Qualitative Possibilistic Influence Diagrams

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Abstract— Influence diagrams (IDs) are one of the most commonly used graphical decision models for reasoning under uncertainty. The quantification of IDs which consists in defining conditional probabilities for chance nodes and utility functions for value nodes is not always obvious. In fact, decision makers cannot always provide exact numerical values and in some cases, it is more easier for them to specify qualitative preference orders. This work proposes an adaptation of standard IDs to the qualitative framework based on possibility theory.

Key words: decision making, influence diagrams, qualitative utility, possibility theory.

I. Introduction

Influence diagrams introduced by Howard and Matheson in 1981 [8] provide an efficient tool for decision making in intelligent systems, they allow to model and evaluate complex decision problems via a compact graphical representation.

Influence diagrams are quantified by conditional probabilities relative to uncertain variables and utility functions evaluating decision maker's satisfaction. Nevertheless, the specification of this numerical component is not always obvious. In fact, experts may be enable to provide exact conditional probabilities. Moreover, it is generally more easier for them to provide a preferential relation between different consequences that expresses utilities rather than numerical values.

To overcome these limitations, this paper proposes a new approach extending standard IDs to a qualitative framework based on the possibility theory [5]. In fact, this theory can be interpreted in two ways: qualitatively, if the handled values reflect only an ordering between different states of the world or quantitatively, if the handled values make sense in the ranking scale.

Our idea is to use the qualitative aspect of this theory in order to specify qualitative relations between chance nodes.

Concerning utilities, in the literature we can distinguish two kinds of utility: cardinal utility [1] when the decision maker is able to express his satisfaction by exact numerical values and ordinal utility [1][17] otherwise.

Different combinations of the quantification between chance and utility nodes offer several kinds of possibilistic influence diagrams. In this paper, we are interested in qualitative ones.

Our new approach named, qualitative possibilistic influence diagrams, benefits from the simplicity and efficacity of standard influence diagrams and from the suitability of the possibility theory for modeling qualitative uncertainty.

We also propose an evaluation method to generate optimal decisions maximizing the expected utility. The proposed method is based on the transformation of qualitative possibilistic influence diagrams into qualitative possibilistic causal networks recently proposed in [2] and on making inference in this secondary structure.

This paper is organized as follows: Section 2 provides a brief description of the basics of influence diagrams. The necessary background on possibility theory is recalled in Section 3. Ordinal utility theory is briefly presented in section 4. Then in Section 5, we will define qualitative possibilistic influence diagrams. Finally, section 6 presents an evaluation method relative to these graphical decision models.

II. Influence Diagrams

An influence diagram is a directed acyclic graph (DAG) denoted by G=(N, A) where A is the set of arcs and $N = \{C, D, V\}$ such that:

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- $C = \{C_1, .., C_n\}$ is the set of chance nodes represented by circles.
- $D = \{D_1, .., D_m\}$ is the set of decision nodes represented by rectangles.
- $V = \{V_1, .., V_k\}$ is the set of value nodes represented by lozenges.

In what follows c_{ij} (resp. d_{ij}, v_{ij}) denotes an instance of the variable C_i (resp. D_i, V_i).

Different links existing between chance nodes are quantified by conditional probability tables representing dependencies between them. Namely, each chance node C_i will be attached to its probability in the context of its parents, denoted by $Pa(C_i)$ ($pa(C_i)$ denoted any instance of $Pa(C_i)$). Whereas, each value node V_i is characterized by a utility function U in the context of its parents. In fact, the set of combinations of different parents of value nodes represent the set of consequences relative to the decision problem represented by the influence diagram.

Decision nodes act differently from remaining nodes, since they are not quantified.

Once the ID constructed, it can be used to identify better decisions via evaluation algorithms which allow to generate the optimal strategy yielding to the highest expected utility.

IDs can be evaluated through direct methods [14] which operate directly on the original structure or indirect methods [4][16][18] which transform them into Bayesian networks [9] [10] [11] and reduce the ID evaluation problem into a Bayesian networks inference one. Direct evaluation requires a lot of probabilistic calculations which justify the great development of indirect methods initiated by Cooper [4] for the particular case of influence diagrams with a unique value node.

The key idea of indirect methods is to transform decision and value nodes into chance nodes. Indeed, each decision node D_i is converted into a chance node with an equi-probability distribution. The value node V is transformed into a binary chance node with two values False (F) and True (T) and its utility function is converted into a probability distribution by rescaling it in the unit interval [0, 1].

Finally, the maximal expected utility is computed begining by the last decision D_m to the first one D_1, by considering for each node a set of evidence E updated in the light of the previous step.

More formally, for each decision node D_i, the maximal expected utility is computed as follows:

$$MEU(D_i, E) = K_1.max_{d_{ij}}[P(v = T \mid d_{ij}, E)] - K_2 \quad (1)$$

where $\forall pa(V) \in Pa(V)$:
$K_1 = max_{pa(V)}(U(pa(V))) - min_{pa(V)}(U(pa(V)))$
and $K_2 = -min_{pa(V)}U(pa(V))$.

III. Possibility theory

This section briefly recalls basic elements of possibility theory, for more details see [6] [7].

The basic buildings block in the possibility theory is the notion of possibility distribution denoted by π, it is a mapping from the universe of discourse denoted by Ω to the unit interval [0,1].
This scale has two interpretations, a quantitative one when the handled values have a real sense and a qualitative one when the handled values reflect only an ordering between the different states of the world. In the first case, the product operator can be applied and in the second one, the min operator is used.

A possibility degree is the value from the interval [0,1] associated to each element ω of Ω. The possibility measure of any subset $\psi \subseteq \Omega$ is defined as follows:

$$\Pi(\psi) = max_{\omega \in \psi}\pi(\omega) \quad (2)$$

A possibility distribution is said to be normalized, if $max_{\omega \in \psi}\pi(\omega) = 1$.

In the possibilistic framework, extreme forms of partial knowledge can be represented as follows:

- Complete knowledge:

$$\exists\omega_i \in \Omega, \; s.t \; \pi(\omega_i) = 1 \; and \; \omega_j \neq \omega_i, \pi(\omega_j) = 0 \quad (3)$$

- Total ignorance:

$$\forall\omega_i \in \Omega, \pi(\omega_i) = 1 \quad (4)$$

The two interpretations of the possibilistic scale induce two definitions of the conditioning [5]:

- min-based conditioning relative to the ordinal setting:

$$\pi(\omega|_m\psi) = \begin{cases} 1 & if \; \pi(\omega) = \Pi(\psi) \; and \; \omega \in \psi \\ \pi(\omega) & if \; \pi(\omega) < \Pi(\psi) \; and \; \omega \in \psi \\ 0 & otherwise. \end{cases}$$
$$(5)$$

- product-based conditioning relative to the numerical setting:

$$\pi(\omega|_p\psi) = \begin{cases} \frac{\pi(\omega)}{\Pi(\omega)} & if \omega \in \psi \\ 0 & otherwise. \end{cases} \quad (6)$$

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IV. Ordinal utility theory

As mentioned above, each value node in the influence diagram is quantified by a utility function in the context of its parents. Both cardinal and ordinal utility can be used for this quantification.

The problem with cardinal utility comes from the difficulty in expressing the utilities by numerical values, and in finding the appropriate measurement index (metric). This is not the case in the context of ordinal utility, when we suppose that the decision maker can only provide a total preference relation on the different states of the world.

Let A, B and C be three states and $\succeq$ be the relative preference relation:

- $A \succeq B$ means that A is more preferred than B,
- $A \preceq B$ means that A is less preferred than B,
- $A \sim B$ means that A and B are equally preferred.

Von Neumann and Morgenstern [17] have defined the following axiom's system for any preference relation:

- Axiom 1. Completeness (Orderability): It is always possible to state either that $A \succeq B$ or that $B \succeq A$ or $A \sim B$.
- Axiom 2. Reflexivility: any consequence A is always at least as preferred as itself: $A \succeq A$.
- Axiom 3. Transitivity: If $A \succeq B$ and $B \succeq C$ then $A \succeq C$.

If a preference relation $\succeq$ verifies these axioms then there exists a utility function U such that:

$$A \succeq B \quad iff \quad U(A) \geq U(B) \qquad (7)$$

and

$$A \sim B \quad iff \quad U(A) = U(B) \qquad (8)$$

V. Qualitative possibilistic influence diagrams

A qualitative possibilistic influence diagram, denoted by ΠID_{min}^{O}, is defined by two components:

Graphical component represented by a DAG $G = (N, A)$ having the same structure than stantard influence diagrams (c.f section 2).

Numerical component evaluating different links in the graph concerning chance nodes and utilities for value nodes in the following way:

For each chance node C_i, experts should provide conditional possibility degrees $\Pi(c_{ij} \mid pa(C_i))$ of each instance c_{ij} of C_i and each instance $pa(C_i)$ of $Pa(C_i)$

of its parents. In order to satisfy the normalization constraint, these conditional distributions should satisfy:

$$max_{c_{ij}}\Pi(c_{ij} \mid pa(C_i)) = 1, \ \forall pa(C_i) \qquad (9)$$

Note that for root chance nodes (i.e $(Pa(C_i) = \emptyset)$, this will correspond to an a prior possibility distribution $\Pi(c_{ij})$ satisfying $max_{c_{ij}}\Pi(c_{ij}) = 1$.

For the value node V, we assume that ordinal utilities are used for the quantification. Namely, the decision maker can express a preferential relation $\succeq$ between different consequences.

To develop possibilistic chain rule, utility nodes and links into decision nodes in possibilistic influence diagrams must be ignored, as in standard influence diagrams [10].

As mentioned before, decision nodes act differently from chance nodes, thus it is meaningless to specify prior possibility distribution on them. Moreover, it has no meaning to attach a possibility distribution to children nodes of a decision node D_i unless a decision d_{ij} has been taken.

Therefore what is meaningful is $\Pi(c_{ij} \mid do(d_{ij}))$, where $do(d_{ij})$ is the particular operator defined by Pearl [12], and not $\Pi(c_{ij}, d_{ij})$. When iterating this reasoning we can bunch the whole decision nodes together and express the joint possiblity distribution of different chance nodes conditioned by decision nodes. This means that if we fix a particular configuration of decision nodes, say d, we get a min-based possibilistic causal network [2] representing $\Pi(C \mid do(d))$ i.e the joint possibility relative to C, in the context of the decision d.

In other words, the joint distribution relative to C remains the same when varying d. Thus, using the possibilistic chain rule relative to min-based causal network [2], we can infer the following chain rule relative to qualitative possibilistic IDs:

$$\pi(C \mid_m D) = min_{C_i \in C}\Pi(C_i \mid_m Pa(C_i)) \qquad (10)$$

Example 1: Let ΠID_{min}^{O} be a qualitative influence diagram with three chance nodes (A, B and C), one decision node (D) and one value node (V)(see Figure 1). The a priori and conditional possibility distributions for A, B and C are presented in Table 1.

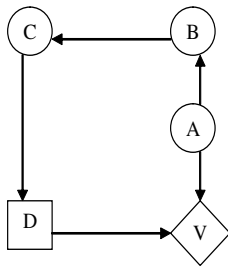The preference order between different combinations of parents of the value node V (i.e A and D) is

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Fig. 1.   Example of an influence diagram

TABLE I

Possibility distributions relative to chance nodes

A	$\Pi(A)$	A	B	$\Pi(B\|A)$	B	C	$\Pi(C\|B)$
T	1	T	T	0.9	T	T	1
F	0.6	F	T	0.2	F	T	0.3
		T	F	1	T	F	0.2
		F	F	1	F	F	1

expressed by:
$(D = Act2 \wedge A = F) \succeq (D = Act1 \wedge A = T) \succeq$
$(D = Act1 \wedge A = F) \succeq (D = Act2 \wedge A = T)$.

VI. Evaluation of qualitative influence diagrams

To evaluate qualitative possibilistic influence diagrams, we propose to define the possibilistic counterpart of the indirect Cooper's method [4]. This choice allows us to avoid heavy computations of direct methods. In addition, we will use the possibilistic counterpart of Bayesian networks, recently proposed in [2]. The principal steps of our evaluation's method are detailed in what follows.

A. Decision nodes transformation

Each decision node D_i in ΠID_{min}^O is transformed into a chance node which will be quantified by the following conditional possibility distribution in the context of its parents:

$$\Pi(d_{ij} \mid pa(D_i)) = 1, \quad \forall d_{ij}, \forall pa(D_i) \qquad (11)$$

This distribution expresses the state of total ignorance with respect to the new chance node D_i. Note that this quantification is more appropriate than the one used in standard influence diagram, since equiprobability represents randomness rather than total ignorance.

B. Value node transformation

The value node V is converted into a new binary chance node having two values: False (F) and True (T). The new chance node is characterized by a

possibility distribution deduced from the original preference relation.

The following proposition gives a transformation function of the preference relation between different consequences which satisfies VNM axioms into numerical utilities.

Proposition 1: Let $\succeq$ be a preference relation between different consequences. Then, the order induced by $\succeq$ can be transformed into a numerical scale as follows:

$$U(pa(V)) = card(Pa(V)) - rank(pa(V)) + 1 \quad (12)$$

where card(Pa(V)) is a function that determines the number of combinations of $pa(V) \in Pa(V)$.
rank(pa(V)) is the rank of pa(V) in the preference order.

Proof 1: Let $pa_1(V)$ and $pa_2(V)$ be two consequences such that $pa_1(V) \succeq pa_2(V)$, we have:
$rank(pa_1(V)) \le rank(pa_2(V))$
$\Rightarrow -rank(pa_1(V)) + 1 \ge -rank(pa_2(V)) + 1$
$\Rightarrow card(Pa(V)) - rank(pa_1(V)) + 1 \ge$
$card(Pa(V)) - rank(pa_2(V)) + 1$
$\Rightarrow U(pa_1(V)) \ge U(pa_2(V))$.

The set of utilities assigned to the different consequences according to the preference relation will be transformed into a possibility distribution. Namely, we are interested in computing $\Pi(v = T \mid Pa(V))$ and $\Pi(v = F \mid Pa(V))$ which can be obtained as follows:

$$\Pi(v = T \mid Pa(V)) = \frac{U(Pa(V)) - U_{min}}{U_{max} - U_{min}} \qquad (13)$$

$$\Pi(v = F \mid Pa(V)) = \frac{U(Pa(V)) - U_{max}}{U_{min} - U_{max}} \qquad (14)$$

Where U_{max} and U_{min} are the maximal utility level and the minimal utility level, respectively.

The obtained qualitative possibilistic distribution $\Pi(V \mid Pa(V))$ is sub-normalized. Then, to satisfy the normalization constraint, the qualitative possibilistic distribution should be transformed as follows:
$\forall pa(V) \in Pa(V) \ and \ v \in \{T, F\}$,
$\Pi(v|_m pa(V)) =$

$$\begin{cases} 1 & if \ max(\Pi(v|_m pa(V)), \Pi(\neg v|_m pa(V))) = \Pi(v|_m pa(V)) \\ \Pi(v|_m pa(V)) & otherwise. \end{cases}$$

$$(15)$$

After the transformation phase, we should determine for each decision node D_i the Maximal

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Expected Utility with respect to the set of evidences E in order to generate the optimal decision strategy.

This computation is based on the max and min operators instead of standard addition and multiplication ones as follows:

$$MEU(D_i, E) =$$

$$max_{d_{ij}}[max_{pa'(V)}min(\Pi(v = T \mid pa(V)), \Pi(pa'(V) \mid d_{ij}, E))] \quad (16)$$

where Pa'(V) denotes the set of chance nodes in Pa(V) and pa'(V) is an instance of Pa'(V).

Note that $\Pi(v = T \mid pa(V))$ is a transformation of U(Pa(V)) into the unit interval [0,1] using (13) in order to let $U(Pa(V))$ and $\Pi(pa'(V) \mid d_{ij}, E)$ commensurable.

To compute $\Pi(pa'(V) \mid d_{ij}, E)$ and $\Pi(v = T \mid pa(V))$, a min-based propagation algorithm for qualitative possibilistic networks can be applied. Indeed, two min-based propagation algorithms have been defined according to the nature of the DAG in the possibilistic causal network [3]. Namely, the qualitative possibilistic adaptation of the centralized version of Pearl's algorithm is used when the DAG is singly connected, and the qualitative possibilistic adaptation of junction trees propagation are appropriate for multiply connected DAGs. In the case that these two algorithms are blocked, the anytime algorithm [2] can be used.

Example 2: Let us continue with Example 1, after the transformation of the influence diagram presented in Figure 1, we will have the possibilistic network represented in Figure 2.
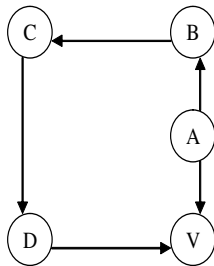

Fig. 2.   Obtained possibilistic network

Table 1 presents the a priori and conditional possibility distributions for A, B and C.
The decision node D is transformed into a chance node using (11) and conditional possibility degrees $\Pi(D \mid C)$ are presented in Table 2.

TABLE II
The possibility distribution of the node D

C	D	$\Pi(D \mid C)$
T	Act1	1
F	Act2	1
T	Act2	1
F	Act1	1

The preference relation will be transformed into numerical utilities using (12) as presented in Table 3.

TABLE III
Ordinal utilities

A	D	U(A,D)
T	Act1	3
F	Act2	4
T	Act2	1
F	Act1	2

To compute possibility degrees for the value node presented in Table 4, (13) and (14) are used.

TABLE IV
The possibility distribution of the value node

V	A	D	$\Pi(V \mid A, D)$	$\Pi(V\mid A, D)$ normalized
T	T	Act1	2/3	1
T	F	Act2	1	1
T	T	Act2	0	0
T	F	Act1	1/3	1/3
F	T	Act1	1/3	1/3
F	F	Act2	0	0
F	T	Act2	1	1
F	F	Act1	2/3	1

Suppose that we receive a certain information saying that the variable C takes the value T, then $\Pi(pa'(V) \mid d_{ij}, E))$ is computed using a min-based propagation algorithm in Junction Trees.
We have:
$min(\Pi(v = T \mid A = T, D = Act1)$,
$\Pi(A = T \mid D = Act1, C = T)) = min(0.666, 0.666)$,
$min(\Pi(v = T \mid A = F, D = Act1)$,
$\Pi(A = F \mid D = Act1, C = T)) = min(0.6, 0.3)$,
$min(\Pi(v = T \mid A = T, D = Act2)$,
$\Pi(A = T \mid D = Act2, C = T)) = min(0, 0.9)$,
$min(\Pi(v = T \mid A = F, D = Act2)$,
$\Pi(A = F \mid D = Act2, C = T)) = min(0.333, 0.3)$,

Thus, $MEU(D, C = T) = 0.666$ and $D^* = Act_1$.

VII. Conclusion

In this paper we have developed a new approach for decision making under uncertainty, namely qualitative influence diagrams. This new approach allows to experts to quantify the dependencies between chance nodes qualitatively by means of possibility distributions. Moreover, utility nodes are supposed to be quantified by qualitative utilities. An indirect evaluation's method was proposed allowing the determination of the optimal strategy which offers the maximal expected utility. As a future work, we will investigate to optimize indirect evaluation's method to deal with IDs with more than one value node. Alternative methods to quantify value nodes are under study, namely, binary qualitative utility [14].

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