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Analysis of Periodic Solution of Delay Fuzzy BAM Neural Networks

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Abstract—In this paper, by employing a new Lyapunov functional and an elementary inequality analysis technique, some sufficient conditions are derived to ensure the existence and uniqueness of periodic oscillatory solution for fuzzy bi-directional memory (BAM) neural networks with time-varying delays, and all other solutions of the fuzzy BAM neural networks converge the uniqueness periodic solution. These criteria are presented in terms of system parameters and have important leading significance in the design and applications of neural networks. Moreover an example is given to illustrate the effectiveness and feasible of results obtained.

Keywords—Fuzzy BAM neural networks, Periodic solution, Global exponential stability, Time-varying delays

I. Introduction

BI-DERECTIONAL associative memory(BAM) neural network was first introduced by Kosto [1,2]. These models generalize the single layer auto-associative Hebbian correlator to a two layer pattern-matched hetero-associative circuits. BAM neural networks is composed of neurons arranged in two layers, the X-layer and the Y-layer. Due to the BAM neural networks has been used in many fields such as image processing, pattern recognition, and automatic control [3]. Recently many researchers [1-16,18-26] have investigated the dynamics of BAM neural networks, including stability and periodic solutions. There are many studying results about the BAM neural networks with and without axonal signal transmission delays [4-16,18-20]. Recently, there are some authors [16,20,26] studied the BAM neural networks with distributed delays, which are more appropriate when neural networks have a multitude of parallel pathways with a variety of axon sizes and lengths. In this paper, we would like to integrate fuzzy operations into BAM neural networks. Speaking of fuzzy operations, T.Yang and L.B.Yang [27] first introduced fuzzy cellular neural networks (FCNNs) combining those operations with cellular neural networks. So far researchers have founded that FCNNs are useful in image processing, and some results have been reported on stability and periodicity of FCNNs [27-35]. However, to the best of our knowledge, few author investigated the stability of fuzzy BAM neural networks with time-varying delays.

In this paper, we investigate global exponential stability of equilibrium point for the following fuzzy BAM neural networks:

$$\begin{cases} x_i'(t) = -a_i x_i(t) + \sum_{j=1}^m c_{ji} f_j(y_j(t-\tau_{ji}(t))) \\ \quad + \bigwedge_{j=1}^m \alpha_{ji} f_j(y_j(t-\tau_{ji}(t))) \\ \quad + \bigvee_{j=1}^m \beta_{ji} f_j(y_j(t-\tau_{ji}(t))) \\ \quad + \bigwedge_{j=1}^m T_{ji} u_j + \bigvee_{j=1}^m H_{ji} u_j + I_i(t) \\ y_j'(t) = -b_j y_j(t) + \sum_{i=1}^n d_{ij} g_i(x_i(t-\sigma_{ij}(t))) \\ \quad + \bigwedge_{i=1}^n p_{ij} g_i(x_i(t-\sigma_{ij}(t))) \\ \quad + \bigvee_{i=1}^n q_{ij} g_i(x_i(t-\sigma_{ij}(t))) \\ \quad + \bigwedge_{i=1}^n S_{ij} u_i + \bigvee_{i=1}^n L_{ij} u_i + J_j(t) \end{cases} \tag{1}$$

where n and m correspond to the number of neurons in X-layer and Y-layer, respectively. For $i = 1, 2, \cdots, n; j = 1, 2, \cdots, m, x_i(t)$ and $y_j(t)$ are the activations of the ith neuron and the jth neurons, respectively. $a_i > 0, b_j > 0$, they denote the rate with which the ith neuron and jth neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs; $\alpha_{ji}, \beta_{ji}, T_{ji}$ and H_{ji} are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template in X-layer, respectively; p_{ij}, q_{ij}, S_{ij} and L_{ij} are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template in Y-layer, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation , respectively; u_j, u_i denote external input of the ith neurons in X-layer and external input of the jth neurons in Y-layer, respectively. external bias $I_i : R^+ \to R, i = 1, 2, \cdots, n$, and $J_j : R^+ \to R, j = 1, 2, \cdots, m$, are continuously periodic functions with period ω. i.e.,$I_i(t+\omega) = I_i(t), J_j(t+\omega) = J_j(t)$. The delays $\tau_{ji}(t)$ and $\sigma_{ij}(t)$ correspond to finite speed of axonal signal transmission; they are nonnegative, differential and periodic functions with periodic ω. i.e., $\tau_{ji}(t+\omega) = \tau_{ji}(t), \sigma_{ij}(t+\omega) = \sigma_{ij}(t)$. $\sup_{t\in[0,+\infty)} \tau_{ji}'(t) = \gamma_1 < 1, \sup_{t\in[0,+\infty)} \sigma_{ij}'(t) = \gamma_2 < 1, \tau = \sup_{t\in[0,+\infty)} \max_{1\le i\le n} \max_{1\le j\le m} \tau_{ji}(t), \sigma = \sup_{t\in[0,+\infty)} \max_{1\le i\le n} \max_{1\le j\le m} \sigma_{ij}(t)$. $f_j(\cdot), g_i(\cdot)$ are signal transmission functions.

The initial conditions associated with system (1) are of the form

$$\begin{cases} x_i(s) = \phi_i(s), s \in [-\sigma, 0], \quad i = 1, 2, \cdots, n \\ y_j(s) = \varphi_j(s), s \in [-\tau, 0], \quad j = 1, 2, \cdots, m \end{cases} \tag{2}$$

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where $\phi_i(\cdot)$ and $\psi_j(\cdot)$ are continuous bounded functions defined on $[-\sigma, 0]$ and $[-\tau, 0]$, respectively.

Throughout the paper, we give the following assumptions

(A1) The signal transmission functions $f_j(\cdot), g_i(\cdot)(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$ are Lipschtiz continuous on R with Lipschtiz constants μ_j and ν_i, namely , for any $x, y \in R$

$$|f_j(x) - f_j(y)| \leq \mu_j|x - y|, \ |g_i(x) - g_i(y)| \leq \nu_i|x - y|,$$

and $f_j(0) = g_i(0) = 0$

(A2) $f_j(\cdot)$ and $g_i(\cdot)$ are bounded on R.

Definition 1.1. *If $f(t) : R \to R$ is a continuous function, then the upper right derivative of $f(t)$ is defined as*

$$D^+f(t) = \lim_{h\to 0^+} \sup \frac{1}{h}(f(t + h) - f(t)).$$

Lemma 1.1.[27] *Suppose x and y are two states of system (1), then we have*

$$\left|\bigwedge_{j=1}^{n} \alpha_{ij} g_j(x) - \bigwedge_{j=1}^{n} \alpha_{ij} g_j(y)\right| \leq \sum_{j=1}^{n} |\alpha_{ij}||g_j(x) - g_j(y)|,$$

and

$$\left|\bigvee_{j=1}^{n} \beta_{ij} g_j(x) - \bigvee_{j=1}^{n} \beta_{ij} g_j(y)\right| \leq \sum_{j=1}^{n} |\beta_{ij}||g_j(x) - g_j(y)|.$$

The remainder of this paper is organized as follows. In Section 2, we will give the sufficient conditions to ensure the existence of periodic oscillatory solution for fuzzy BAM neural networks with time-varying delays, and show that all other solutions converge exponentially to it as $t \to \infty$. In Section 3 an example will be given to illustrate effectiveness of our results obtained. We will give a general conclusion in Section 4.

II. Periodic oscillatory solutions

In this section, we will consider the periodic oscillatory solutions of system (1) and give their proofs.

Theorem 2.1. *Under assumptions (A1) and (A2), there exists exactly one $\omega-$periodic solution of system (1.1) and all other solutions of system (1.1) converge exponentially to it as $t \to \infty$. if there exist constants $\lambda_i > 0, \lambda_{n+j} > 0 (i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$ such that*

$$\begin{cases} -\lambda_i a_i + \frac{1}{2(1-\gamma_1)} \sum_{j=1}^{m} \lambda_i \mu_j (|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|) \\ +\frac{1}{2} \sum_{j=1}^{m} \lambda_{n+j} \nu_i (|d_{ij}| + |p_{ij}| + |q_{ij}|) < 0, \\ \\ -\lambda_{n+j} b_j + \frac{1}{2(1-\gamma_2)} \sum_{i=1}^{n} \lambda_{n+j} \nu_i (|d_{ij}| + |p_{ij}| + |q_{ij}|) \\ +\frac{1}{2} \sum_{i=1}^{n} \lambda_i \mu_j (|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|) < 0, \end{cases} \tag{3}$$

Proof. Let $C = C([-\sigma, 0] \times [-\tau, 0], R^{n+m})$ be the Banach space of continuous functions with the topology of uniform convergence. For any $(\phi_x, \varphi_y)^T \in C$, we define

$$\|(\phi_x, \varphi_y)^T\| = \sup_{-\tau\leq\theta\leq 0} |\phi_x(\theta)| + \sup_{-\sigma\leq\theta\leq 0} |\varphi_y(\theta)|,$$

where $|\phi_x(\theta)| = \sum_{i=1}^{n}(\phi_{xi}(\theta))^2, |\varphi_y(\theta)| = \sum_{j=1}^{m}(\varphi_{yj}(\theta))^2$.

For any $(\phi_x, \varphi_y)^T, (\phi_x^*, \varphi_y^*)^T \in C$, we denote the solution of system (1.1) through the initial value $((0, 0)^T, (\phi_x, \varphi_y)^T)$ and $((0, 0)^T, (\phi_x^*, \varphi_y^*)^T)$ as

$$x(t, \phi_x) = (x_1(t, \phi_x), x_2(t, \phi_x), \cdots, x_n(t, \phi_x))^T,$$

$$y(t, \varphi_y) = (y_1(t, \varphi_y), y_2(t, \varphi_y), \cdots, y_m(t, \varphi_y))^T.$$

$$x(t, \phi_x^*) = (x_1(t, \phi_x^*), x_2(t, \phi_x^*), \cdots, x_n(t, \phi_x^*))^T,$$

$$y(t, \varphi_y^*) = (y_1(t, \varphi_y^*), y_2(t, \varphi_y^*), \cdots, y_m(t, \varphi_y^*))^T.$$

respectively. Define

$$x_t(\phi_x) = x(t + \theta, \phi_x), \quad \theta \in [-\tau, 0], \quad t \geq 0.$$

$$y_t(\varphi_y) = y(t + \theta, \varphi_y), \quad \theta \in [-\sigma, 0], \quad t \geq 0.$$

From system (1), we get

$$\begin{aligned} (x_i(t, \phi_x) \ &- \ x_i(t, \phi_x^*))' \\ &= -a_i(x_i(t, \phi_x) - x_i(t, \phi_x^*)) \\ &\quad + \sum_{j=1}^{m} c_{ji}(f_j(y_j(t - \tau_{ji}(t), \varphi_y) \\ &\quad - f_j(y_j(t - \tau_{ji}(t), \varphi_y^*))) \\ &\quad + \bigwedge_{j=1}^{m} \alpha_{ji} f_j(y_j(t - \tau_{ji}(t)), \varphi_y) \\ &\quad - \bigwedge_{j=1}^{m} \alpha_{ji} f_j(y_j(t - \tau_{ji}(t)), \varphi_y^*) \\ &\quad + \bigvee_{j=1}^{m} \beta_{ji} f_j(y_j(t - \tau_{ji}(t)), \varphi_y) \\ &\quad - \bigvee_{j=1}^{m} \beta_{ji} f_j(y_j(t - \tau_{ji}(t)), \varphi_y^*) \end{aligned} \tag{4}$$

and

$$\begin{aligned} (y_j(t, \varphi_y) \ &- \ y_j(t, \varphi_y^*))' \\ &= -b_j(y_j(t, \varphi_y) - y_j(t, \varphi_y^*)) \\ &\quad + \sum_{i=1}^{n} d_{ij}(g_i(x_i(t - \sigma_{ij}(t), \phi_x) \\ &\quad - g_i(x_i(t - \sigma_{ij}(t), \phi_x^*)))) \\ &\quad + \bigwedge_{i=1}^{n} p_{ij} g_i(x_i(t - \sigma_{ij}(t)), \varphi_y) \\ &\quad - \bigwedge_{i=1}^{n} p_{ij} g_i(x_i(t - \sigma_{ij}(t)), \phi_x^*) \\ &\quad + \bigvee_{i=1}^{n} q_{ij} g_i(x_i(t - \sigma_{ij}(t)), \phi_x) \\ &\quad - \bigvee_{i=1}^{n} q_{ij} g_i(x_i(t - \sigma_{ij}(t)), \phi_x^*) \end{aligned} \tag{5}$$

Since (3) hold, we can choose a small $\varepsilon > 0$, such that

$$\begin{cases} \lambda_i(\frac{\varepsilon}{2} - a_i) + \frac{1}{2(1-\gamma_1)} \sum_{j=1}^m \lambda_i \mu_j (|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|) \\ +\frac{1}{2} e^{\varepsilon\sigma} \sum_{j=1}^m \lambda_{n+j} \nu_i (|d_{ij}| + |p_{ij}| + |q_{ij}|) < 0, \\ \\ \lambda_{n+j}(\frac{\varepsilon}{2} - b_j) + \frac{1}{2(1-\gamma_2)} \sum_{i=1}^n \lambda_{n+j} \nu_i (|d_{ij}| + |p_{ij}| + |q_{ij}|) \\ +\frac{1}{2} e^{\varepsilon\tau} \sum_{j=1}^m \lambda_i \mu_j (|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|) < 0, \end{cases} \tag{6}$$

Consider the following Lyapunov functional

$$\begin{aligned} V(t) &= \frac{1}{2} \sum_{i=1}^n \lambda_i \Big[(x_i(t,\phi_x) - x_i(t,\phi_x^*))^2 e^{\varepsilon t} \\ &\quad + \sum_{j=1}^m \int_{t-\tau_{ji}(t)}^t \mu_j (|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|) \\ &\quad \times (y_j(s,\varphi_y) - y_j(s,\varphi_y^*))^2 e^{\varepsilon(s+\tau)} ds \Big] \\ &\quad + \frac{1}{2} \sum_{j=1}^m \lambda_{n+j} \Big[(y_j(t,\varphi_y) - y_j(t,\varphi_y^*))^2 e^{\varepsilon t} \\ &\quad + \sum_{i=1}^n \int_{t-\sigma_{ij}(t)}^t \nu_i (|d_{ij}| + |p_{ij}| + |q_{ij}|) \\ &\quad \times (x_i(s,\phi_x) - x_i(s,\phi_x^*))^2 e^{\varepsilon(s+\sigma)} ds \Big] \end{aligned} \tag{7}$$

Calculate the right upper derivative $D^+V(t)$ of V along the solutions of (4) and (5). we get

$$\begin{aligned} D &\ ^+V(t) \\ = &\sum_{i=1}^n \lambda_i \Big[\frac{1}{2} (x_i(t,\phi_x) - x_i(t,\phi_x^*))^2 \varepsilon e^{\varepsilon t} \\ &+ (x_i(t,\phi_x) - x_i(t,\phi_x^*))(x_i(t,\phi_x) - x_i(t,\phi_x^*))' e^{\varepsilon t} \\ &+ \frac{1}{2} \sum_{j=1}^m \mu_j (|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|)(y_j(t,\varphi_y) - y_j(t,\varphi_y^*))^2 \\ &\times e^{\varepsilon(t+\tau)} - \frac{1}{2} \sum_{j=1}^m (1 - \tau_{ji}'(t)) \mu_j (|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|) \\ &\times (y_j(t-\tau_{ji}(t),\varphi_y) - y_j(t-\tau_{ji}(t),\varphi_y^*))^2 e^{\varepsilon(t-\tau_{ji}(t)+\tau)} \Big] \\ &+ \sum_{j=1}^m \lambda_{n+j} \Big[\frac{1}{2} (y_j(t,\varphi_y) - y_j(t,\varphi_y^*))^2 \varepsilon e^{\varepsilon t} \\ &+ (y_j(t,\varphi_y) - y_j(t,\varphi_y^*))(y_j(t,\varphi_y) - y_j(t,\varphi_y^*))' e^{\varepsilon t} \\ &+ \frac{1}{2} \sum_{i=1}^n \nu_i (|d_{ij}| + |p_{ij}| + |q_{ij}|)(x_i(t,\phi_x) - x_i(t,\phi_x^*))^2 \\ &\times e^{\varepsilon(t+\sigma)} - \frac{1}{2} \sum_{i=1}^n (1 - \sigma_{ij}'(t)) \nu_i (|d_{ij}| + |p_{ij}| + |q_{ij}|) \\ &\times (x_i(t-\sigma_{ij}(t),\phi_x) - x_i(t-\sigma_{ij}(t),\phi_x^*))^2 e^{\varepsilon(t-\sigma_{ij}(t)+\sigma)} \Big] \\ \le &\ e^{\varepsilon t} \sum_{i=1}^n \lambda_i \Big[(\frac{\varepsilon}{2} - a_i)(x_i(t,\phi_x) - x_i(t,\phi_x^*))^2 \\ &+ \sum_{j=1}^m \mu_j (|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|) |x_i(t,\phi_x) - x_i(t,\phi_x^*)| \\ &\times |y_j(t-\tau_{ji}(t),\varphi_y) - y_j(t-\tau_{ji}(t),\varphi_y^*)| \\ &+ \frac{1}{2} e^{\varepsilon\tau} \sum_{j=1}^m \mu_j (|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|)(y_j(t,\varphi_y) - y_j(t,\varphi_y^*))^2 \\ &- \frac{1-\gamma_1}{2} \sum_{j=1}^m \mu_j ((|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|) \\ &\times (y_j(t-\tau_{ji}(t),\varphi_y) - y_j(t-\tau_{ji}(t),\varphi_y^*))^2 \Big] \\ &+ e^{\varepsilon t} \sum_{j=1}^m \lambda_{n+j} \Big[(\frac{\varepsilon}{2} - b_j)(y_j(t,\varphi_y) - y_j(t,\varphi_y^*))^2 \\ &+ \sum_{i=1}^n \nu_i (|d_{ij}| + |p_{ij}| + |q_{ij}|) |y_j(t,\varphi_y) - y_j(t,\varphi_y^*)| \\ &\times |x_i(t-\sigma_{ij}(t),\phi_x) - x_i(t-\sigma_{ij}(t),\phi_x^*)| \\ &+ \frac{1}{2} e^{\varepsilon\sigma} \sum_{i=1}^n \nu_i (|d_{ij}| + |p_{ij}| + |q_{ij}|)(x_i(t,\phi_x) - x_i(t,\phi_x^*))^2 \\ &- \frac{1-\gamma_2}{2} \sum_{i=1}^n \nu_i ((|d_{ij}| + |p_{ij}| + |q_{ij}|) \\ &\times (x_i(t-\sigma_{ij}(t),\phi_x) - x_i(t-\sigma_{ij}(t),\phi_x^*))^2 \Big] \end{aligned}$$

Applying the elementary inequality $2ab \le a^2 + b^2$, we obtain

$$\begin{aligned} D &\ ^+V(t) \\ \le &\ e^{\varepsilon t} \sum_{i=1}^n \lambda_i \Big[(\frac{\varepsilon}{2} - a_i)(x_i(t,\phi_x) - x_i(t,\phi_x^*))^2 \\ &+ \sum_{j=1}^m \mu_j (|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|) \\ &\times \Big(\frac{1}{2(1-\gamma_1)} |x_i(t,\phi_x) - x_i(t,\phi_x^*)|^2 \\ &+ \frac{1-\gamma_1}{2} |y_j(t-\tau_{ji}(t),\varphi_y) - y_j(t-\tau_{ji}(t),\varphi_y^*)|^2 \Big) \\ &+ \frac{1}{2} e^{\varepsilon\tau} \sum_{j=1}^m \mu_j (|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|)(y_j(t,\varphi_y) - y_j(t,\varphi_y^*))^2 \\ &- \frac{1-\gamma_1}{2} \sum_{j=1}^m \mu_j ((|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|) \\ &\times (y_j(t-\tau_{ji}(t),\varphi_y) - y_j(t-\tau_{ji}(t),\varphi_y^*))^2 \Big] \\ &+ e^{\varepsilon t} \sum_{j=1}^m \lambda_{n+j} \Big[(\frac{\varepsilon}{2} - b_j)(y_j(t,\varphi_y) - y_j(t,\varphi_y^*))^2 \\ &+ \sum_{i=1}^n \nu_i (|d_{ij}| + |p_{ij}| + |q_{ij}|) \\ &\times \Big(\frac{1}{2(1-\gamma_2)} |y_j(t,\varphi_y) - y_j(t,\varphi_y^*)|^2 \\ &+ \frac{1-\gamma_2}{2} |x_i(t-\sigma_{ij}(t),\phi_x) - x_i(t-\sigma_{ij}(t),\phi_x^*)|^2 \Big) \\ &+ \frac{1}{2} e^{\varepsilon\sigma} \sum_{i=1}^n \nu_i (|d_{ij}| + |p_{ij}| + |q_{ij}|)(x_i(t,\phi_x) - x_i(t,\phi_x^*))^2 \\ &- \frac{1-\gamma_2}{2} \sum_{i=1}^n \nu_i ((|d_{ij}| + |p_{ij}| + |q_{ij}|) \\ &\times (x_i(t-\sigma_{ij}(t),\phi_x) - x_i(t-\sigma_{ij}(t),\phi_x^*))^2 \Big] \end{aligned}$$

$$
\begin{aligned}
&= e^{\varepsilon t}\sum_{i=1}^{n}\lambda_i\Big[(\frac{\varepsilon}{2}-a_i)(x_i(t,\phi_x)-x_i(t,\phi_x^*))^2\\
&\quad+\frac{1}{2(1-\gamma_1)}\sum_{j=1}^{m}\mu_j(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\\
&\quad\times(x_i(t,\phi_x)-x_i(t,\phi_x^*))^2+\frac{1}{2}e^{\varepsilon\tau}\sum_{j=1}^{m}\mu_j\\
&\quad\times(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)(y_j(t,\varphi_y)-y_j(t,\varphi_y^*))^2\Big]\\
&\quad+e^{\varepsilon t}\sum_{j=1}^{m}\lambda_{n+j}\Big[(\frac{\varepsilon}{2}-b_j)(y_j(t,\varphi_y)-y_j(t,\varphi_y^*))^2\\
&\quad+\frac{1}{2(1-\gamma_2)}\sum_{i=1}^{n}\nu_i(|d_{ij}|+|p_{ij}|+|q_{ij}|)\\
&\quad\times(y_j(t,\varphi_y)-y_j(t,\varphi_y^*))^2+\frac{1}{2}e^{\varepsilon\sigma}\sum_{i=1}^{n}\nu_i\\
&\quad\times(|d_{ij}|+|p_{ij}|+|q_{ij}|)(x_i(t,\phi_x)-x_i(t,\phi_x^*))^2\Big]\\
&= e^{\varepsilon t}\sum_{i=1}^{n}\Bigg[\lambda_i(\frac{\varepsilon}{2}-a_i)+\frac{1}{2(1-\gamma_1)}\sum_{j=1}^{m}\lambda_i\mu_j\\
&\quad\times(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)+\frac{1}{2}e^{\varepsilon\sigma}\sum_{j=1}^{m}\lambda_{n+j}\nu_i\\
&\quad\times(|d_{ij}|+|p_{ij}|+|q_{ij}|)\Big](x_i(t,\phi_x)-x_i(t,\phi_x^*))^2\\
&\quad+e^{\varepsilon t}\sum_{j=1}^{m}\Bigg[\lambda_{n+j}(\frac{\varepsilon}{2}-b_j)+\frac{1}{2(1-\gamma_2)}\sum_{i=1}^{n}\lambda_{n+j}\nu_i\\
&\quad\times(|d_{ij}|+|p_{ij}|+|q_{ij}|)+\frac{1}{2}e^{\varepsilon\tau}\sum_{i=1}^{n}\lambda_i\mu_j\\
&\quad\times(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\Big](y_j(t,\varphi_y)-y_j(t,\varphi_y^*))^2\\
&\le 0
\end{aligned}
$$

Therefore $V(t)\le V(0),\ t\ge 0$. From (7), we have

$$
\begin{aligned}
V(t) &\ge \frac{1}{2}e^{\varepsilon t}\left(\min_{1\le i\le n+m}\lambda_i\right)\left(\sum_{i=1}^{n}|x_i(t,\phi_x)-x_i(t,\phi_x^*)|^2\right.\\
&\quad\left.+\sum_{j=1}^{m}|y_j(t,\varphi_y)-y_j(t,\varphi_y^*)|^2\right)
\end{aligned}
$$

On the other hand

$$
\begin{aligned}
V(0) &= \frac{1}{2}\sum_{i=1}^{n}\lambda_i\Bigg[(\phi_{xi}-\phi_{xi}^*)^2+\sum_{j=1}^{m}\int_{-\tau_{ji}(0)}^{0}\mu_j\\
&\quad\times(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)(y_j(s,\varphi_y)-y_j(s,\varphi_y^*))^2\\
&\quad\times e^{\varepsilon(s+\tau)}ds\Big]+\frac{1}{2}\sum_{j=1}^{m}\lambda_{n+j}\Big[(\varphi_{yj}-\varphi_{yj}^*)^2\\
&\quad+\sum_{i=1}^{n}\int_{-\sigma_{ij}(0)}^{0}\nu_i(|d_{ij}|+|p_{ij}|+|q_{ij}|)\\
&\quad\times(x_i(s,\phi_x)-x_i(s,\phi_x^*))^2e^{\varepsilon(s+\sigma)}ds\Big]\\
&\le \frac{1}{2}\Bigg[\max_{1\le i\le n}\lambda_i+\max_{1\le i\le n}(\nu_i)e^{\varepsilon\sigma}\sigma\sum_{j=1}^{m}\lambda_{n+j}\\
&\quad\times\max_{1\le i\le n}(|d_{ij}|+|p_{ij}|+|q_{ij}|)\Bigg]\|\phi_x-\phi_x^*\|\\
&\quad+\frac{1}{2}\Bigg[\max_{1\le j\le m}\lambda_{n+j}+\max_{1\le j\le m}(\mu_j)e^{\varepsilon\tau}\tau\sum_{i=1}^{n}\lambda_i\\
&\quad\times\max_{1\le j\le m}(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\Bigg]\|\varphi_y-\varphi_y^*\|
\end{aligned}
$$

Then it follows easily that, for all $t\ge 0$.

$$
\begin{aligned}
\sum_{i=1}^{n}(x_i(t,\phi_x) &- x_i(t,\phi_x^*))^2+\sum_{j=1}^{m}(y_j(t,\varphi_y)-y_j(t,\varphi_y^*))^2\\
&\le M(\|\phi_x-\phi_x^*\|+\|\varphi_y-\varphi_y^*\|)e^{-\varepsilon t} \qquad (8)
\end{aligned}
$$

where

$$
\begin{aligned}
M &= \max\Bigg\{\frac{1}{\min_{1\le i\le n+m}(\lambda_i)}\Bigg(\max_{1\le i\le n}\lambda_i+\max_{1\le i\le n}(\nu_i)e^{\varepsilon\sigma}\sigma\\
&\quad\sum_{j=1}^{m}\lambda_{n+j}\max_{1\le i\le n}(|d_{ij}|+|p_{ij}|+|q_{ij}|)\Bigg),\\
&\quad\frac{1}{\min_{1\le i\le n+m}(\lambda_i)}\Bigg(\max_{1\le j\le m}\lambda_{n+j}+\max_{1\le j\le m}(\mu_j)e^{\varepsilon\tau}\tau\\
&\quad\sum_{i=1}^{n}\lambda_i\max_{1\le j\le m}(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\Bigg)\Bigg\}\\
&\ge 1
\end{aligned}
$$

Hence we have

$$\sum_{i=1}^{n}(x_i(t,\phi_x)-x_i(t,\phi_x^*))^2\le M(\|\phi_x-\phi_x^*\|+\|\varphi_y-\varphi_y^*\|)e^{-\varepsilon t}$$

$$\sum_{j=1}^{m}(y_j(t,\varphi_y)-y_j(t,\varphi_y^*))^2\le M(\|\phi_x-\phi_x^*\|+\|\varphi_y-\varphi_y^*\|)e^{-\varepsilon t}$$

Therefore, for all $t\ge 0$,

$$|x_t(\phi_x)-x_t(\phi_x^*)|\le Me^{-\varepsilon(t-\sigma)}(\|\phi_x-\phi_x^*\|+\|\varphi_y-\varphi_y^*\|)$$

$$|y_t(\varphi_y)-y_t(\varphi_y^*)||\le Me^{-\varepsilon(t-\tau)}(\|\phi_x-\phi_x^*\|+\|\varphi_y-\varphi_y^*\|)$$

We can choose a positive integer k such that

$$Me^{-\varepsilon(k\omega-\sigma)}\le\frac{1}{4},\quad Me^{-\varepsilon(k\omega-\tau)}\le\frac{1}{4}.$$

Define a Poincare mapping $P:C\to C$ by $P((\phi_x,\varphi_y)^T)=(x_\omega(\phi_x),y_\omega(\varphi_y))^T$. Then we can derive from system (1) that

$$\|P^k((\phi_x,\varphi_y)^T)-P^k((\phi_x^*,\varphi_y^*)^T)\|\le\frac{1}{2}\|(\phi_x,\varphi_y)^T-(\phi_x^*,\varphi_y^*)^T\|$$

This implies that P^k is a contraction mapping, hence there exists a unique fixed point $(\phi_x^{**},\varphi_y^{**})^T\in C$ such that

$$P^k((\phi_x^{**},\varphi_y^{**})^T)=(\phi_x^{**},\varphi_y^{**})^T.$$

Note that

$$P^k(P((\phi_x^{**},\varphi_y^{**})^T))=P(P^k((\phi_x^{**},\varphi_y^{**})^T))=P((\phi_x^{**},\varphi_y^{**})^T).$$

This shows that $P((\phi_x^{**},\varphi_y^{**})^T)\in C$ is also a fixed point of P^k. and so $P((\phi_x^{**},\varphi_y^{**})^T)=(\phi_x^{**},\varphi_y^{**})^T$. i.e., $(x_\omega(\phi_x^{**}),y_\omega(\varphi_y^{**})^T=(\phi_x^{**},\varphi_y^{**})^T$

Let $(x(t,\phi_x^{**}), y(t,\varphi_y^{**})^T$ be the solution of system (1.1) through $((0,0)^T, (\phi_x^{**}, \varphi_y^{**})^T)$, then $(x(t+\omega,\phi_x^{**}), y(t+\omega,\varphi_y^{**}))^T$ is also a solution of system (1.1). Obviously

$$\begin{aligned}(x_{t+\omega}(\phi_x^{**}), y_{t+\omega}(\varphi_y^{**}))^T &= (x_t(x_\omega(\phi_x^{**})), y_t(y_\omega(\varphi_y^{**})))^T \\ &= (x_t(\phi_x^{**}), y_t(\varphi_y^{**}))^T.\end{aligned}$$

for $t \geq 0$, hence

$$(x(t+\omega,\phi_x^{**}), y(t+\omega,\varphi_y^{**}))^T = (x(t,\phi_x^{**}), y(t,\varphi_y^{**}))^T.$$

This implies $(x(t,\phi_x^{**}), y(t,\varphi_y^{**}))^T$ is exactly one $\omega-$periodic solution of system (1), and it follows from (8) that all other solutions of system (1) converge exponentially to it as $t \to \infty$. The proof is completed.

Remark 2.1 *If we don't consider fuzzy AND and fuzzy OR operations in system (1), then system (1) becomes traditional BAM neural networks with time-varying delays. it is clear that Theorem 1 [36] is corollary of Theorem 2.1. Therefore our results generalizes the known results.*

III. An illustrative example

In this section, we will give an example to illustrate feasible of our result.

Example 3.1 Consider the following fuzzy BAM neural networks with time-varying delays

$$\begin{cases} x_i'(t) = -a_i x_i(t) + \sum_{j=1}^2 c_{ji} f_j(y_j(t-\tau_{ji}(t))) \\ \quad + \bigwedge_{j=1}^2 \alpha_{ji} f_j(y_j(t-\tau_{ji}(t))) \\ \quad + \bigvee_{j=1}^2 \beta_{ji} f_j(y_j(t-\tau_{ji}(t))) \\ \quad + \bigwedge_{j=1}^2 T_{ji} u_j + \bigvee_{j=1}^2 H_{ji} u_j + I_i(t) \\ y_j'(t) = -b_j y_j(t) + \sum_{i=1}^2 d_{ij} g_i(x_i(t-\sigma_{ij}(t))) \\ \quad + \bigwedge_{i=1}^2 p_{ij} g_i(x_i(t-\sigma_{ij}(t))) \\ \quad + \bigvee_{i=1}^n q_{ij} g_i(x_i(t-\sigma_{ij}(t))) \\ \quad + \bigwedge_{i=1}^2 S_{ij} u_i + \bigvee_{i=1}^2 L_{ij} u_i + J_j(t) \end{cases} \tag{9}$$

where

$$\tau_{ji}(t) = \frac{1}{3}(\sin t + 1), \sigma_{ij}(t) = \frac{1}{4}(\cos t + 1), i, j = 1, 2.$$

$$f_1(r) = f_2(r) = g_1(r) = g_2(r) = \frac{1}{2}(|r+1| - |r-1|)$$

$I_1(t) = I_2(t) = \sin t, J_1(t) = J_2(t) = \cos t$. It is obvious that $f(\cdot), g(\cdot)$ satisfy assumption (A1) and (A2), $\gamma_1 = \frac{1}{3}, \gamma_2 = \frac{1}{4}$, $\mu_1 = \mu_2 = \nu_1 = \nu_2 = 1$. Let $T_{ji} = H_{ji} = S_{ij} = L_{ij} = u_i = u_j = 1 (i, j = 1, 2)$.

$$\alpha_{11} = \frac{5}{3}, \alpha_{21} = \frac{1}{3}, \alpha_{12} = -\frac{1}{4}, \alpha_{22} = \frac{3}{4}; \beta_{11} = \frac{1}{3}, \beta_{21} = \frac{2}{3},$$
$$\beta_{12} = -\frac{1}{4}, \beta_{22} = \frac{3}{4}; p_{11} = \frac{4}{3}, p_{21} = -\frac{1}{4}, p_{12} = -\frac{2}{3}, p_{22} = \frac{3}{4};$$
$$q_{11} = \frac{2}{3}, q_{21} = \frac{2}{3}, q_{12} = -\frac{1}{3}, q_{22} = \frac{7}{3}; a_1 = 5.2, a_2 = 5,$$
$$b_1 = 4.7, b_2 = 5.3, c_{11} = \frac{2}{3}, c_{21} = \frac{1}{3}, c_{12} = \frac{1}{4}, c_{22} = \frac{3}{4},$$
$$d_{11} = -\frac{1}{3}, d_{12} = \frac{2}{3}, d_{21} = \frac{2}{5}, d_{22} = \frac{3}{5}, \lambda_i = 1, i = 1, 2, 3, 4.$$

By simply calculating, we can get

$$-a_1 + \frac{1}{2(1-\gamma_1)}\sum_{j=1}^2(|c_{j1}| + |\alpha_{j1}| + |\beta_{j1}|) + \frac{1}{2}\sum_{j=1}^2(|d_{1j}| + |p_{1j}| + |q_{1j}|) = -0.2 < 0$$

$$-a_2 + \frac{1}{2(1-\gamma_1)}\sum_{j=1}^2(|c_{j2}| + |\alpha_{j2}| + |\beta_{j2}|) + \frac{1}{2}\sum_{j=1}^2(|d_{2j}| + |p_{2j}| + |q_{2j}|) = -0.25 < 0$$

$$-b_1 + \frac{1}{2(1-\gamma_2)}\sum_{i=1}^2(|d_{i1}| + |p_{i1}| + |q_{i1}|) + \frac{1}{2}\sum_{i=1}^2(|c_{1i}| + |\alpha_{1i}| + |\beta_{1i}|) = -0.514 < 0$$

$$-b_2 + \frac{1}{2(1-\gamma_2)}\sum_{i=1}^2(|d_{i2}| + |p_{i2}| + |q_{i2}|) + \frac{1}{2}\sum_{i=1}^2(|c_{2i}| + |\alpha_{2i}| + |\beta_{2i}|) = -0.275 < 0$$

Since the all conditions of Theorem 2.1 are satisfied, therefore the system (9) has an unique 2π-periodic solution, which is exponential stable.

IV. Conclusion

In this paper, we have studied the existence, uniqueness and exponential stability of the periodic solution for fuzzy BAM neural networks with time-varying delays. Some sufficient conditions set up here are easily verified and these conditions are correlated with parameters and time delays of the system (1). The obtained criteria can be applied to design globally exponentially periodic oscillatory fuzzy BAM neural networks.

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