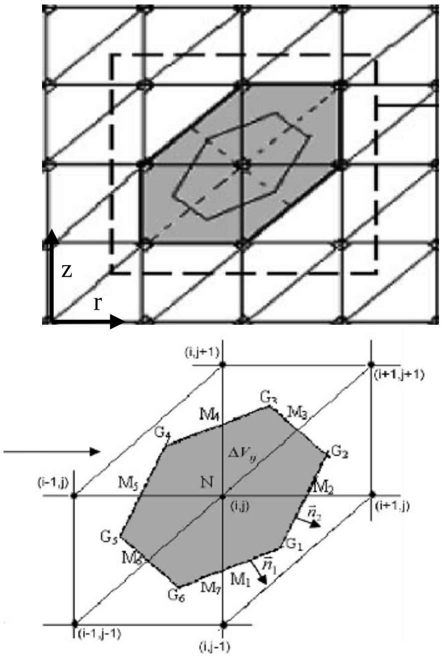

Fig. 3 Spatial discretization in ($\vec{e}_r, \vec{e}_z$) plane and control volume ΔV_{ik}

The surface A_{ik}^{l} of a sub-volume δV_{ik}^{l} is formed by four faces .

In the first, the radiative transfer equation integrated over both control volume and control solid angle gives:

$$\begin{aligned}&\int_{\Delta v_{ik}}\int_{\Delta\Omega^{mn}} \vec{\nabla}.(I(s,\vec{\Omega}).\vec{\Omega})d\Omega dv = \\ &-\int_{\Delta\Omega^{mn}}\int_{\Delta v_{ik}} (k_a+k_d)I(s,\vec{\Omega})dvd\Omega \\ &+\int_{\Delta\Omega^{mn}}\int_{\Delta v_{ik}} k_a I_b(s)d\Omega dV \\ &+\int_{\Delta\Omega^{mn}}\int_{\Delta v_{ik}} \frac{k_d}{4\pi}\int_{\Omega'=4\pi} I(s,\vec{\Omega'})P(\vec{\Omega'}\to\vec{\Omega})d\Omega' d\Omega dV\end{aligned} \quad (5)$$

To approximate the integrals that represent the extinction, emission and in-scattering contributions, the radiation intensity is considered constant within ΔV_{ik} and $\Delta\Omega^{mn}$ and is evaluated at the centroid of the control volume and at the centre direction of the control solid angle.

For the term on the left-hand side in eq. 5, the divergence theorem, the skew positive coefficient up wind (SPCU), and step schemes are used to cal cu late the corresponding quantity.

The final algebraic equation of the RTE is given by the following expression [7]:

$$\begin{aligned}&\gamma_{1ik}^{mn} I_{ik-1}^{mn} + \gamma_{2ik}^{mn} I_{i+1k}^{mn} + \gamma_{3ik}^{mn} I_{i+1k+1}^{mn} + \sum_{(m',n')=(1,1)}^{(N_\theta,N_\Psi)} \alpha_{ik}^{mnm'n'} I_{ik}^{m'n'} \\ &+\gamma_{4ik}^{mn} I_{ik+1}^{mn} + \gamma_{5ik}^{mn} I_{i-1k}^{mn} + \gamma_{6ik}^{mn} I_{i-1k-1}^{mn} = \beta_{ik}^{mn}\end{aligned} \quad (6)$$

Then, the algebraic eq. (6) is writ ten in the following matrix form [42]

$$AI = b \quad (7)$$

The obtained matrix sys tem is solved using the conditioned conjugate gradient squared method (CCGS). A de tailed calculation can be found in ref. [42]. The obtained equations are coupled and must be solved iteratively to yield the radiation and the temperature fields.

IV. Lattice Boltzmann method (LBM) formulation

The starting point of the LBM is the kinetic equation which for a two dimensional enclosure is given by [1-10]

$$\frac{\partial f_i(\vec{r},t)}{\partial t} + \vec{c}_i.\nabla f_i(\vec{r},t) = \Omega_i, \qquad i = 0,1,2,....,8 \quad (8)$$

where f_i is the particle distribution function denoting the number of particles at the lattice node $\vec{r} = \vec{r}(r,z)$ and time t moving in direction i with velocity $\vec{c}_i$ along the lattice link $\Delta\vec{r} = \vec{c}_i\Delta t$ connecting the nearest neighbours. The term Ω_i represents the local change in f_i due to particle collisions. For 2-D cylindrical geometry and taking into account the single time relaxation model of the Bhatanagar–Gross–Krook (BGK) approximation, the discrete Boltzmann equation is given by

$$\frac{\partial f_i(\vec{r},t)}{\partial t} + \vec{c}_i.\nabla f_i(\vec{r},t) = -\frac{1}{\tau}[f_i(\vec{r},t) - f_i^{eq}(\vec{r},t)] \quad (9)$$

where τ is the relaxation time and f^{eq} is the equilibrium distribution function.

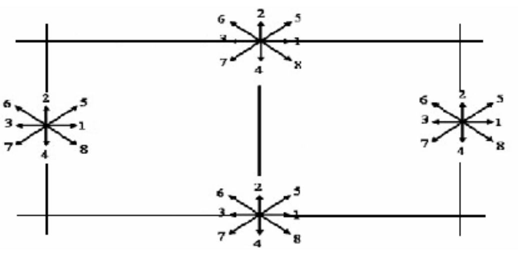

Fig. 5 Boundary conditions with known and unknown populations.

In heat transfer problems, the relaxation time τ for the D2Q9 lattice (Fig. 5) is computed from [5-9]

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$$\tau = \frac{3k}{(\rho c_p)c^2} + \frac{\Delta t}{2} \tag{10}$$

After discretization, Eq. (71) can be written as

$$f_i(\vec{r} + \vec{c}_i\Delta t, t + \Delta t) = f_i(\vec{r}, t) - \frac{\Delta t}{\tau}[f_i(\vec{r}, t) - f_i^{eq}(\vec{r}, t)] \tag{11}$$

$$f_i^{eq}(\vec{r}, t) = \omega_i T(\vec{r}, t) \tag{12}$$

In case of heat transfer problems, the temperature is obtained after summing f_i over all direction , i.e,

$$T(\vec{r}, t) = \sum_{i=0}^{8} f_i(\vec{r}, t) \tag{13}$$

To incorporate the volumetric radiation in the energy equation and the axisymmetric configuration, in the LBM formulation, Eq. (11) gets modified to

$$\begin{aligned} f_i(\vec{r} + \vec{c}_i\Delta t, t + \Delta t) &= f_i(\vec{r}, t) - \frac{\Delta t}{\tau}[f_i(\vec{r}, t) - f_i^{(0)}(\vec{r}, t)] \\ &+ \omega_i\left(\frac{\Delta t}{\rho c_p}\right)\left(\frac{\lambda}{r}\frac{\partial T}{\partial r} + g - div(\vec{q}_R)\right) \end{aligned} \tag{14}$$

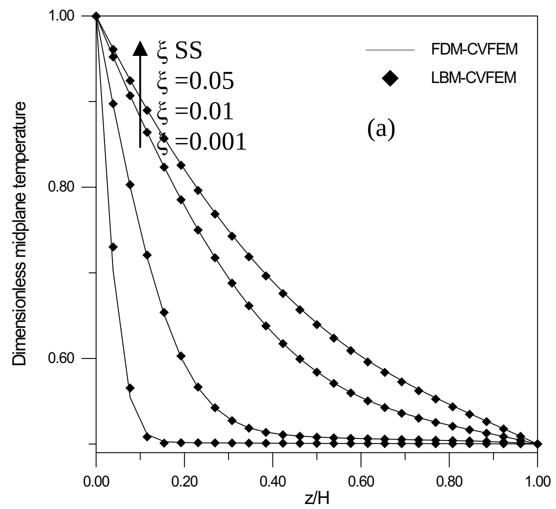
Eq. (14) is the equivalent form of the energy equation Eq. (1) in the LBM formulation, taking into account the presence of the volumetric radiation and the axisymmetric configuration. The boundary conditions are based on the properties of the known and unknown populations on each side as shown on figure 5. To express these conditions the bounce-back concept in the LBM in which particle fluxes are balanced at any point on the boundary was used.

V. RESULTS AND DISCUSSION

To illustrate the robustness of the present coupled formulation LBM-CVFEM and the FDM-CVFEM, we assume the following test case of cylindrical axisymmetric enclosure. It consists of a solid cylinder with a height H and a radius R containing an absorbing and emitting medium at constant temperature and with $\omega = 0.0$ and $\beta = 1.0$. The walls are black ($\varepsilon_w = 1$). The number of spatial nodes is fixed on $(N_r, N_z) = (17, 33)$ and the number of control angles is $(N_\theta, N_\psi) = (6,8)$. In the used new hybrid numerical approach, non-dimensional time step $\Delta\xi = 10^{-4}$ $(\xi = k\beta^2 t / \rho Cp)$ was considered and steady-state condition was assumed to have been achieved when the maximum variation in temperature at any location between two consecutive time levels satisfies the following constraint $\left|\theta_{ik}^{mn} - \theta_{ik}^{oldmn}\right| \le 10^{-4}$. In the test problem considered here, it is assumed that the bottom surface of the cylinder is hot and kept at a dimensionless temperature $T_b = T_{ref}$ of unity, the inner cylindrical surface is kept insulated, and the remaining surface are kept at temperature $T_i = T_b / 2$. We now present numerical results to demonstrate the effects of the conduction radiation parameter N, scattering albedo ω, asymmetry factor, and extinction coefficient β on temperature distribution and heat transfer. The surface emissivity plays an important role in the transport phenomena with regard to the contribution from surface radiation and during mathematical modeling it appears in the nonlinear boundary condition. So, the LBM-CVFEM and the FDM-CVFEM dimensionless mid-plane ($r / R = 0.5$) temperature results are compared in figure 6a-b for the effect of surface emissivity (ε). In figure 6, results have been compared for (a) $\varepsilon_1 = 0.1$, (b) $\varepsilon_1 = 1.0$ and $\varepsilon_2 = \varepsilon_3 = 1.0$. For these results $N = 0.10$, $\beta = 1.0$ and $\omega = 0.0$. For the same parameters, dimensionless mid-plane ($z / H = 0.5$) temperature results are compared in figure 7a-b for the effect of surface emissivity (ε). It is observed from figure 6 and 7 that with increase in emissivity, both the radiative transfer and core temperature increases as obvious. In this benchmark we notice that, in the vicinity of the hot surface and the cold surface the temperature increases with the increase of the emissivity. The effects of the reflection effects are pronounced both in vicinity of the cold and hot surface. Table I highlight the number of iterations required to obtain steady-state dimensionless temperature T / T_{ref} with FDM-CVFEM and LBM-CVFEM, for different dimensionless heat generation parameter $g*$. In the present case, results of the FDM-CVFEM are in excellent agreement with the LBM-CVFEM results.

TABLE I
COMPARISON OF THE NUMBER OF ITERATIONS REQUIRED TO OBTAIN STEADY-STATE DIMENSIONLESS TEMPERATURE WITH FDM-CVFEM AND LBM-CVFEM, WITH ($\beta = 1.0$, $\omega = 0.5$, $N = 0.1$) FOR DIFFERENT $g*$

g^*	r/R = 0.50, z/H = 0.5		r/R = 0.75, z/H = 0.5	
	FDM-CVFEM	LBM-CVFEM	FDM CVFEM	LBM -CVFEM
0	0.658 (955)	0.656 (948)	0.574	0.572
2	0.785 (1119)	0.784 (1116)	0.689	0.689
5	0.94 (1172)	0.945 (1170)	0.841	0.841



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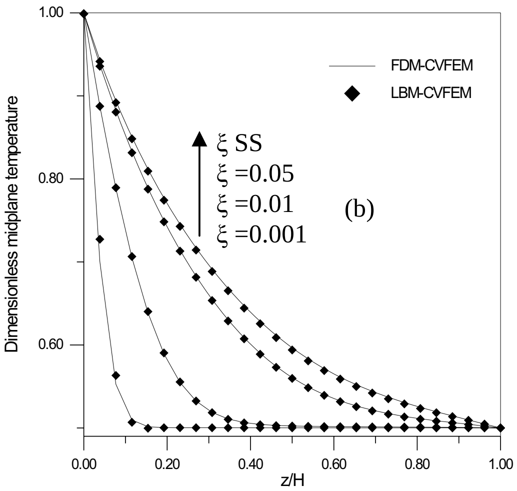


Fig. 6 Comparison of dimensionless mid-plane ($r/R = 0.5$) temperature for $N = 0.10$, $\beta = 1.0$, $\omega = 0.0$, (a) $\varepsilon_1 = 0.1$,(b) $\varepsilon_1 = 1.0$ and $\varepsilon_2 = \varepsilon_3 = 1.0$

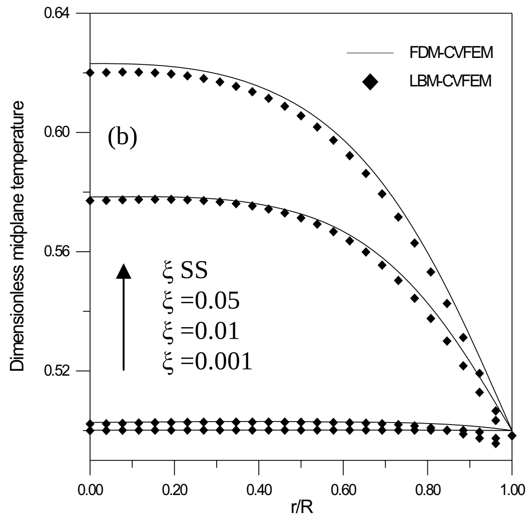
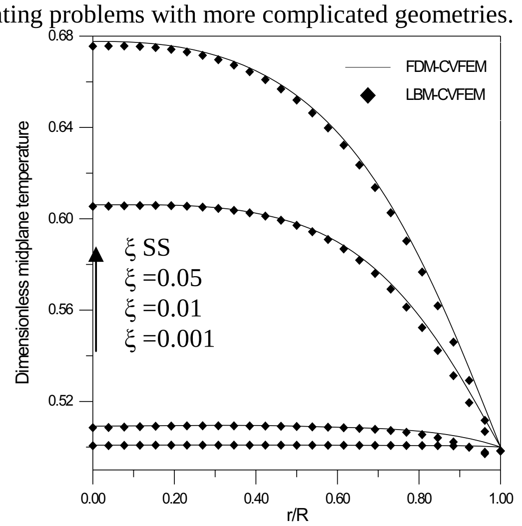


Fig. 7 Comparison of dimensionless mid-plane ($z/H = 0.5$) temperature for $N = 0.10$, $\beta = 1.0$, $\omega = 0.0$,(a) $\varepsilon_1 = 0.1$,(b) $\varepsilon_1 = 1.0$ and $\varepsilon_2 = \varepsilon_3 = 1.0$

VI. Conclusion

In this paper, the lattice Boltzmann method-control volume finite element method (LBM-CVFEM) is successfully applied to solve and analyse the transient combined conductive radiative heat transfer problem in an axisymmetric cylindrical geometry with absorbing, emitting and scattering medium. To compare the performance and the workability of the LBM-CVFEM hybrid method, the problems were also solved using the FDM-CVFEM combinations. The present work provides a solution of the heat transfer in a cylindrical enclosure with black or gray diffuse emitting and reflecting walls containing an absorbing–emitting–scattering medium. The CVFEM model was applied to study the influence of wall emissivity and heat generation on the temperature distribution in the emitting–absorbing–scattering medium. The obtained results show that the present coupled CVFEM with the LBM could not only predict the coupled transient radiative conductive heat transfer in participating media accurately but also be flexible in treating problems with more complicated geometries.



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