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An Anisotropic Model of Damage and Unilateral Effect for Brittle Materials

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***Abstract*—**This work deals with the initial applications and formulation of an anisotropic plastic-damage constitutive model proposed for non-linear analysis of reinforced concrete structures submitted to a loading with change of the sign. The original constitutive model is based on the fundamental hypothesis of energy equivalence between real and continuous medium following the concepts of the Continuum Damage Mechanics. The concrete is assumed as an initial elastic isotropic medium presenting anisotropy, permanent strains and bimodularity (distinct elastic responses whether traction or compression stress states prevail) induced by damage evolution. In order to take into account the bimodularity, two damage tensors governing the rigidity in tension or compression regimes are introduced. Then, some conditions are introduced in the original version of the model in order to simulate the damage unilateral effect. The three-dimensional version of the proposed model is analyzed in order to validate its formulation when compared to micromechanical theory. The one-dimensional version of the model is applied in the analyses of a reinforced concrete beam submitted to a loading with change of the sign. Despite the parametric identification problems, the initial applications show the good performance of the model.

***Keywords*—**Damage model, plastic strain, unilateral effect.

I. Introduction

THE Continuum Damage Mechanics (CDM) has already proved to be a suitable tool for simulating the material deterioration in equivalent continuous media due exclusively to microcracking process. In this work, for modeling the concrete behaviour, it can be assumed that the concrete belongs to the category of materials which can be considered initially isotropic and unimodular presenting different behaviours in tension and compression when damaged. A formulation of constitutive laws for isotropic and anisotropic elastic materials presenting different behaviours in tension and compression under small deformations was proposed in [1] for two and three-dimensional cases. The authors have considered a bimodular hyperelastic material defining an elastic potential energy density W which must be once continuously differentiable (whole wise), but only piecewise twice continuously differentiable. In this way, the model is able to produce different response in tension and compression. Reference [2] has extended the formulation [1] in order to take into account the damage effects. Accordingly with, the bulk (λ_{ab}) and shear (μ_a) modulus are considered as functions of the damage state, so that the stress-strain relationship would be influenced by damage variables. Moreover, the hypersurface $g(\varepsilon, D_i)$ adopted as the criterion for identification of the constitutive responses in compression or tension would be also influenced by the damage variables. Then, a constitutive model for the concrete was derived from the formulation [2]. The concrete is initially considered as an isotropic continuous medium with anisotropy (transverse isotropy) and bimodularity induced by the damage. On one side the class of anisotropy induced and considered in the model elapses from the assumption that locally the loaded concrete always presents damage distribution oriented diffusely as appointed by experimental observations.

In this work, an improved version of the damage model taking into accounting the unilateral effect is proposed and critically discussed. The original version of the damage model is bimodular in the sense that presents different elasticity tensors in tension and compression. Thus, the model is potentially capable to simulate the stiffness recovery when the medium is submitted to a loading with change of the sign that evidences the transition from predominant regimes of tension to compression, i. e., the so-called unilateral behavior of the damaged concrete. However, the model is not capable to simulate the influence of the previous damage processes in compression (diffuse damage) when there is the transition from predominant regimes of compression to tension [3]. Therefore, to avoid this problem a new elasticity tensor is proposed in this work. Many different strategies are possible and have been proposed in the literature to model the stiffness recovery as [3]-[8]. From a micromechanics point of view this is due to the partial closure of micro-cracks loaded in compression which affect less the elasticity moduli in compression than in tension [6]. On the other hand, [7] and [8] suggest that despite the recent progresses in the macroscopic modeling of the unilateral effect, this subject still remains as an open research field when it deals with induced anisotropy damage models. Indeed, this work intends to contribute to the modeling of damage unilateral effect. However, it must be noted that the proposed model is not capable to take into account the friction effects, namely blocking and dissipative sliding of closed microcrack lips. This feature can be discussed in future works.

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II. UNILATERAL DAMAGE MODEL

The original damage model formulation is built from the formalism presented in [2]. Moreover, the model respects the principle of energy equivalence between damaged real medium and equivalent continuous medium established in the CDM. In this work the definition of that tensor follows a so-called scalar form expressed as: $\boldsymbol{D} = f_j(D_i)\,\boldsymbol{M_j}$, where $f_j(D_i)$ are scalar valued functions of the damage scalar variables D_i and $\boldsymbol{M_j}$ are anisotropic tensors. In the case of this model, the particular adopted tensors to $\boldsymbol{M_j}$ are the ones that allow representing the transverse isotropy. Then, for dominant tension states, a scalar damage tensor is proposed:

$$\boldsymbol{D_T}=f_1(D_1,D_4,D_5)\,(\boldsymbol{A}\otimes\boldsymbol{A})+2f_2(D_4,D_5)\,[(\boldsymbol{A}\overline{\underline{\otimes}}\boldsymbol{I}+\boldsymbol{I}\overline{\underline{\otimes}}\boldsymbol{A})-(\boldsymbol{A}\otimes\boldsymbol{A})] \qquad (1)$$

where $f_1(D_1, D_4, D_5) = D_1 - 2\,f_2(D_4, D_5)$ and $f_2(D_4, D_5) = 1 - (1\text{-}D_4)\ (1\text{-}D_5)$. The variable D_1 represents the damage in direction orthogonal to the transverse isotropy local plane of the material, while D_4 is representative of the damage due to the sliding movement between the crack faces. The third damage variable, D_5, is only activated if a previous compression state accompanied by damage has occurred. In (1), the tensor $\boldsymbol{I}$ is the second-order identity tensor and the tensor $\boldsymbol{A}$, is formed by dyadic product of the unit vector perpendicular to the transverse isotropy plane for himself. Those products are given in [2]. For dominant compression states, it is proposed the other damage tensor:

$$\boldsymbol{D_C}= f_1^{*}(D_2,D_4,D_5)\,(\boldsymbol{A}\otimes\boldsymbol{A})+f_2(D_3)\,[(\boldsymbol{I}\overline{\otimes}\boldsymbol{I})-(\boldsymbol{A}\otimes\boldsymbol{A})] +2f_3(D_4,D_5)\,[(\boldsymbol{A}\overline{\underline{\otimes}}\boldsymbol{I}+\boldsymbol{I}\overline{\underline{\otimes}}\boldsymbol{A})-(\boldsymbol{A}\otimes\boldsymbol{A})] \qquad (2)$$

where $f_1^{*}(D_2, D_4, D_5) = D_2 - 2f_3(D_4, D_5)$,$f_2(D_3) = D_3$ and $f_3(D_4,D_5)=1-(1\text{-}D_4)(1\text{-}D_5)$. Note that the compression damage tensor introduces two additional scalar variables in its composition: D_2 and D_3. The variable D_2 (damage perpendicular to the transverse isotropy local plane) reduces the Young's modulus in that direction and in conjunction to D_3 (that represents the damage in the transverse isotropy plane) degrades the Poisson's ratio throughout the perpendicular planes to the one of transverse isotropy.

On the other hand, the constitutive tensor is written as:

$$\boldsymbol{E}\,(\boldsymbol{\varepsilon}) := \begin{cases} E_{-}(\varepsilon)\ \text{if}\ \ g\,(\varepsilon, D_T, D_C) < 0, \\ E_{+}(\varepsilon)\ \text{if}\ \ g\,(\varepsilon, D_T, D_C) > 0, \end{cases} \qquad (3)$$

$$\boldsymbol{E}_{+}(\boldsymbol{\varepsilon}) = \lambda_{11}[I\otimes I] + 2\mu_1[I\overline{\underline{\otimes}}I] - \lambda_{22}^{+}(D_1,D_4,D_5)\,[A\otimes A] - \lambda_{12}^{+}(D_1)\,[\boldsymbol{A}\otimes\boldsymbol{I}+\boldsymbol{I}\otimes\boldsymbol{A}] - \mu_2(D_4,D_5)\ [\boldsymbol{A}\overline{\underline{\otimes}}\boldsymbol{I}+\boldsymbol{I}\overline{\underline{\otimes}}\boldsymbol{A}] \qquad (4)$$

$$\boldsymbol{E}_{-}(\boldsymbol{\varepsilon}) = \lambda_{11}[\boldsymbol{I}\otimes\boldsymbol{I}] + 2\mu_1[\boldsymbol{I}\overline{\underline{\otimes}}\boldsymbol{I}] - \lambda_{22}^{-}(D_2,D_3,D_4,D_5)\,[A\otimes A] - \lambda_{12}^{-}(D_2,D_3)\,[\boldsymbol{A}\otimes\boldsymbol{I}+\boldsymbol{I}\otimes\boldsymbol{A}] - \lambda_{11}^{-}(D_3)\,[\boldsymbol{I}\otimes\boldsymbol{I}] - \frac{(1-2\nu_0)}{\nu_0}\lambda_{11}^{-}(D_3)\,[\boldsymbol{I}\overline{\otimes}\boldsymbol{I}] - \mu_2(D_4,D_5)\ [\boldsymbol{A}\overline{\underline{\otimes}}\boldsymbol{I}+\boldsymbol{I}\overline{\underline{\otimes}}\boldsymbol{A}] \qquad (5)$$

Note that for null values of the damage variables, the material behaves as isotropic and unimodular medium, where $\lambda_{11} = \lambda_0$ and $\mu_1 = \mu_0$ are Lamè constants. The remaining parameters will only exist for no-null damage [2].

The hyperplane $g(\boldsymbol{\varepsilon},\boldsymbol{D})$ is defined by the unit normal $\boldsymbol{N}$ and characterized by its dependence of both the strain and damage states [2]. To simplify the presentation, the hyperplane will be here expressed as the one obtained by enforcing the direction *1* in the strain space to be perpendicular to the transverse isotropy local plane. Thus, the hyperplane is given by:

$$g(\boldsymbol{\varepsilon},\boldsymbol{D_T},\boldsymbol{D_C})=\boldsymbol{N}(\boldsymbol{D_T},\boldsymbol{D_C}).\boldsymbol{\varepsilon}^{e}=\gamma_1(D_1,D_2)\,\varepsilon_V^{e}+\gamma_2(D_1,D_2)\,\varepsilon_{11}^{e} \qquad (6)$$

where $\gamma_1(D_1,D_2)=\{1+H(D_2)[H(D_1)\text{-}1]\}\,\eta(D_1)+\{1+H(D_1)[H(D_2)\text{-}1]\}\,\eta(D_2)$ and $\gamma_2(D_1,D_2)=D_1+D_2$. The $H(D_i)$ are Heaveside functions ($H(D_i)=1$ for $D_i > 0$ and $H(D_i) = 0$ for $D_i = 0$ $(i = 1, 2)$). The $\eta(D_i)$ functions are defined, respectively, for the tension and compression cases, assuming for the first one that there was no previous damage of compression affecting the present damage variable D_1 and analogously, for the second one that has not had previous damage of tension affecting variable D_2 and they are given by:

$$\eta\,(D_1) = \frac{-D_1+\sqrt{3-2D_1^2}}{3}\ ;\ \eta\,(D_2) = \frac{-D_2+\sqrt{3-2D_2^2}}{3} \qquad (7)$$

Due to anisotropy induced by damage, it is convenient to separate the damage criteria into two: the first one is only used to indicate damage beginning, or that the material is no longer isotropic and the second one is used for loading and unloading when the material is already considered as transverse isotropic. This second criterion identifies if there is or not evolution of the damage variables. That division is justified by the difference between the complementary elastic strain energies of isotropic and transverse isotropic material. For more details see [2] and [9].

If there is damage evolution, i. e., when $\dot{\boldsymbol{D}}_{\boldsymbol{T}} \neq \boldsymbol{0}$ or $\dot{\boldsymbol{D}}_{\boldsymbol{C}} \neq \boldsymbol{0}$, the evolution laws of the damage variables are written as associated variables functions. Considering just the case of monotonic loading, the evolution laws proposed for the scalar damage variables are resulting of fittings on experimental results and present similar characteristics to those one described in [4]. The general form proposed is:

$$D_i = 1 - \frac{1 + A_i}{A_i + exp\left[B_i(Y_i - Y_{0i})\right]} \quad i = 1,\, 5 \tag{8}$$

where A_i, B_i and Y_{0i} are parameters of the model that must be identified through the uniaxial tension and compression tests and biaxial compression tests.

When the damage process is activated, the formulation starts to involve the tensor $\boldsymbol{A}$ that depends on the normal to the transverse isotropy plane. Therefore, it is necessary to establish some rules to identify its location for an actual strain state. Initially, it is established a general criterion for the existence of the transverse isotropy plane. In [9] is proposed that the transverse isotropy due to damage only arises if positive strain rates exist at least in one of the principal directions. After assuming such proposition as valid, some rules to identify its location are defined.

The one-dimensional version of the model takes into account permanent strains induced by damage evolution. Assuming, for simplicity, that the permanent strains are composed exclusively by volumetric strains, as it has already been considered in others works [3], and taking into account the unilateral effect, the evolution law results:

$$\dot{\varepsilon}^p = \left(\frac{\beta_1}{\left(1 - D_1\right)^2}\dot{D}_1 + \frac{\beta_2}{\left(1 - D_2\right)^2}\dot{D}_2 \right)\boldsymbol{I} \tag{9}$$

Observe that β_1 and β_2 are parameters directly related to the evolutions of permanent strains induced by damage in tension and in compression, respectively.

The original version of the damage model [2] is bimodular, however it is necessary to take into account the diffuse damage generated in previous compression regimes when dealing to tension regimes. This problem can be solved by introduction of a new elasticity tensor in dominant tension regimes. Therefore, respecting the principle of energy equivalence, the new constitutive tensor is written as:

$$\boldsymbol{E_T} = (\boldsymbol{I} - \boldsymbol{D_C^*})(\boldsymbol{I} - \boldsymbol{D_T})\boldsymbol{E}_0(\boldsymbol{I} - \boldsymbol{D_T})(\boldsymbol{I} - \boldsymbol{D_C^*}) \tag{10}$$

Note that the damage tensor in compression $\boldsymbol{D_C^*}$ should be in the composition of the constitutive tensor in dominant tension regimes. In this work, an alternative form for the damage tensor $\boldsymbol{D_C^*}$ is presented in order to take into account the damage processes generated in previous compression. This tensor is given by:

$$\boldsymbol{D_C^*} = f_1(D_2)\,(\boldsymbol{A} \otimes \boldsymbol{A}) + f_2(D_3)\,[(\boldsymbol{I} \bar{\otimes} \boldsymbol{I}) - (\boldsymbol{A} \otimes \boldsymbol{A})] \tag{11}$$

where $f_1(D_2) = D_2$ and $f_2(D_3) = D_3$. It is important to observe that the damage tensor $\boldsymbol{D_C^*}$ provides the diffuse damage in previous compression states through the changing of the volumetric modulus, as proposed in [3]. Then, following the formalism presented in [2], the unilateral damage model proposed in this work is written as:

$$W(\boldsymbol{\varepsilon}) = \rho\psi(\boldsymbol{\varepsilon}) := \begin{cases} W_-(\boldsymbol{\varepsilon}) \text{ if } \boldsymbol{g}(\boldsymbol{\varepsilon}, \boldsymbol{D_T}, \boldsymbol{D_C}) < 0, \\ W_+(\boldsymbol{\varepsilon}) \text{ if } \boldsymbol{g}(\boldsymbol{\varepsilon}, \boldsymbol{D_T}, \boldsymbol{D_C}) > 0, \end{cases} \tag{12}$$

$$\begin{aligned} W_+ = \rho\psi_+(\boldsymbol{\varepsilon}) = {} & \frac{\lambda_{11}}{2} tr^2(\boldsymbol{\varepsilon}) + \mu_1 tr(\boldsymbol{\varepsilon}^2) - \frac{\lambda_{22}^+(D_1, D_2, D_3, D_4, D_5)}{2} tr^2(\boldsymbol{A\varepsilon}) \\ & - \lambda_{12}^+(D_1, D_2, D_3) tr(\boldsymbol{\varepsilon}) tr(\boldsymbol{A\varepsilon}) - \frac{\lambda_{11}^-(D_3)}{2} tr^2(\boldsymbol{\varepsilon}) \\ & - \frac{(1 - 2\nu_0)}{2\nu_0} \lambda_{11}^-(D_3) tr[(\boldsymbol{I} \bar{\otimes} \boldsymbol{I})\boldsymbol{\varepsilon}]^2 - \mu_2(D_4, D_5)\, tr(\boldsymbol{A\varepsilon}^2) \end{aligned} \tag{13}$$

$$\begin{aligned} W_- = \rho\psi_-(\boldsymbol{\varepsilon}) = {} & \frac{\lambda_{11}}{2} tr^2(\boldsymbol{\varepsilon}) + \mu_1 tr(\boldsymbol{\varepsilon}^2) - \frac{\lambda_{22}^-(D_2, D_3, D_4, D_5)}{2} \\ & tr^2(\boldsymbol{A\varepsilon}) - \lambda_{12}^-(D_2, D_3) tr(\boldsymbol{\varepsilon}) tr(\boldsymbol{A\varepsilon}) - \frac{\lambda_{11}^-(D_3)}{2} tr^2(\boldsymbol{\varepsilon}) \\ & - \frac{(1 - 2\nu_0)}{2\nu_0} \lambda_{11}^-(D_3) tr[(\boldsymbol{I} \bar{\otimes} \boldsymbol{I})\boldsymbol{\varepsilon}]^2 - \mu_2(D_4, D_5)\, tr(\boldsymbol{A\varepsilon}^2) \end{aligned} \tag{14}$$

Now, the parameters λ_{ij} and μ_I are given by:

$$\begin{aligned} & \lambda_{22}^+(D_1, D_2, D_3, D_4, D_5) = (\lambda_0 + 2\mu_0)(2D_1 - D_1^2) \\ & - 2\lambda_{12}^+(D_1, D_2, D_3) - 2\mu_2(D_4, D_5) + \frac{(\nu_0 - 1)}{\nu_0}\lambda_{11}^-(D_3) \\ & + (\lambda_0 + 2\mu_0)\left[(1 - D_1)^2 - (1 - D_1)^2(1 - D_2)^2\right] \\ & \lambda_{12}^+(D_1, D_2, D_3) = \lambda_0\left[(1 - D_3)^2 - (1 - D_1)(1 - D_2)(1 - D_3)\right] \\ & \mu_2(D_4, D_5) = 2\mu_0[1 - (1 - D_4)^2(1 - D_5)^2] \\ & \lambda_{22}^-(D_2, D_3, D_4, D_5) = (\lambda_0 + 2\mu_0)(2D_2 - D_2^2) - 2\lambda_{12}^-(D_2, D_3) \\ & + \frac{(\nu_0 - 1)}{\nu_0}\lambda_{11}^-(D_3) - 2\mu_2(D_4, D_5) \\ & \lambda_{12}^-(D_2, D_3) = \lambda_0[(1 - D_3)^2 - (1 - D_2)(1 - D_3)] \\ & \lambda_{11}^-(D_3) = \lambda_0(2D_3 - D_3^2) \end{aligned} \tag{15}$$

The stress tensor is obtained from the gradient of the elastic potential, as follows:

$$\boldsymbol{\sigma}(\boldsymbol{\varepsilon}) = \begin{cases} \sigma_-(\varepsilon) = \nabla_\varepsilon \rho\psi_-(\varepsilon) \text{ if } g(\boldsymbol{\varepsilon}, \boldsymbol{D_T}, \boldsymbol{D_C}) < 0, \\ \sigma_+(\varepsilon) = \nabla_\varepsilon \rho\psi_+(\varepsilon) \text{ if } g(\boldsymbol{\varepsilon}, \boldsymbol{D_T}, \boldsymbol{D_C}) > 0, \end{cases} \tag{16}$$

$$\begin{aligned} \boldsymbol{\sigma}_+(\boldsymbol{\varepsilon}) = {} & \lambda_{11} tr(\boldsymbol{\varepsilon})\boldsymbol{I} + 2\mu_1\boldsymbol{\varepsilon} - \lambda_{22}^+(D_1, D_2, D_3, D_4, D_5)\, tr(\boldsymbol{A\varepsilon})\boldsymbol{A} \\ & - \lambda_{12}^+(D_1, D_2, D_3)\,(tr(\boldsymbol{\varepsilon})\boldsymbol{A} + tr(\boldsymbol{A\varepsilon})\boldsymbol{I}) - \lambda_{11}^-(D_3)\, tr(\boldsymbol{\varepsilon})\boldsymbol{I} \\ & - \frac{(1 - 2\nu_0)}{\nu_0}\lambda_{11}^-(D_3)\,(\boldsymbol{I} \bar{\otimes} \boldsymbol{I})\boldsymbol{\varepsilon} - \mu_2(D_4, D_5)\,(\boldsymbol{A\varepsilon} + \boldsymbol{\varepsilon A}) \end{aligned} \tag{17}$$

$$\begin{aligned} \boldsymbol{\sigma}_-(\boldsymbol{\varepsilon}) = {} & \lambda_{11} tr(\boldsymbol{\varepsilon})\boldsymbol{I} + 2\mu_1\boldsymbol{\varepsilon} - \lambda_{22}^-(D_2, D_3, D_4, D_5)\, tr(\boldsymbol{A\varepsilon})\boldsymbol{A} \\ & - \lambda_{12}^-(D_2, D_3)\,(tr(\boldsymbol{\varepsilon})\boldsymbol{A} + tr(\boldsymbol{A\varepsilon})\boldsymbol{I}) - \lambda_{11}^-(D_3)\, tr(\boldsymbol{\varepsilon})\boldsymbol{I} \end{aligned}$$

$$-\frac{(1-2\nu_0)}{\nu_0}\lambda^-_{11}(D_3)\,(\mathbf{I}\overline{\otimes}\mathbf{I})\boldsymbol{\varepsilon}-\mu_2(D_4,D_5)\,(\mathbf{A}\boldsymbol{\varepsilon}+\boldsymbol{\varepsilon}\mathbf{A}) \quad (18)$$

The constitutive tensor is also obtained from the elastic potential, i. e.:

$$\mathbf{E}\,(\boldsymbol{\varepsilon}):=\begin{cases}\mathbf{E}_-(\boldsymbol{\varepsilon})=\nabla^2_{\varepsilon}\rho\psi_-\,(\boldsymbol{\varepsilon}) & \text{if}\quad g\,(\boldsymbol{\varepsilon},\mathbf{D_T},\mathbf{D_C})<0,\\ \mathbf{E}_+(\boldsymbol{\varepsilon})=\nabla^2_{\varepsilon}\rho\psi_+\,(\boldsymbol{\varepsilon}) & \text{if}\quad g\,(\boldsymbol{\varepsilon},\mathbf{D_T},\mathbf{D_C})>0,\end{cases} \quad (19)$$

$$\mathbf{E}_+(\boldsymbol{\varepsilon})=\mathbf{E_T}=\lambda_{11}[\mathbf{I}\otimes\mathbf{I}]+2\mu_1[\mathbf{I}\underline{\overline{\otimes}}\mathbf{I}]$$
$$-\lambda^+_{22}(D_1,D_2,D_3,D_4,D_5)\,[\mathbf{A}\otimes\mathbf{A}]$$
$$-\lambda^+_{12}(D_1,D_2,D_3)\,[\mathbf{A}\otimes\mathbf{I}+\mathbf{I}\otimes\mathbf{A}]-\lambda^-_{11}(D_3)\,[\mathbf{I}\otimes\mathbf{I}]$$
$$-\frac{(1-2\nu_0)}{\nu_0}\lambda^-_{11}(D_3)[\mathbf{I}\overline{\otimes}\mathbf{I}]-\mu_2(D_4,D_5)\,[\mathbf{A}\underline{\overline{\otimes}}\mathbf{I}+\mathbf{I}\underline{\overline{\otimes}}\mathbf{A}] \quad (20)$$

$$\mathbf{E}_-(\boldsymbol{\varepsilon})=\mathbf{E_C}=\lambda_{11}[\mathbf{I}\otimes\mathbf{I}]+2\mu_1[\mathbf{I}\underline{\overline{\otimes}}\mathbf{I}]$$
$$-\lambda^-_{22}(D_2,D_3,D_4,D_5)\,[\mathbf{A}\otimes\mathbf{A}]$$
$$-\lambda^-_{12}(D_2,D_3)[\mathbf{A}\otimes\mathbf{I}+\mathbf{I}\otimes\mathbf{A}]-\lambda^-_{11}(D_3)\,[\mathbf{I}\otimes\mathbf{I}]$$
$$-\frac{(1-2\nu_0)}{\nu_0}\lambda^-_{11}(D_3)[\mathbf{I}\overline{\otimes}\mathbf{I}]-\mu_2(D_4,D_5)\,[\mathbf{A}\underline{\overline{\otimes}}\mathbf{I}+\mathbf{I}\underline{\overline{\otimes}}\mathbf{A}] \quad (21)$$

Taking into account the unilateral effects and assuming that direction 1 in the strain space be perpendicular to the transverse isotropy local plane, the complementary elastic energy of the damaged medium in dominant tension regimes is now expressed by:

$$W^*_{e+}=\frac{\sigma^2_{11}}{2E_0(1-D_1)^2(1-D_2)^2}+\frac{(\sigma^2_{22}+\sigma^2_{33})}{2E_0(1-D_3)^2}$$
$$-\frac{\nu_0(\sigma_{11}\sigma_{22}+\sigma_{11}\sigma_{33})}{E_0(1-D_1)(1-D_2)(1-D_3)}-\frac{\nu_0\sigma_{22}\sigma_{33}}{E_0(1-D_3)^2}$$
$$+\frac{(1+\nu_0)}{E_0(1-D_4)^2(1-D_5)^2}(\sigma^2_{12}+\sigma^2_{13})+\frac{(1+\nu_0)}{E_0}\sigma^2_{23} \quad (22)$$

The variables associated to damage variables in tension with damage activated in previous compression will also be modified, because they are obtained from the elastic potential (22). Therefore, the following relationships are valid:

$$Y_T=\frac{\partial W^*_{e+}}{\partial D_1}+\frac{\partial W^*_{e+}}{\partial D_4}=Y_1+Y_4 \quad (23)$$

$$Y_1=\frac{\sigma^2_{11}}{E_0(1-D_1)^3(1-D_2)^2}-\frac{\nu_0(\sigma_{11}\sigma_{22}+\sigma_{11}\sigma_{33})}{E_0(1-D_1)^2(1-D_2)(1-D_3)} \quad (24)$$

$$Y_4=\frac{(1+\nu_0)}{E_0(1-D_4)^3(1-D_5)^2}(2\sigma^2_{12}+2\sigma^2_{13}) \quad (25)$$

Note that just Y_1 must to take into account the diffuse damage represented by D_2 and D_3. In this case, that damage variables are constants because there is no energy release rates during the damage evolution in dominant tension regimes related to D_2 and D_3.

In the case of dominant tension regimes without activation of damage processes in previous compression the original version of the damage model [2] is recovered.

It can be verified that the unilateral damage model satisfies two basic requirements of this modeling kind:

1) The model does not produce spurious energy dissipation upon closed load paths which do not activate damage [4].
2) The continuity of the stress-strain law across the tension-compression interface is assured (hiperplano $g(\boldsymbol{\varepsilon},\mathbf{D_T},\mathbf{D_C})$), because the damage model is derived from the formulation proposed in [2], following the requirements of [1] and [8]. The continuity of the stress-strain law between two damage states imposes that the elastic potential must be once continuously differentiable (whole wise), but only piecewise twice continuously differentiable.

According to [1], other problem related to this kind of modeling concerns the loss of isotropy of the elasticity tensor in the transition through the tension-compression interface. The isotropy is preserved only if the interface is defined in the same group of symmetry of the elasticity tensor. In the proposed model the hyperplane and elasticity tensor belong to the group of isotropic material if there is not damage process. On the other hand, if there is activation of damage processes, the hyperplane starts to present the symmetry of the transverse isotropic material as well as the elasticity tensor. Anyway, the model always preserves the isotropy of the elasticity tensor.

III. Discussion about Micromechanical Theory

Despite the proposed model has been based on the macromechanical behaviour of the concrete, it presents strong connection to the micromechanical theory. The description of the damage activation-deactivation process as part of macroscopic modelling requires to know when the transition between these two states of damage occur and how damage deactivation affect the elastic properties of the material [8]. It is noted that the formulation for bimodular anisotropic damaged media proposed in [2] replies the first question (see (6)). Besides, the continuity of the stress-strain law was assured. In this context, this section aims to point out the influence of the opening-closure of microdefects on the elastic properties of the microcracked concrete.

Consider the simple case of a material weakened by a single array of parallel microcracks with unit normal $\boldsymbol{n}$ as in Fig. 1 and parameter $A=16\left(1-\nu_0^2\right)/\left(6-3\nu_0\right)$. This case is interesting for the damage model proposed in this work because the effective medium exhibits the symmetry associated with the geometric shape of the microcracks with the privileged direction $\boldsymbol{n}$ (transverse isotropic material).

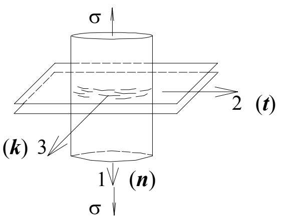

Fig. 1 Material weakened by an array of parallel microcracks

Then, the elastic moduli are fully determined by five independent coefficients for any vectors ***t*** and ***k*** forming with ***n*** an orthonormal basis of $\boldsymbol{R}^3$.

$$E(n) = E_0 \left[1 + \frac{A}{V} \sum_{K=1}^{N} a_k^3 (2 - \nu_0) H\left(\sigma_n^k\right) \right]^{-1} \quad (26)$$

$$\nu(n,t) = \nu_0 \left[1 + \frac{A}{V} \sum_{K=1}^{N} a_k^3 (2 - \nu_0) H\left(\sigma_n^k\right) \right]^{-1} \quad (27)$$

$$E(\boldsymbol{t}) = E_0 \quad (28)$$

$$\nu(\boldsymbol{t},\boldsymbol{k}) = \nu_0 \quad (29)$$

$$\mu(n,t) = \mu_0 \left[1 + \frac{A}{(1+\nu_0)V} \sum_{K=1}^{N} a_k^3 \right]^{-1} \quad (30)$$

The Heaviside function *H* depending on the normal stress to each microcrack is open ($\sigma_n^k \geq 0$) or closed ($\sigma_n^k < 0$).

The particular nature of the microdefects contribution allows to extend these considerations for any of *N* microcracks with different normal vectors. In [8] are described some conclusions about (26)-(30) that are useful for a discussion about the proposed model. In general way, the damage unilateral effect should no longer be considered only by the single restoration of the Young modulus in the direction normal to closed microcracks. In this context, let us compare the damaged elastic moduli given by the proposed model to those ones given by the micromechanical equations. Then, considering Fig. 1 and assuming, for instance, that the transversal isotropy local plane is coincident with the 2-3 plane, the elastic moduli given by the proposed model in dominant compression (subscript *C*) and in tension (subscript *T*) regimes are written as:

$$E_{T1} = E_0(1-D_1)^2(1-D_2)^2 \; ; \; E_{C1} = E_0(1-D_2)^2 \quad (31)$$

$$\nu_{T12} = \nu_{T13} = \nu_0 \frac{(1-D_1)(1-D_2)}{(1-D_3)} ; \nu_{C12} = \nu_{C13} = \nu_0 \frac{(1-D_2)}{(1-D_3)} \quad (32)$$

$$E_{T2} = E_{T3} = E_0(1-D_3)^2 \; ; \; E_{C2} = E_{C3} = E_0(1-D_3)^2 \quad (33)$$

$$\nu_{T23} = \nu_{C23} = \nu_0 \quad (34)$$

$$\mu_{T12} = \mu_{T13} = \mu_{C12} = \mu_{C13} = \mu_0(1-D_4)^2(1-D_5)^2 \quad (35)$$

The longitudinal elastic moduli in tension and in compression in the direction 1 depend on the dominant state, i. e., of the opening-closure criterion. This is also valid for the Poisson ratio in the 12 and 13 planes. On the other hand, the Poisson ratio in the 23 plane (transversal isotropy local plane) is not affected by the damage process. The shear moduli are not changed in the transition from the tension to compression regimes and vice-versa. Observe (33) and consider the transition from dominant tension regime (damage process in tension activated or not) to the compression regime without previous compression. In this case one has: $E_{T2} = E_{T3} = E_{C2} = E_{C3} = E_0$. This result is in correspondence with the form described by (28). Indeed, the $(1-D_3)^2$ coefficient is necessary to take into account the diffuse damage in previous compression when the current dominant state is tension.

Finally, it is observed that despite the proposed model has macromechanical motivations in the macroscopic behaviour of the concrete, the model assists to the requirements suggested by [8] for the micromechanical analysis of the unilateral effect in materials.

IV. Numerical Applications

Initially, Fig. 2 shows that the consideration of the permanent strains improves the capture of the transverse strains by the model. Besides, the model predicts the change in sign of the volumetric strain.

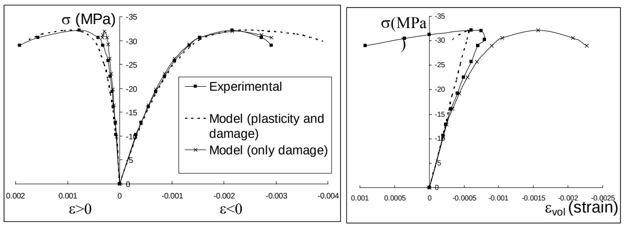

Fig. 2 Uniaxial compression test: a) axial e transverse strains; b) volumetric strain

In the second application, the unilateral model is used in the simulation of a uniaxial test in concrete specimens upon reversal load. Observe that the permanent strains are important in the definition of the hyperplane, in the sense that the total strains start to compose the criterion (6). The initial stiffness recovery can be clearly observed taking into account permanent strain in the dominant tension regime. It is noted the contribution of the diffuse damage generated in previous compression regimes when dealing to tension regimes.

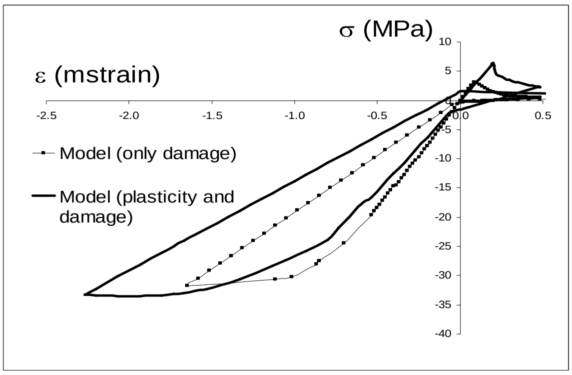

Fig. 3 Uniaxial test in concrete specimens upon reversal load

The one-dimensional version of the unilateral damage model was implemented in a program for bars structures analysis with finite layered elements. The damage model is assumed to simulate the concrete layers behaviour and for the longitudinal reinforcement bars, an elastoplastic behaviour is admitted. This example deals with a test [4] that corresponds to a reinforced concrete beam, in a configuration of three point cyclic flexion. The beam is subject cyclic loading at the mid span. Table I contains the parameters values.

In the numerical analysis, displacements increments were enforced in the mid span. Using the advantage of symmetry, only half of the beam is discretized into 20 finite elements. The transversal sections were divided into 16 layers. The numerical and experimental responses are displayed in Fig. 5.

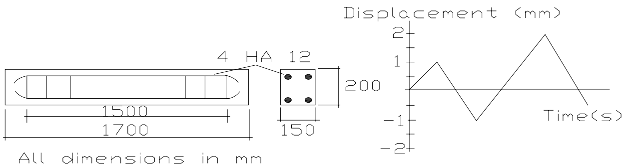

Fig. 4 Geometry, reinforcement details and history of loading

TABLE I
PARAMETERS OF THE UNILATERAL DAMAGE MODEL

Tension	Compression
$Y_{01}=6.0x10^{-5}MPa$	$Y_{02}=3.0x10^{-3}MPa$
$A_1=-0.93$	$A_2=1.50$
$B_1=110\ MPa^{-1}$	$B_2=10.01\ MPa^{-1}$
$\beta_1=8x10^{-5}MPa$	$\beta_2=1.0x10^{-3}MPa$

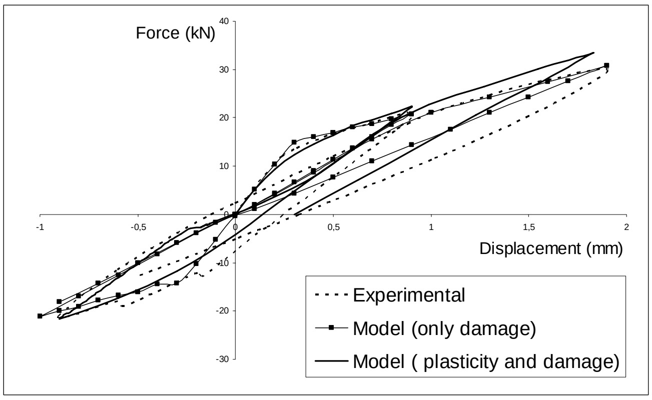

Fig. 5 Global response of the reinforced concrete beam

The results obtained by the model have shown to be satisfactory despite the limited parametric identification of the parameters related to permanent strains. Therefore, the ultimate loads are computed more accurately than the permanent strains in the unloading processes. Note that the damage processes in the compression regimes are not so important in this example, according to observations in [6]. Besides, the damage profile is also close to test observations. The obtained results encourage us to proceed in the improvement of the model to deal with more complex phenomena in future works, e. g., blocking and dissipative sliding of closed microcracks lips, non-local version of the model and a more efficient parametric identification of β_1 and β_2, among others.

REFERENCES

[1] A. Curnier, Q. He, and P. Zysset, “Conewise linear elastic materials,” *J. of Elasticity*, vol. 37, pp. 1–38, 1995.

[2] J. J. C. Pituba, “On the formulation of damage constitutive models for bimodular and anisotropic media,” in *Proc. 3th European Conference on Computational Mechanics: Solids, Structures and Coupled Problems in Engineering*, Lisbon, 2006.

[3] C. Comi, “A nonlocal damage model with permanent strains for quasi-brittle materials,” in *Proc. Continuous Damage and Fracture*, Cachan, 2000, pp. 221–232.

[4] C. La Borderie, “Phenomenes unilateraux dans un materiau endommageable: Modelisation et application a l'analyse de structures en beton,” Ph.D. thesis, Université Paris 6, Cachan, 1991.

[5] I. Carol, and K. Willam, “Spurious Energy dissipation/generation in stiffness recovery models for elastic degradation and damage,” *Int. J. of Solids and Structures*, vol. 33, no. 20-22, pp. 2939–257, 1996.

[6] R. Desmorat, “Strain localization and unilateral conditions for anisotropic induced damage model,” in *Proc. Continuous Damage and Fracture*, Cachan, 2000, pp. 71–79.

[7] A. Dragon, and D. Halm, “Doubly dissipative model for quasi-brittle solids: anisotropic damage and frictional sliding on microcracks,” in *Proc. Continuous Damage and Fracture*, Cachan, 2000, pp. 207–215.

[8] H. Welemane, and F. Cormery, “Some remarks on the damage unilateral effect modeling for microcracked materials,” *Int. J. of Damage Mech.*, vol. 11, pp. 65–86, January 2002.

[9] J. J. C. Pituba, “An anisotropic damage model for concrete: first two-dimensional applications,” *Int. J. of Damage Mech.*, submitted for publication.