Statistical Analysis of Stresses in Rigid Pavement

Aleš Florian, Lenka Ševelová, and Rudolf Hela

Abstract—Complex statistical analysis of stresses in concrete slab of the real type of rigid pavement is performed. The computational model of the pavement is designed as a spatial (3D) model, is based on a nonlinear variant of the finite element method that respects the structural nonlinearity, enables to model different arrangement of joints, and the entire model can be loaded by the thermal load. Interaction of adjacent slabs in joints and contact of the slab and the subsequent layer are modeled with help of special contact elements. Four concrete slabs separated by transverse and longitudinal joints and the additional subgrade layers and soil to the depth of about 3m are modeled. The thickness of individual layers, physical and mechanical properties of materials, characteristics of joints, and the temperature of the upper and lower surface of slabs are supposed to be random variables. The modern simulation technique Updated Latin Hypercube Sampling with 20 simulations is used for statistical analysis. As results, the estimates of basic statistics of the principal stresses $\sigma 1$ and $\sigma 3$ in 53 points on the upper and lower surface of the slabs are obtained.

Keywords—concrete, FEM, pavement, simulation.

I. INTRODUCTION

COMPLEX analysis of pavements is often very difficult for design practice. Rheological properties of materials, cracking, joints, contact of concrete slabs in joints, contact of slab and subsequent layer, temperature changes, non-homogenity of pavement base, water regime in the subgrade, environmental changes, etc. influence serviceability of the structure in a decisive way. The problem is moreover complicated by the fact that the input data are generally random variables. Further source of uncertainties stems from vagueness of input data.

The combination of the proper analytical model taking into account specific properties of the particular type of structure with modern simulation techniques seems to be an effective tool for solution of the problem [1] - [3]. The analysis of a pavement using these methods provides the designer with reliability limits of the structural response and enables the determination of possible critical development.

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The results of the analysis also enable finding out which input variables require special attention due to their random variability dominantly influencing the structural behavior.

The behavior of the older type of rigid pavement is analyzed. This type of pavement is made from plain concrete, no dowels are used, and joints are made during laying of concrete. Dimensions of individual concrete slabs are 7,5 x 3,75 m, see Fig. 1. The structure is loaded by the self-weight of concrete slabs, by the thermal loading due to the temperature difference between the upper and lower surface of the slab, and by the load of intensity 50 kN at a distance of 0,25 m from the edge of slab - see point 26 in Fig. 1. Thus the total state of stress in the slab results from all three different sources of load acting together.

The computational model is based on the nonlinear finite element analysis. Four concrete slabs, all other layers and longitudinal and transverse joints are modeled as 3D space. Joints, contact of slabs in joints, contact of slabs and subsequent layer, and the thermal loading are modeled in detail.

Total 17 basic random input variables describing layer thicknesses, mechanical properties of materials, characteristics of joints and temperature on both surfaces of concrete slabs are used in the study. They are described by the assumed cumulative distribution functions (generally three-parametric) and by the appropriate statistical parameters. The influence of uncertainties in input variables on the behavior of the pavement is respected in the analysis with help of numerical simulation techniques [4]. The modern simulation technique Updated Latin Hypercube Sampling with 20 simulations is used [5], [6].

The statistical analysis of principal stresses $\sigma 1$ and $\sigma 3$ in concrete slabs is performed to show possibilities of reliability methods in analysis of real pavement structures. The stresses are evaluated in 53 points on the upper and lower surface (totally 106 points) of the concrete slabs, see Fig. 1. Sign convention is chosen so that positive stresses are tensile, while the negative stresses are compressive. Principal stress $\sigma 1$ represents an extreme value of tensile stress in the given point of the structure, while $\sigma 3$ is an extreme value of compressive stress, which arises due to the spatial stress state. Although the calculation and measurement of deflections on the pavement plays in today's practice the most important role, the calculation of stresses seems to be in fact much more important. Principal stress represents the extreme normal stress
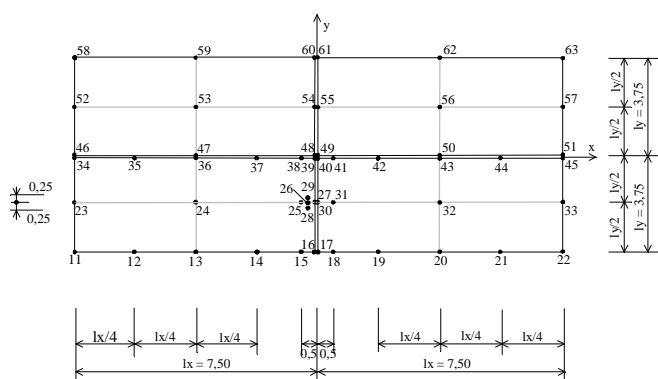
Fig. 1 Points for evaluation of stresses

at a given point of the structure and thus it is the crucial characteristic which would be used in dimensioning process. If the principal stress exceeds the tensile strength of the material, a local tensile crack is created. If it exceeds the compressive strength of the material (for pavements only hypothetically), the material is locally crushed.

The result example of statistical analysis of principal stresses is provided. These results are compared with results obtained from deterministic analysis. The aim is to demonstrate the practical applicability of reliability methods in analyzing the behavior of pavements, their possibilities to supplement or even replace experimental methods, the nature of the information obtained, and finally, the possibility to quantify the reliability of pavements.

II. ANALYTICAL MODEL

The computational model of the pavement is developed in ANSYS system [7]. Its main features are:

- a parametric model, open and flexible,
- a nonlinear model that respects the structural nonlinearity,
- designed as a spatial (3D) model that models the four adjacent concrete slabs with subsequent layers and the surrounding soil,
- allows to model different arrangement of joints,
- the interaction of adjacent slabs is realized with help of special contact elements which prevent the transmission of tensile stress,
- contact between a slab and a subsequent layer is also realized with the contact elements, which allow realistic modeling of lifting of corners and the center of the slab,
- each layer can be of any thickness and of any material,
- the model can be loaded by thermal loading,
- external load can be applied at any point of the slab,
- displacements, rotations, strains, normal or tangential stresses etc. can be evaluated at any point in any layer of the pavement structure.

The geometry of the structure is modeled by elements BRICK45. It is the eight-mode element with three degrees of freedom (UX, UY, UZ) in each node.

Attention is paid to modeling of concrete slab contact with subsequent layer and interaction of adjacent slabs in joints. In these areas the so-called contact problem occurs, in which

tensile stresses can not be transmitted. This is the case of structural nonlinearity and the pavement modeling therefore becomes nonlinear. Thus the solution is divided into individual iteration steps and the Newton-Raphson method is used in each step. The actual contacts are modeled with help of special contact elements. The contact element connects appropriate nodes of finite element mesh and operates in compression as the imaginary spring with given stiffness, but if it is pulled, the nodes behave independently.

The proposed model of joints tries to simulate at least some of the complex phenomena that occur here. The adjacent slabs interact due to their mutual contact, due to the material in joints, and also due to stress transmitted through the other layers of the structure.

III. SIMULATION TECHNIQUE

The modern simulation technique Updated Latin Hypercube Sampling [5], [6] with 20 simulations is used for statistical analysis. It is an improved variant of Latin Hypercube Sampling [4]. The method keeps the methodology of Latin Hypercube Sampling, but uses the improved strategy of generating input samples based on specially modified tables of random permutations of rank numbers. The modified tables consist of random permutations that are mutually statistically independent. Using of Updated Latin Hypercube Sampling generally results to the further increase of accuracy, quality and reliability of the results obtained from reliability analysis. The detailed description of Updated Latin Hypercube Sampling can be found in [5], [6].

IV. INPUT RANDOM VARIABLES

Total of 17 variables are considered as random input variables, see Table I and Table II (units in MPa, mm, ^{0}C). Their statistical parameters are carefully evaluated taking into account the data obtained from the in-situ measurements, experimental tests, data from technological handbooks and scientific publications and the corresponding standards. The influence of the current level of construction process and technological discipline is also taken into account. To derive appropriate statistical parameters of input variables, the following procedure is utilized. At first, the limits are specified (minimum, maximum and mean value) in which input variables will occur with a high probability, see Table I. Then, based on the assumption that values smaller than the minimum value and higher than the maximum value can occur only with a low probability, and choosing the appropriate cumulative distribution function (CDF) (N – normal, LN – three-parametric lognormal, TN - truncated normal), the other required statistical parameters are determined – coefficient of variation (COV) and skewness, see Table II. The normal and truncated normal CDF are used for symmetrically distributed variables, the three-parametric lognormal CDF for the other case. For simplicity, the mutual statistical independence of input variables is considered with the following exception - the temperatures of the upper and lower surface of the slabs are supposed to be fully statistically dependent.

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TABLE I
LIMITS OF INPUT VARIABLES

No.	Layer	Input variable	mean	min	max	CDF
X1	concrete slab	thickness	220	180	250	LN
X2		Young modulus	37500	30000	45000	N
X3		Poisson's coefficient	0,20	0,19	0,21	N
X4	top road base	thickness	200	150	250	N
X5		Young modulus	5000	500	15000	LN
X6		Poisson's coefficient	0,25	0,20	0,30	N
X7	bottom road base	thickness	250	150	650	LN
X8		Young modulus	120	80	200	LN
X9		Poisson's coefficient	0,30	0,25	0,35	N
X10	subgrade	Young modulus	80	30	150	LN
X11		Poisson's coefficient	0,35	0,30	0,45	LN
X12	joints	width transversal	20	15	25	N
X13		width longitudinal	1,5	0	3	TN
X14		Young modulus	150	50	1000	LN
X15		coefficient of friction	0,5	0,1	0,9	N
X16	temperature	upper surface	11	-10	40	LN
X17		lower surface	10	5	20	LN

The derived statistical parameters of input variables used in the presented study take into account uncertainties due to their random nature and also the uncertainties due to our incomplete knowledge of the structure, insufficient experimental research, modeling errors, vagueness of input data etc.

TABLE II
DERIVED STATISTICAL PARAMETERS OF INPUT VARIABLES

No.	Layer	Input variable	mean	COV	skewness
X1	concrete slab	thickness	220	0,09	-0,6
X2		Young modulus	37500	0,12	0,0
X3		Poisson's coefficient	0,20	0,02	0,0
X4	top road base	thickness	200	0,15	0,0
X5		Young modulus	5000	0,44	0,9
X6		Poisson's coefficient	0,25	0,08	0,0
X7	bottom road base	thickness	250	0,24	1,0
X8		Young modulus	120	0,18	0,9
X9		Poisson's coefficient	0,30	0,07	0,0
X10	subgrade	Young modulus	80	0,32	0,5
X11		Poisson's coefficient	0,35	0,075	0,9
X12	joints	width transversal	20	0,15	0,0
X13		width longitudinal	1,5	0,75	2*
X14		Young modulus	150	0,39	1,0
X15		coefficient of friction	0,5	0,34	0,0
X16	temperature	upper surface	11	0,95	0,5
X17		lower surface	10	0,25	0,9

* - truncation parameter

V. RESULTS

The statistical analysis provides us with the following statistics of principal stresses in individual points in concrete slabs:

- mean (MPa),
- standard deviation (MPa),
- coefficient of variation (dimensionless),
- skewness (dimensionless),
- minimum and maximum value (MPa),
- suitable type of probability distribution.

The mean value describes the average tendency of the stresses, the standard deviation and the coefficient of variation their variability, skewness the asymmetry of their population under and above the mean value, and finally the minimum and maximum values describe the possible interval within the stresses can occur. In addition, the deterministic analysis (DA) with input variables set to their nominal (mean) values is performed. The positive sign corresponds to the tensile stresses, the negative sign to the compression ones.

The suitable type of probability distribution is chosen with the help of the comparative tests from a set of competing distributions [8]. In our study this set includes normal (N) three-parametric lognormal (LN), truncated normal (TN), three-parametric Weibull (W) and three-parametric Pearson III (P3) probability distribution. Based on the chosen distribution, the 1, 5, 95 and 99% quantiles are determined, which are usually the most important in civil engineering practice. Quantiles allow making probabilistic conclusions. E.g. for 1% quantile, the 1% probability exists that the value is less than the quantile, while the 99% probability exists that the value is larger than the quantile.

A. Principal stress σ1

Illustrative results of statistical analysis of principal stress σ1 (maximal tensile stress) in some important points on the upper and lower surface of concrete slabs are shown in Fig. 2 and Fig. 3.

Nominal values of stress obtained from deterministic analysis (DA) on the lower surface have the character of tensile stress at all points, while on the upper surface there are both tensile and compressive stresses as well. Nominal values are within the interval 0,3 to 1,6 MPa on the lower surface and in the interval -0,05 to 1,1 MPa on the upper surface. In both cases, the nominal values are very close to the minimum values obtained from statistical analysis. Mean values of stress on both surfaces in all points have the character of tensile stress and are generally always greater than the nominal values. They are within the interval 0,4 to 2,0 MPa on the lower surface and in the interval 0,2 to 1,4 MPa on the upper surface. The largest nominal as well as mean values reach their maximum at points near the external load application.

The interval in which principal stress σ1 may occur is considerable high - see the interval between the minimum and maximum values in Fig. 2 and Fig. 3. Larger tensile stresses

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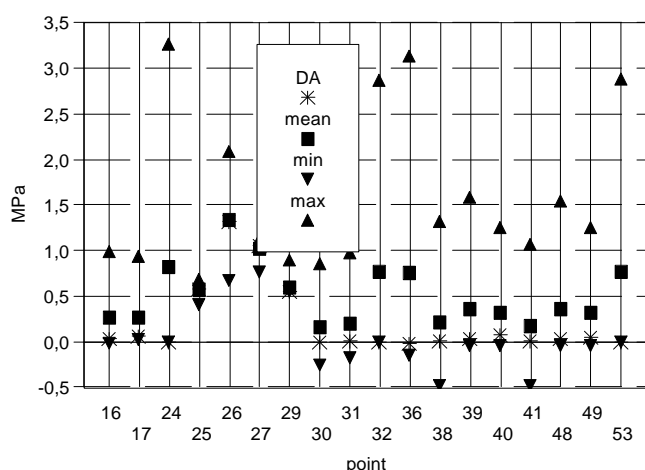International Journal of Civil and Environmental Engineering
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Fig. 2 Principal stress σ1 in points on upper surface

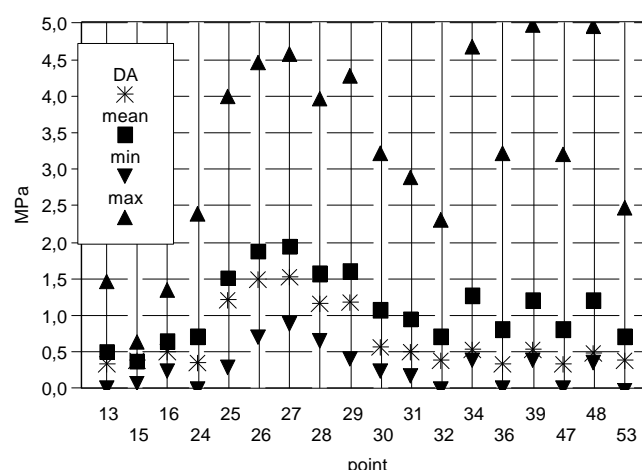
Fig. 3 Principal stress σ1 in points on lower surface

and thus higher probability of tensile cracks occurring arise on the lower surface of concrete slabs. In the case of lower surface, the maximum values are up to 5,3 MPa, in the case of upper surface they are up to 3,3 MPa. The largest tensile stresses on the lower surface arise in points near the external load application (points 25 - 29), in points on all slabs near the longitudinal joint (points 33 - 51) and especially in the corners of slabs (points 34, 39, 40, 45, 46, 48 , 49, 51). The stresses in some points reach magnitude where the creation of tensile cracks is highly probable.

The largest tensile stresses on the upper surface arise in points near the external load application (points 26, 30) and in points on the transverse axis of symmetry of all slabs (points 13, 14, 20, 24, 36, 37, 43, 47, 50, 53, 56, 59, 62).

The stresses in all points show considerable variability and also nonzero skewness. Skewness has (with some exception) the positive sign and in some points is quite large. As a result of significant positive skewness, the often used normal probability distribution (a priori assuming zero skewness) is not suitable to describe random variability of principal stress σ1. Weibull probability distribution seems to be the most appropriate.

B. Principal stress σ3

Illustrative results of statistical analysis of principal stress σ3 (maximal compressive stress) in some important points on the upper and lower surface of concrete slabs are shown in Fig. 4 and Fig. 5. The nominal values of stress obtained from deterministic analysis (DA) on the upper surface have the character of compressive stress in all points, while on the lower surface there are also minimal tensile stresses in points near the external load application (points 28, 30). Nominal values are within the interval -0,1 to -2,2 MPa on the upper surface and in the interval from 0,1 to -0,5 MPa on the lower surface. In both cases, the nominal values are very close to the minimum values obtained from statistical analysis. Mean values of stresses on both surfaces have the same character as nominal values but they are generally greater than the nominal

values. They are within the interval -0,7 to -2,8 MPa on the upper surface and in the interval 0,1 to -1,0 MPa on the lower surface.

The interval in which principal stresses σ3 may occur is considerably high - see the interval between the minimum and maximum values in Fig. 4 and Fig. 5. Larger compressive stresses arise on the upper surface of concrete slabs. In this case, the maximum values are up to -7,3 MPa, in the case of lower surface they are up to -4,9 MPa. The largest compressive stresses on the upper surface arise in points on the transverse axis of symmetry of all slabs and in points close the contact of all slabs (points 13, 20, 24, 32, 38, 41, 47, 50, 53, 56, 59, 62). On the lower surface the largest compressive stresses arise in some corners of slabs (points 11, 22, 58, 60, 61, 63).

The stresses in all points show considerable variability and also nonzero skewness. Skewness in points on the upper surface has (with some exception) the negative sign and in some points is quite large. Weibull probability distribution seems to be the most appropriate to describe random variability of principal stress σ3.

VI. CONCLUSION

Combination of the accurate computational models (e.g. based on non-linear FEM) and the effective simulation methods is able to analyze the behavior of structures with respect to the uncertainties of the input variables. Simulation methods allow to obtain more realistic view on the behavior of structures, allow to design structure for a predetermined reliability level, to set limits (extremes), in which the structure will behave, and finally allow to supplement or even replace experimental research.

The presented statistical analysis of stresses in concrete slabs of rigid pavement shows that the principal stresses in the slabs are of positive as well as negative sign, i.e. tensile and compressive stresses occur despite the fact that dynamic loading is not considered. This is caused by the thermal loading due to the temperature difference between the upper and lower surface of the slabs.
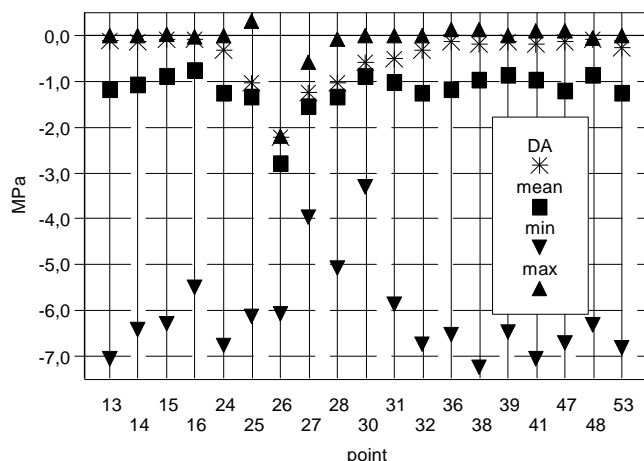
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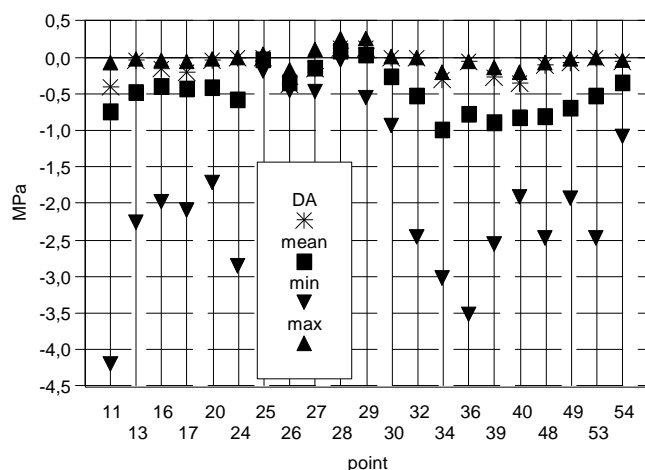
Fig. 4 Principal stress σ3 in points on upper surface

Fig. 5 Principal stress σ3 in points on lower surface

In some points of the lower (but also of the upper) surface of concrete slabs the large tensile stresses occur, that reach a maximum value of about 5,3 MPa. Especially on the lower surface in the points near the external load application and in all corners of the slabs the creation of tensile cracks is highly probable. The presented study also shows that the standard deterministic analysis with the input variables set to their nominal (mean) values generally does not provide information about the average behavior of the structure.

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