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Amplification of Compression Waves in Clean and Bubbly Liquid

Robert I. Nigmatulin, Raisa Kh. Bolotnova, Nailya K. Vakhitova, Andrey S. Topolnikov, Svetlana I. Konovalova, and Nikolai A. Makhota

***Abstract*—**The theoretical investigation is carried out to describe the effect of increase of pressure waves amplitude in clean and bubbly liquid. The goal of the work is to capture the regime of multiple magnification of acoustic and shock waves in the liquid, which enables to get appropriate conditions to enlarge collapses of micro-bubbles. The influence of boundary conditions and frequency of the governing acoustic field is studied for the case of the cylindrical acoustic resonator. It has been observed the formation of standing waves with large amplitude at resonant frequencies. The interaction of the compression wave with gas and vapor bubbles is investigated for the convergent channel. It is shown theoretically that the chemical reactions, which occur inside gas bubbles, provide additional impulse to the wave, that affect strongly on the collapses of the vapor bubbles.

***Keywords*—** acoustics, cavitation, detonation, shock waves

I. Introduction

When the pressure wave interacts with liquid there can be both the decreasing and amplification of its amplitude. The last one is caused by the influence of geometrical factor and interaction of the wave with bubbles, which exist in the liquid [1], [2]. It is known that during focusing of the spherical wave in the centre of bubble cluster its amplitude reaches 500 MPa or even higher, and the collapse of the bubbles in the centre of the cluster is about 500 times more intensive than the collapse of a single bubble at the same conditions [3].

The effect of interaction of bubble clusters with wave impulses, which causes the multiple magnification of the amplitude, is widely used in practice [4]. This effect is applied in medicine to diagnose and treat internal organs [5]. It has been obtained experimentally [6], that the rapid flow of hydrocarbon liquid (benzene) through the nozzle during cavitation with the presence of shock waves leads to the formation of nanostructures, which contain solid carbon.

After the discovery of single bubble sonoluminescence phenomenon [7] and successful experiments of bubble fusion in deuterated liquids during acoustic cavitation [8], [9] the scientific interest to the bubble, as the object of the energy cumulating, rises up essentially. As it was proved in [10], one of the main factors, which determine the intensity of gas compression inside bubbles, is the amplitude of the external force. Among all it can be increased by interaction between bubbles in the clusters [8], [9]. Thus, the actual question is to find conditions to maximize the effect of bubble detonation.

At the present paper the results of theoretical research of intensification of compression waves, which spread through the clean and bubbly liquid, are presented. Two effects are studied dealing with main mechanisms of increase of wave amplitude.

The first effect − the influence of geometrical factor on the increase of the pressure wave amplitude − is studied for the problem of acoustic waves propagation in cylindrical vessel. The periodical oscillations of the ring, which is placed on the vessel wall, produce spherically-symmetric standing acoustic waves, which spread over the liquid and interact with each other. The aim of the research is to investigate resonant conditions, when the acoustic wave amplitude becomes large and there occur the periodical regime in the vessel.

The second problem deals with spreading of the compression wave in the channel of various cross section area, which is filled with liquid with gas and vapor bubbles. During interaction of the wave with gas bubbles, containing chemically active components, the bubbles collapses, their temperature increases and initiates the chemical reactions with outcoming heat. As a result of interaction of the wave with gas bubbles its amplitude becomes much higher. Vapor bubbles, which are also forced by the pressure wave, get the additional impulse during compression stage. The goal of the research is to model the process of propagation of the compression wave in the liquid filled with bubbles of two different types and to investigate the influence of the initial and boundary conditions on the degree of the energy cumulating inside vapor bubbles.

R.I. Nigmatulin, Dr. Sc., Real Member (Academician) of Russian Academy of Sciences, Director of P.P. Shirsov Institute of Oceanology, 117997 Russia, Moscow, Nahimovski prospect 36 (e-mail nigmar@ocean.ru)

R.Kh. Bolotnova, Dr. Sc., Principal scientist of the Institute of Mechanics Ufa Branch of the Russian Academy of Sciences, 450054 Russia, Ufa, Prospect Oktyabrya 71 (e-mail: bolotnova@anrb.ru).

N.K. Vakhitova, PhD, Principal scientist of the Institute of Mechanics Ufa Branch of the Russian Academy of Sciences, 450054 Russia, Ufa, Prospect Oktyabrya 71 (e-mail: vakhitova@imech.anrb.ru).

A.S. Topolnikov, PhD, Senior scientist of the Institute of Mechanics Ufa Branch of the Russian Academy of Sciences, 450054 Russia, Ufa, Prospect Oktyabrya 71 (corresponding author, tel./fax: +7-347-2355255, e-mail: tandrew@anrb.ru).

S.I. Konovalova, PhD, Junior scientist of the Institute of Mechanics Ufa Branch of the Russian Academy of Sciences, 450054 Russia, Ufa, Prospect Oktyabrya 71 (e-mail: sveta@imech.anrb.ru).

N.A. Makhota, postgraduate student (e-mail: kolya1002@yandex.ru).

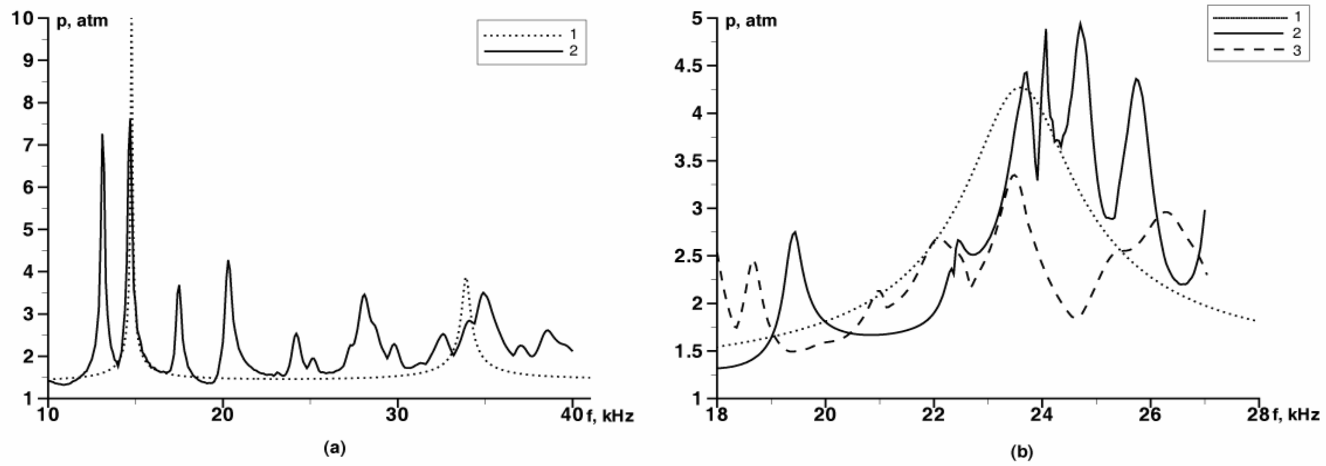

Fig.1 Maximum pressure vs. frequency under boundary conditions (4) (a) and (5) (b). Line 1 corresponds to a one-dimensional case, line 2 corresponds to a two-dimensional case, line 3 corresponds to the case taking into account the wall oscillations.

II. The Formation of High Amplitude Standing Acoustic Waves in the Cylindrical Vessel

Two-dimensional waves in a cylindrical vessel of radius R=0.031 m and height Z=0.2 m are considered. The vessel is located on a motionless surface, the top border of liquid is free. The liquid is acoustically disturbed by means of a pulsing ring ($z_1 \le z \le z_2$) on a vessel wall. Distribution of acoustic waves in the liquid is described by the wave equation in cylindrical coordinates:

$$\frac{\partial^2 p}{\partial t^2} = C_L^2 \left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{\partial^2 p}{\partial z^2} \right), \qquad (1)$$

where t is time, r, z are the cylindrical coordinates, $p = p(r,z,t)$ is the pressure in the liquid, C_L is the sound velocity in the liquid. The following initial and boundary conditions are used:

$$p(r,z,0) = p(r,Z,t) = p_0, \quad \frac{\partial p}{\partial t}(r,z,0) = \frac{\partial p}{\partial r}(0,z,t) = \\ = \frac{\partial p}{\partial z}(r,0,t) = 0, \quad \frac{\partial p}{\partial r}(R,z,t) = -\frac{1}{\rho_L}\frac{\partial^2 w}{\partial t^2}, \qquad (2)$$

where p_0 is the atmospheric pressure, ρ_L is the liquid density, $w = w(z,t)$ is the vessel wall displacement.

For the modeling of the wall oscillations the following set of equations [11] is used:

$$D\frac{\partial^4 w}{\partial z^4} + \frac{Eh}{R}w + m\frac{\partial^2 w}{\partial t^2} = p_W, \quad w(r,z,0) = \frac{\partial w}{\partial t}(r,z,0) = 0, \\ w(r,0,t) = \frac{\partial w}{\partial z}(r,0,t) = \frac{\partial^2 w}{\partial z^2}(r,Z,t) = \frac{\partial^3 w}{\partial z^3}(r,Z,t) = 0, \qquad (3) \\ \frac{\partial w}{\partial t}(r,z_1 \le z \le z_2,t) = v_r(t).$$

Here $D = Eh^3\left(12\left(1-\nu^2\right)\right)^{-1}$, E is the wall elastic modulus, ν is Poisson's ratio, h is the wall thickness, $m = \rho_{ob}h$, ρ_{ob} is the wall density, $v_r(t) = \Delta\nu(1-\cos\omega t)$, $\omega = 2\pi/f$ is the frequency, $\Delta\nu = \Delta p/(\rho_L C_L)$, Δp is the pressure amplitude, $p_W = p(R,z,t) - p_0$.

Without wall oscillations the equation (1) was solved under the following boundary conditions:

$$p(R,z_1 \le z \le z_2,t) = p_0 + \Delta p \sin(\omega t), \\ \frac{\partial p}{\partial r}(R,z < z_1,t) = \frac{\partial p}{\partial r}(R,z > z_2,t) = 0, \qquad (4)$$

or

$$\frac{\partial p}{\partial r}(R,z_1 \le z \le z_2,t) = -\frac{1}{\rho_L}\frac{\partial v_r}{\partial t}, \\ \frac{\partial p}{\partial r}(R,z < z_1,t) = \frac{\partial p}{\partial r}(R,z > z_2,t) = 0. \qquad (5)$$

The calculations were carried out for the following values of parameters: $p_0 = 1\,\text{atm}$, $C_L = 1189\,\text{m/s}$ (D-acetone), $E = 6.5\cdot 10^{10}\,\text{Pa}$ (glass), $\nu = 0.2$, $h = 1\,\text{mm}$, $\rho_{ob} = 2200\,\text{kg/m}^3$, $\Delta p = 0.2\,\text{atm}$, $z_1 = 0.05\,\text{m}$, $z_2 = 0.075\,\text{m}$, $f = 10 \div 50\,\text{kHz}$. The implicit scheme was employed to solve equations (1) and (3) numerically.

As the test calculations of resonant frequencies in an one-dimensional case have been carried out. The calculated resonant frequencies ($f_1 \approx 15\,\text{kHz}$, $f_2 \approx 35\,\text{kHz}$, $f_1' \approx 24\,\text{kHz}$, $f_2' \approx 43\,\text{kHz}$) satisfy to the deduced formulas under boundary conditions (4) and (5) correspondingly:

$$f_{\mathrm{Res}} = \frac{\mu_m C_L}{2\pi R}, \quad f'_{\mathrm{Res}} = \frac{\nu_m C_L}{2\pi R}, \qquad (6)$$

where μ_m, ν_m are the roots of Bessel functions of the first kind J_0 and J_1 correspondingly.

In Fig.1 the maximum pressure on the central vessel axis versus the frequency on the ring is shown. Under boundary condition (4) the maximum increase in amplitude has been observed at frequency 14670 Hz. Under boundary condition (5) the maximum has been observed at frequency 24220 Hz, taking into account the wall oscillations the maximum has been observed at frequency 46400 Hz, the resonances being less expressed than in case of (4).

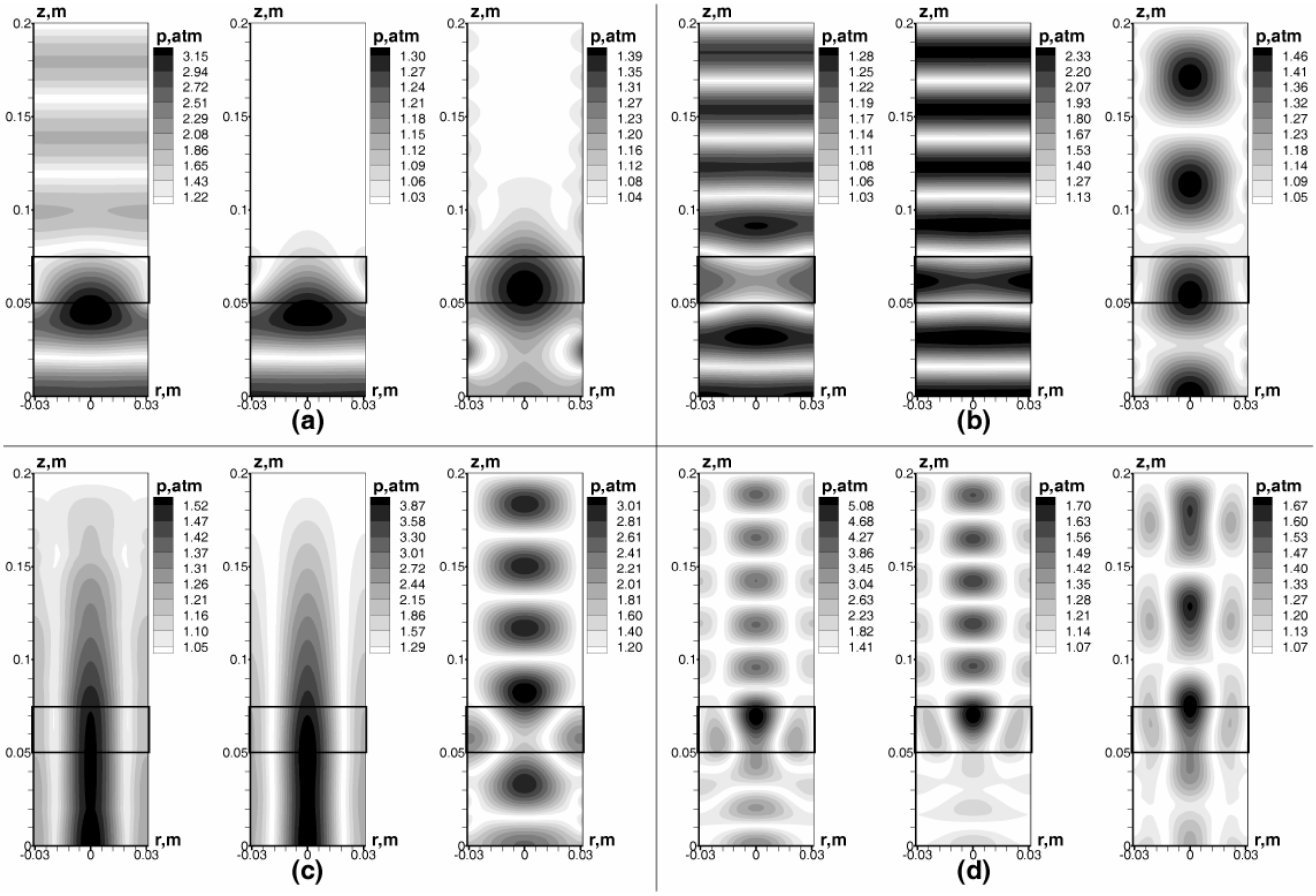

Fig.2 Maximum pressure distribution in the vessel at $f = 14700$ Hz (a), $f = 19300$ Hz (b), $f = 23600$ Hz (c), $f = 34900$ Hz (d). Left column corresponds to boundary condition (4), central column corresponds to (5), right one corresponds to the case accounting for wall oscillations.

The calculations show that periodic standing waves are formed in the liquid. In Fig. 2 distributions of the maximum pressure amplitude are shown at various frequencies and modes on the ring and at the wall. The area of the maximum pressure may be above, below or at the ring level, depending on the frequency and conditions on the wall. The account of the wall oscillations leads to that the pressure fluctuations fade at the wall.

III. Bubble Detonation in the Convergent Channel

Let us consider a channel of variable cross section (see Fig.3) filled with liquid, which contains vapor and gas bubbles. As for liquid the deuterated acetone (C_3D_6O) is used, which was employed in the bubble fusion experiments [8], [9]. The gas is the mixture of acetylene (C_2H_2) and oxygen (O_2), which react with generation of carbonic acid (CO_2) and steam (H_2O). The length of channel is $L = 1.5$ м, the radii of input and output cross sections are $R_1 = 0.02$ м и $R_2 = 0.01$ м, the length of parabolic part of the channel equals $L^* = 0.5$ м.

To describe the distribution of compression waves in the polydisperse bubble liquid the set of partial differential equations, which express the conservation of mass for the mixture components and momentum for the mixture in the one-velocity isothermal assumption, is used:

$$\frac{\partial}{\partial t}\left(\alpha_L \rho_L S\right) + \frac{\partial}{\partial x}\left(\alpha_L \rho_L u S\right) = J\,S, \qquad (7)$$

$$\frac{\partial\left(N_G S\right)}{\partial t} + \frac{\partial}{\partial x}\left(N_G u S\right) = 0, \qquad (8)$$

$$\frac{\partial\left(N_V S\right)}{\partial t} + \frac{\partial}{\partial x}\left(N_V u S\right) = 0, \qquad (9)$$

$$\frac{\partial}{\partial t}\left(\alpha_L \rho_L\, u S\right) + \frac{\partial}{\partial x}\left(\alpha_L \rho_L\, u^2 S\right) = -S\frac{\partial p}{\partial x} - \tau\Sigma\,. \qquad (10)$$

Here t is the time, x is the dimensional coordinate, ρ_L and p are the density and pressure in the liquid, u is the cross-section averaged velocity, α_L is the volume concentration of the liquid, S is the area of cross section, J is the mass flow caused by phase exchange in the vapor-liquid system, N_G and N_V are the numbers of gas and vapor bubbles correspondingly in the liquid per unit volume, $\Sigma = 2\sqrt{\pi S}$ is the perimeter of cross section, τ is the wall stress tension coefficient for mixture.

The following expressions are fulfilled:

$$\tau = \frac{1}{2} f \alpha_L \rho_L u \left| u \right|, \qquad (11)$$

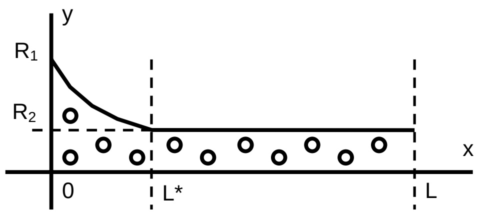

Fig. 3 The sketch of the channel.

where

$$f = \begin{cases} 16/\mathrm{Re}, & \mathrm{Re} = 2\sqrt{\pi S}u/\mu_L < 2300, \\ 0.0791\,\mathrm{Re}^{-0.25}, & \mathrm{Re} \geq 2300, \end{cases} \quad (12)$$

μ_L is the dynamic viscosity of the liquid,

$$\alpha_G = \frac{4}{3}\pi a_G^3 N_G, \quad \alpha_V = \frac{4}{3}\pi a_V^3 N_V, \quad (13)$$

where α_G (α_V) and a_G (a_V) are the volume concentration and radius of gas (vapor) bubble,

$$J = -4\pi N_V a_V^2 j, \quad (14)$$

where j is the intensity of mass inflow per unit bubble surface, which is calculated according to the saturation conditions [10].

The generalized Rayleigh-Plesset equation is employed to describe evolution of gas and vapor bubble's radii:

$$\left(1 - \frac{\dot{a}_k}{C_L}\right) a_k \ddot{a}_k + \frac{3}{2}\left(1 - \frac{\dot{a}_k}{3C_L}\right)\dot{a}_k^2 = \\ = \left(1 + \frac{\dot{a}_k}{C_L}\right)\frac{p_{Lk} - p}{\rho_{L0}} + \frac{a_k}{\rho_{L0} C_L}\frac{d}{dt}\left(p_{Lk} - p\right), \quad (15)$$

where ρ_{L0} is the density of undisturbed liquid, C_L is the speed of sound in the liquid, p_{Lk} is the pressure in the liquid at the gas ($k = G$) and vapor ($k = V$) bubble interface:

$$p_{Lk} = p_k - \frac{2\sigma}{a_k} - \frac{4\mu_L \dot{a}_k}{a_k}, \quad (16)$$

where σ is the surface tension coefficient at the bubble interface.

The set of equations (7)-(16) are closed with equations of state for liquid, gas and vapor.

The acoustically compressible liquid is described by the following equation:

$$p = p_0 + C_L^2\left(\rho_L - \rho_{L0}\right). \quad (17)$$

Gas and vapor densities depend not only from the pressure, but also from the temperature due to the surface mass transfer and chemical reactions. Let us suppose that the distribution of gas and vapor density, pressure, temperature and volume concentration are spatially uniform. To describe the variation of parameters inside bubbles the set of ordinary differential equations is employed [12]. The temperature at the gas bubble interface is assumed to be equal to the liquid temperature and differs from the temperature inside the bubble. The surface temperature of vapor bubble otherwise will be equal to the interior temperature and differs from the liquid temperature [13]. To obtain liquid temperature gradient near the vapor bubble interface the heat conductivity equation is solved [12].

At the initial time moment the liquid with bubbles stays at the dynamical equilibrium:

$$u(x,0) = 0, \quad p(x,0) = p_0, \\ N_G(x,0) = N_{G0}(x), \quad N_V(x,t) = N_{V0}(x). \quad (18)$$

To describe the spreading of compression waves in the convergent channel from the left boundary to the right the following boundary conditions are used:

$$u(0,t) = u_1(t), \quad p(L,t) = p_0, \\ N_G(0,t) = N_{G1}(t), \quad N_V(0,t) = N_{V1}(t). \quad (19)$$

To investigate the high temperature processes inside vapor bubbles during collapse the complicated model is used instead of homogeneous model [10], which accounts for propagation of spherically shock waves inside the bubble and in the surrounded liquid. The interior of the bubble and liquid near its interface are described by wide-range equations of state, which account for evaporation and condensation processes at the bubble interface, and also for dissociation and ionization in the bubble centre [14].

The set of equations is solved numerically. For derivation of equations (7)-(10) the non-implicit numerical scheme is used, based upon control volume method [15]. The high order Dorman-Prince method is applied to solve the ordinary differential equations for simulation of gas and vapor bubbles parameters [16]. As for modeling of the final stage of vapor bubble collapse with possibility of shock waves generation the 1st order Godunov scheme is applied [17].

Consider the liquid at the initial time moment filled with uniformly distributed in space bubbles of radii $a_{G0} = a_{V0} = 10^{-3}$ m and volume concentrations $\alpha_{G0} = \alpha_{V0} = 5 \cdot 10^{-4}$. The initial pressure in the liquid equals $p_0 = 1$ atm and the initial liquid temperature equals to the saturation temperature of D-acetone $T_{l0} = 322$ K. The other parameters are: $\rho_{L0} = 858$ kg/m^3, $\mu_L = 4 \cdot 10^{-4}$ Pa·s, $C_L = 1189$ m/s, $\sigma = 0.026$ N/m. The compression wave is generated at the left boundary of the channel, where the liquid velocity changes spasmodic from zero to u_1.

The results of numerical simulation are presented in Fig. 4 for $u_1 = 0.3$ m/s at time moment $t = 1.2$ ms. The amplitude of the compression wave, which is initiated at $x = 0$, grows while the cross section area decreases. The wave has non-monotonic structure because of bubble's radial oscillations. When the compression wave propagates further to the right its amplitude becomes weaker. The oscillations of gas bubbles are more intensive than the vapor bubbles oscillations. The maximum temperature inside vapor bubble is about 400K, while the gas bubble temperature is 600K. At this temperature no chemical reactions occur inside gas bubbles, the volume concentration of products of burning of acetylene does not exceed 1%.

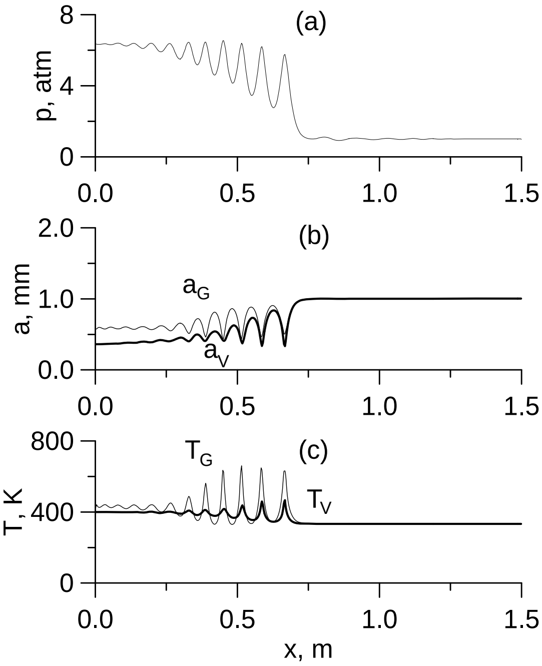

Fig. 4 The spatial distribution of pressure in the liquid (a), gas and vapor bubble radii (b) and temperatures (c). $a_{G0} = a_{V0} = 10^{-3}$ m, $\alpha_{G0} = \alpha_{V0} = 5 \cdot 10^{-4}$, $u_1 = 0.3$ m/s. The results corresponds to the time moment $t = 1.2$ ms.

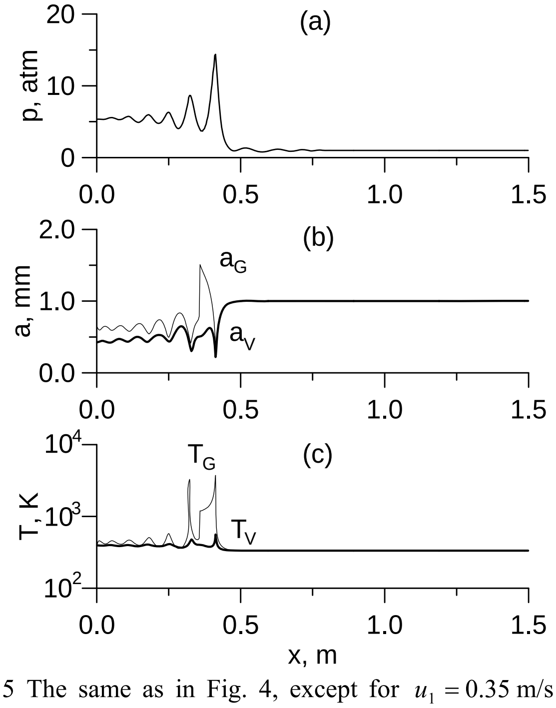

Fig. 5 The same as in Fig. 4, except for $u_1 = 0.35$ m/s and $t = 0.7$ ms.

When the amplitude of the initial compression wave becomes larger the situation changes in principal. In Fig. 5 the spatial distribution of liquid and bubble parameters are shown at time moment $t = 0.7$ ms, when the initial jump of liquid velocity at $x = 0$ equals $u_1 = 0.35$ m/s. In this case the formation of detonation shock wave occurs, that causes the amplification of the pressure wave in the liquid and increases bubble's oscillations. The average temperature inside gas bubbles during compression is higher than the average temperature inside vapor bubbles, nevertheless due to formation of a strong shock wave the peak of temperature in the vapor bubble centre reaches 10^8 K or even higher.

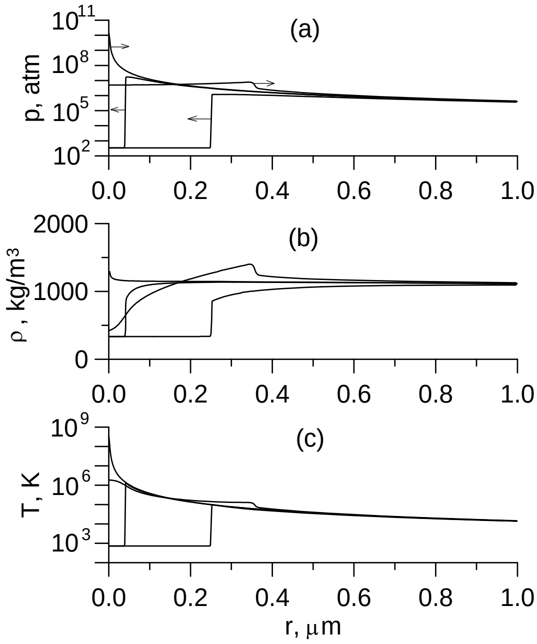

Fig. 6 The spatial profiles of vapor pressure (a), density (b) and temperature (c) at the time moments, when the strong shock wave focuses at the bubble centre and reflects from it.

In Fig. 6 the distribution of parameters inside vapor bubble are presented at the time moments, when the strong shock wave focuses at the bubble centre and reflects from it. The results are obtained by the help of numerical code, which uses complicated mathematical model accounting for the non-uniform spatial distribution of gas and liquid parameters and wide-range equation of state of D-acetone [10]. At the moment of maximum bubble compression a strong converging shock wave is generated inside vapor bubble, which leads to the extreme rise in pressure and temperature.

IV. Conclusion

The theoretical research of pressure wave amplification during its propagation through clean and bubbly liquid has been carried out.

Formation of standing waves in the cylindrical vessel filled with liquid has been observed. The resonant frequencies, at which the maximum increase in amplitude of pressure at the central axis of the vessel is observed, have been received. It has been shown that the area of the maximum pressure can be below, above or at the ring level. The account of the wall oscillations leads to that the pressure fluctuations fade at the wall.

The evolution of the compression wave in the convergent channel filled with liquid D-acetone with gas and vapor bubbles has been studied. It has been obtained that the strong detonation wave forms, which increase in its amplitude substantially and causes the intensive collapses of vapor bubbles.

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