World Academy of Science, Engineering and Technology
International Journal of Aerospace and Mechanical Engineering
Vol:6, No:5, 2012

Multicriteria Synthesis of a Polycentric Knee Prosthesis For Transfemoral Amputees

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Abstract**—**In one of the prosthesis designs for lower limb transfemoral amputations artificial knee joints with polycentric mechanisms are used. Such prostheses are characterized by high stability during the stance phase of the movement. The existing variety of polycentric mechanisms indicates the possibility of finding the optimal prosthesis design satisfying several quality criteria.In this paper we present a multicriteria method for the synthesis of the artifical polycentric knee mechanism based on the uniform systematic study of the design parameters space and on the analysis of Pareto optimal solutions.

Keywords**—**Optimalcriteria, polycentric knee, prosthesis, synthesis,transfemoral amputee.

I. INTRODUCTION

FOR a long period of time mankind has attempted to create artificial devices (prostheses) capable to replace as fully as possible natural human organs lost or damaged due to various reasons. First of all such devices were created to restore motor functions lost due to loss of the lower limbs. On one side, this problem seemed the most simple on a technical level, on the other side - the restoration of motor activity allows a person to carry out vital actions even with quite simple devices. The obvious drawbacks of such devices have stimulated the search for new, more sophisticated designs that could improve the quality of life for amputees. However, despite the huge technological progress made to date in this area, lower limb prostheses are still not correspond to biological analogues from different points of view [1].

Lower limb prostheses for transfemoral amputations differ by the kind of thigh and shank joint (single-axis and polycentric) and by the control methods [2].Single-axis prostheses have a fixed center of rotation of the hip relative to shank, are relatively inexpensive and with high accuracy simulate the motion of the knee. However, these prostheses have low functionality and are not sufficiently stable during the stance phase.In the polycentric prostheses position of the instant center of rotation (ICR) continuously changes with changing of the angle of knee flexion.

Due to design complexity they are quite expensive, but compared to the single-axis provide a high level of stability and functionality. Generally prostheses with polycentric mechanism of the artificial knee joint (PMAK) provide greater maximal angle of knee flexion. In the sitting position the effect of the thigh length reducing is implemented, which makes the appearance of the amputee with a long stump (including amputation with disarticulation of the knee) more natural. Walking with such prostheses is more comfortable and less tiring for amputees [3].

One of the major drawbacks of traditional PMAK is relatively low ability to simulate the natural movement of the knee. That is, increasing the functionality of the prosthesis is achieved by reducing the cosmetic aspect of the reconstructed motion. But, obviously, in the process of designing the optimal from a cosmetic point of view PMAK, this disadvantage can be eliminated or at least minimized.

To date, PMAK studied in detail [3, 4]. Commercially there is available a set of designs offered by the world known manufacturers of prosthetic devices: Ossur, Otto Bock, Hosmer, Endolite, Teh Lin and others. However, it should be noted that almost all of them are significantly different from each other, not only by constructive but also by functional characteristics. As an example, Fig. 1 shows some schemes of the four-bar PMAK and their centrodes at motion the shank relative to the thigh.

Thus, it can be stated that, despite the advantages of these design options, none of them is globally optimal. However, all of them, obviously, satisfy certain quality criteria taken into account when designing.

Approaches to the solution of the PMAK optimization problems which described in the literature are based on traditional methods of the mechanisms theory, the practical realization of which was made possible by the introduction of computers in computational practice.

In [5, 6], considering the biomechanics of walking, the authors decided to set the PMAK optimization problem by minimizing the proximity criterion of the desired and practically implemented centrodes of the relative motion of thigh and shank:

$$\Phi_1 = e_1 U + e_2 x_{HD} + e_3 y_{HD} \qquad (1)$$

where Φ_1 - the value of the objective function; U - the maximum distance from the ICR at any input angle to the desired ICR$_D$ at the same angle; x_{HD} - - the maximum horizontal displacement; y_{HD} - the maximum vertical

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displacement; e_1, e_2, e_3 - variable coefficients.In this approach a variety of optimization methods can be used: Rosenbrock [7], Powell [8], Fletcher and Reeves [9], etc.

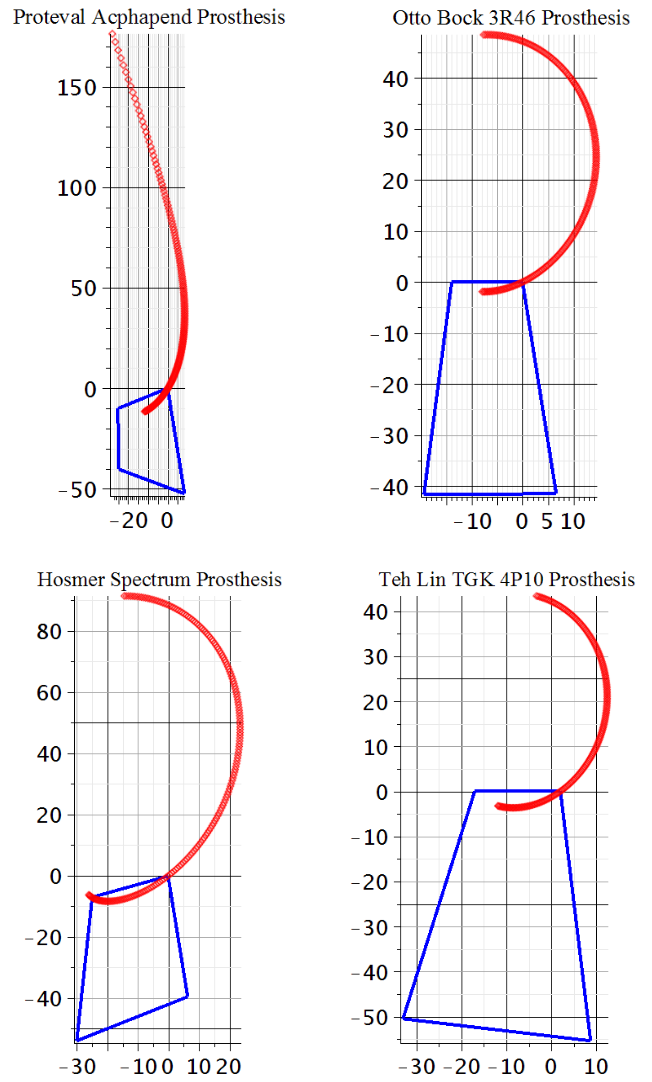

Fig. 1 The schemes some four-bar PMAK (blue) and the corresponding centrodes at motion the shank relative to the thigh (red). (Dimensions are shown in units of length)

As it was noted by the authors, the final mechanism, which can be found by this method, depends on many factors, chief among them are the quality of the initial approximation and the compliance of the realized and desired centrodes. For the final version it is proposed to improve the initial assumptions until the appropriate solution is found [5].

A similar approach can be used in the formulation of other quality criteria. For example, in [10] authors suggest PMAK optimization procedure based on functional and cosmetic requirements.

Thus, unlike [6], the coordinates of the centrode were used in the objective function Φ_2 indirectly in the form of penalty functions f_i, taking into account its location relative to the permitted voluntary control area. Thus, the objective function was presented in the following form:

$$\Phi_2 = \sum_{i=1}^{N}(f_i) + \sum_{i=1}^{N}\left[\left(x_{RP_i} - x_{P_i}\right)^2 + \left(y_{RP_i} - y_{P_i}\right)^2\right], \tag{2}$$

where N – the total number of different knee positions during the stance phase; f_i - penalty functions that are equal to zero if ICR$_i$lies within voluntary control area and taking sufficiently large values otherwise; $\left\{x_{RP_i}, y_{RP_i}\right\}$, $\left\{x_{P_i}, y_{P_i}\right\}$ - experimental and implemented coordinates of the coupler points, tightly associated with the femoral component of the prosthesis, respectively.

The initial data of the optimization process were given:

- the coordinates of the experimental points$\left\{x_{RP_i}, y_{RP_i}\right\}$for each experimental flection angle θ_{F_i}, $i = 1,\dots,N$;

- the feasible values regions of the design parameters

$$x_{\min j} \le x_j \le x_{\max j}, \quad j = 1,\dots,10, \tag{3}$$

limited by minimum $x_{\min j}$ and maximum $x_{\max j}$ values;

- limitations of the voluntary control area

$$\rho_{ICR_i} \ge \rho_{H_i}, \tag{4}$$

$$\left(\alpha_{H_i P_i} - \alpha_{\max_i}\right) \le \beta_{ICR_i} \le \left(\alpha_{H_i P_i} + \alpha_{\max_i}\right), \tag{5}$$

where ρ_{ICR_i}, ρ_{H_i} - current and minimum distances from the ICR to the center of pressure (COP), respectively; $\alpha_{H_i P_i}$ - the angle between the line connecting the center of the hip with the COP and the x-axis of the global coordinate system; $\alpha_{\max_i}$ - the maximum angle between the direction of the vector of the ground reaction force (GRF) and the straight line passing through the center of the hip and the COP, limiting the permitted voluntary control area, defined for the each stage of the stance phase taken into account.

To minimize function (2) it is recommended to use specific algorithms, taking into account the fact that its range has many gaps due to the presence of penalty functions, for example, genetic [11,12], neural networks [13], controlled deviation[14], etc.

Sufficiently detailed analysis of the examples above allows asserting that similar PMAK optimization can be performed in accordance with other criteria of quality. But at the same time, some of the possible criteria may be controversial and as a result will be received mechanisms with contradictory characteristics.

Correct solution of the engineering problems with a given set of criteria (possibly controversial) can be obtained using multicriteria optimization techniques. For example, in [15] a hybrid multicriteria genetic algorithm was used for Pareto optimum synthesis of four-bar linkages considering the minimization of two contradictory objective functions simultaneously: the function of tracking error and the function of transmission angle deviation from 90 degree. The authors found that the hybrid Pareto optimum synthesis of mechanisms could unveil very important design trade-offs between conflicting objective functions which would not have been found by other methods.

For the first time genetic algorithms have been presented by J. Holland [16, 17], and then were successfully applied to solve many optimization problems. For the problems of mechanisms synthesis they were adapted by the methods proposed by W. Fang [18]. As noted in [11], the main advantages of these methods are their simplicity in implementing the algorithms and their low computational cost. At their using no need for a deep knowledge about parameters space, such as whether or not it is continuous, presents local minimums etc. However, these advantages are simultaneously the disadvantages in the solution of problems where the properties of the parameters space are important, when this space should be investigated as fully as possible. In particular, this task is the synthesis of optimal PMAK in accordance with a set of quality criteria.Significant advantages in its decision can be achieved using the Parameter Space Investigation method (PSI method), developed by I. Sobol and R. Statnikov based on the sequences of points uniformly distributed in the multidimensional cube opened by them [19, 20, 21].

The main purpose of this work is to develop a reliable universal algorithm of PMAK multicriteria synthesis on the base of PSI method.

II. FORMULATION OF MULTICRITERIA OPTIMIZATION PROBLEM

A. *PMAK mathematical model*

Let an optimized PMAK has a four bar linkage structure.Its design scheme is shown in Fig. 2.

In this mechanism AB is the input link. It is assumed that it is tightly associated with the hip and moves plane-parallel relatively to arbitrarily still link O_1O_3, fixed to the shank, which is associated with the moving system of coordinates $O_1x_1y_1$. During the motion relative to the joint A, link AB rotates at an angle θ_3 which is taken as an independent generalized coordinate. Thus, $\theta_2 = \theta_2(\theta_3)$ and $\theta_4 = \theta_4(\theta_3)$ - the functions of the generalized coordinate θ_3. Rotation angles of all links are measured from the the positive direction of the x-axis and considered positive if directed counterclockwise.

In order to unify, we introduce the following notation [10]: $l_{O_3O_1} = x_1$, $l_{O_1A} = x_2$, $l_{AB} = x_3$, $l_{BO_3} = x_4$, $x_{O_1} = x_5$, $y_{O_1} = x_6$, $\gamma_1 = x_7$, $\theta_3 - \theta_F = x_8$, $l_{AP} = x_9$, $\gamma_3 = x_{10}$. Here the symbol l denotes the length of the links indicated in the indexes; x_{O_1}, y_{O_1} - the global Cartesian coordinates of the joint O_1; γ_1 - the inclination angle of the link O_1O_3 relative to the x-axis of the global coordinate system; γ_3 - the angle between the segments AB and AP; θ_F - the knee flexion angle.

From the conditions that the circuit $O_1ABO_3O_1$ is closed, we obtain [10]:

$$\theta_2 = 2\arctan\left(\frac{F_1 \pm \sqrt{F_1^2 + F_2^2 - F_3^2}}{F_2 + F_3}\right), \qquad (6)$$

$$\theta_4 = \arcsin\left(\frac{x_2\sin(\theta_2) + x_3\sin(\theta_3)}{x_4}\right), \qquad (7)$$

$$x_{ICR} = x_5 + \frac{\tan(\theta_4 + x_7)\cos(x_7) - \sin(x_7)}{\tan(\theta_4 + x_7) - \tan(\theta_2 + x_7)} \cdot x_1, \qquad (8)$$

$$y_{ICR} = x_6 + \frac{\tan(\theta_2 + x_7)[\tan(\theta_4 + x_7)\cos(x_7) - \sin(x_7)]}{\tan(\theta_4 + x_7) - \tan(\theta_2 + x_7)} \cdot x_1, \qquad (9)$$

$$x_P = x_5 + x_2\cos(\theta_2 + x_7) + x_9\cos(\theta_3 + x_{10} + x_7), \qquad (10)$$

$$y_P = x_6 + x_2\sin(\theta_2 + x_7) + x_9\sin(\theta_3 + x_{10} + x_7), \qquad (11)$$

where

$$F_1 = \sin(\theta_3),\ F_2 = \cos(\theta_3) - \frac{x_1}{x_3},\ F_3 = -\frac{x_1^2 + x_2^2 + x_3^2 - x_4^2}{2x_2x_3} + \frac{x_1}{x_2}\cos(\theta_3)\cdot$$

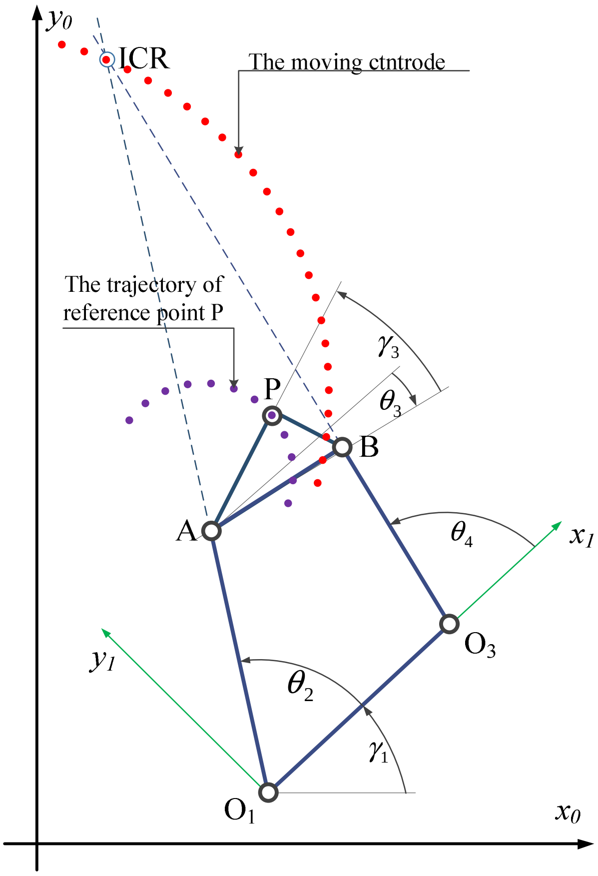

Fig. 2 Calculated scheme of a four-bar PMAK model

B. PMAK quality criteria (objective functions), parametric and functional constraints

Assume that PMAK must satisfy $m = 2$ the quality criteria (1) and (2)simultaneously.At the same time in (2) the penalty functions f_i can be considered constant and equal to zero, while in (1)

$$U = \sum_{i=1}^{N} \left[\left(x_{DICR_i} - x_{ICR_i} \right)^2 + \left(y_{DICR_i} - y_{ICR_i} \right)^2 \right],$$

$$e_1 = 1,\ e_2 = e_3 = 0\ .$$

Taking into account the expressions (6-11), the criteria (1) and (2) can be computed unambiguously if there are given coordinates of the desired centrode $\left\{ x_{DICR_i}, y_{DICR_i} \right\}$ and the coordinates $\left\{ x_{RP_i}, y_{RP_i} \right\}$ of the desired trajectory of the point P for a set of values $\theta_{F\,i}$, $i = 1,...,N$.

At this stage information about the dependence of the criteria not required. In addition, if they depend, they can be coherent or contradictory. When solving more complex problems the greater number of quality criteria can be taken into account.

The exact solution of the multicriteria optimization problem of complex objects, as a rule, cannot be obtained. Therefore, initially, you must specify a valid accuracy of the solution in the form of constraints of the quality criteria from objective point of view

$$\Phi_k(X) \le \Phi_{\max k},\ k = 1,...,m, \qquad (12)$$

where $\Phi_{\max k}$ - is the worst value of $\Phi_k(X)$ acceptable to an expert. These constraints can be repeatedly revised during solving of the problem.

As a design variables we assume $n = 10$ independent parameters x_j, $j = 1,...,n$, which constraints are specified by (3). The set of these parameters represents a point $X(x_1,...,x_n)$ of the n-dimensional design variable space and boundary values $x_{\min j}$ and $x_{\max j}$ define a parallelepiped Π in this space. From the expert's perspective, the boundary values can be modified, if that leads to improvement of the basic criteria.

Functional limitations are usually given in the form of inequalities of the form

$$c_{\min r} \le f_r(X) \le c_{\max r},\ r = 0,...,l\ . \qquad (13)$$

In this task, as functional limitations the condition (4) and (5) are accepted. Functions $f_r(X)$ together with $c_{\min r}$ and $c_{\max r}$ are some requirements of the designed object that sometimes an expert can successively revise in order to improve the basic performance criteria.

C. Multicriteria optimization problem

The design variable (3), functional (13) and criteria (12) constraintsdefine the feasible solution set $D \subset \Pi$. Letusformulatethebasicproblemofmulticriteriaoptimization: *it is necessary to define the feasible solution set* $D \subset \Pi$ *and find a set* $P \subset D$ *such that* $\Phi(P) = \min\limits_{X \in D} \left[\Phi_1(X),...,\Phi_m(X) \right]$ *where* P *is the Pareto optimal set.*

Point $X_i \in D$ is called Pareto optimal if there is no point $X \in D$, such that $\forall k \in [1,m]:\ \Phi_k(X) \le \Phi_k(X_i)$ and $\exists k: \Phi_k(X) < \Phi_k(X_i)$.

III. THE ALGORITHM OF PSI METHOD

The main peculiarity of the PSI method is the possibility of a systematic study of the n-dimensional parameter space of points uniformly distributed in it.

Independent design variables x_j, $j = 1,...,n$ get out as co-ordinates of points X_i, $i = 1,...,s$ in uniformly distributed n-dimensional space P_n. Clearly, for reception of the authentic decision of a task, the space of parameters P_n should be investigated as more as possible full, otherwise is possible loss of really optimum solving, from the point of view of the chosen criterions. Offered by I. Sobol and R. Statnikovthe LP_τ sequences of the uniformly distributed points in the n-dimensional cube, is allow regularly to investigate the space of parameters P_n that gives the chance greatlyto reduce number of trial points at synthesis to the minimum.

Let i - is a number of point X_i, and j - one of its co-ordinates. Then for set i is calculated an auxiliary parameter $M = 1 + \dfrac{\ln i}{\ln 2}$, and then for $j = 1,...,n$ - is calculated co-ordinates of points $S_{i,j}$ of the LP_τ sequence [19]

$$S_{i,j} = \sum_{k=1}^{M} 2^{-k+1} \left\{ \frac{1}{2} \sum_{l=k}^{M} \left[2\left\{ i 2^{-l} \right\} \right] \left[2\left\{ R_j^{(l)} 2^{k-1-l} \right\} \right] \right\}, \qquad (14)$$

where $R_j^{(l)}$ - coefficient of numerators table; z – number in square or figurate brackets; $[z]$– whole part, $\{z\}$– fractional part of number z . Co-ordinates of trial point X_i (variable parameters of synthesis) are calculated then under the formula

$$\alpha_{ij} = x_{\min j} + \left(x_{\max j} - x_{\min j} \right) S_{i,j}\ . \qquad (15)$$

At $X_i = (\alpha_{i1},...,\alpha_{in})$ are calculated the expressions (6-11)and verified functional limitations (13). If they are satisfied, then the point X_i shown as a trial and in it is calculated the values of the quality criteria (12).Otherwise, this point is discarded.

Let the number of selected points is $nsp \le n$. For each of the computed criteria of quality is constructed a table of tests in which

$$\Phi_k(X^{k1}) \le \Phi_k(X^{k2}) \le \ldots \le \Phi_k(X^{knsp}), \qquad (16)$$

Where the numbers $k1, k2, \ldots, knsp$ in general, differ for each $k = 1, \ldots, m$.

According to the study of the parameter space can be constructed the correlation matrix $\|r_{\mu\nu}\|$, где $r_{\mu\nu}$ - the coefficients of pair correlation of the criteria $\Phi_\mu(X)$ and $\Phi_\nu(X)$, $\mu \ne \nu$. This matrix allows us to estimate the degree of linear relationship between any two criteria.If, forexample, $r_{\mu\nu} \approx 1$,then the criteria $\Phi_\mu(X)$ and $\Phi_\nu(X)$ are linearly dependent and, when changing coordinates of the point X: $\Phi_\nu(X) \approx K \cdot \Phi_\mu(X)$, where K - some constant. If $K < 0$, then the above criteria - contradictory i.e., decrease the value of one of them leads to an increase in the other.

The scheme of algorithm for determining the feasible solution set $D \subset \Pi$ is shown in Fig. 3.

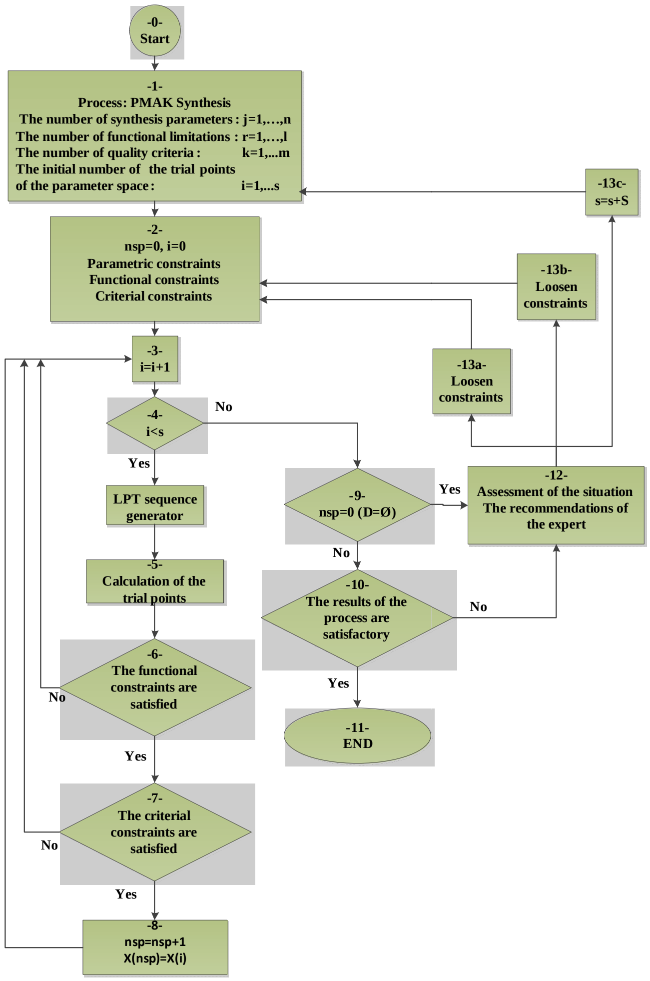

Fig. 3 The scheme of algorithm for determining the feasible solution set

Finally, the analysis of the feasible solution set $D \subset \Pi$ allows us to construct a Pareto optimal solution set $P \subset D$.

IV. CASE STUDY

The program to implement the procedures for the multicriteria synthesis ofPMAK was developed based on the PSI method. The testing of program was carried out in accordance with the objectives set out in [5] and [10]. However, the desired centrode and the reference trajectory of the point P at the knee bending were taken at random in general. It was assumed that the relevant point of the desired centroids belong to the segment of a straight line, and the point of reference trajectories lie on the arc of a circle, oriented in a certain way in the vicinity of the synthesized PMAK. At first supposed to explore the parameters space, which includes 10^4 sampling points.

In addition to the above-described functional constraints (4) and (5) in the program was provided a procedure for calculating the boundary configurations of the mechanism in cases where the Grashof conditions not implemented[23].

The parametric constraints (3) were chosen based on the known dimensions of the realized four-bar PMAK.

In Fig.4 is shown a variety of configurations of the synthesized four-bar PMAK, the moving centrode and the point P trajectory.

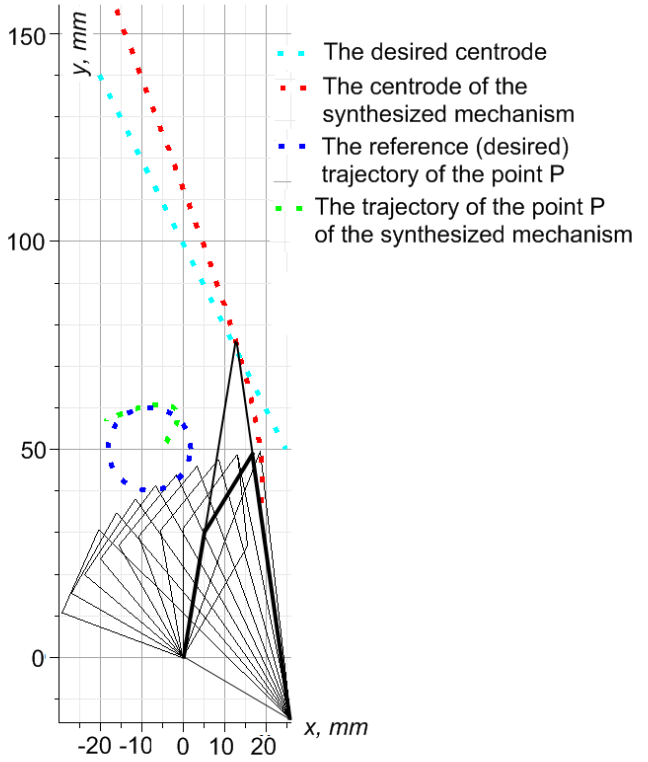

Fig. 4 The test solution of the PMAK multicriteria synthesis

V. CONCLUSIONS

Four-bar mechanism obtained in the result of synthesis can not be considered as optimal in terms of its use in lower limb prosthesis for transfemoral amputees. However, this mechanism is close to optimal, taking into account accepted criteria of quality and the desired functional characteristics, which in this example were chosen arbitrarily.At the synthesis was obtained the set of Pareto optimal solutions and its analysis allowed to select the most appropriate option from the perspective of the authors.

To achieve a truly optimal PMAK at the initial stage of synthesis is necessary to specify the real desired characteristics, which can be obtained as a result of biomechanical gait analysis not only of different categories of disabled persons, but under different conditions of their life.In formulating the quality criteria should take into account the dynamic characteristics of the prosthesis, as well as especially in the design of the proposed control system.

It should be noted ease of implementation of the PSI method and a high level of its informativeness. The dialogue process of solving the problem allows us, finally, to obtain the most appropriate design.Moreover, PSI method, which is very important, allows the designer to analyze and "bad options" that may be optimal for other quality criteria.

The disadvantages are the relatively large computational cost. However, given the growth speed of modern computers and the importance of the tasks which should be solved, their can be considered insignificant.

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