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Genetic Algorithm Approach for Solving the Falkner–Skan Equation

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***Abstract*—**A novel method based on Genetic Algorithm to solve the boundary value problems (BVPs) of the Falkner–Skan equation over a semi-infinite interval has been presented. In our approach, we use the free boundary formulation to truncate the semi-infinite interval into a finite one. Then we use the shooting method based on Genetic Algorithm to transform the BVP into initial value problems (IVPs). Genetic Algorithm is used to calculate shooting angle. The initial value problems arisen during shooting are computed by Runge-Kutta Fehlberg method. The numerical solutions obtained by the present method are in agreement with those obtained by previous authors.

***Keywords*—**Boundary Layer Flow, Falkner–Skan equation, Genetic Algorithm, Shooting method.

I. Introduction

THE concept of boundary layer flows of an incompressible fluid has several engineering applications, such as aerodynamic extrusion of plastic sheets, cooling of a metallic plate in a cooling bath [1]-[4]. Such flows are also encountered in the glass and polymer industries. The nonlinear third-order Falkner–Skan equation is a famous example of these boundary layer flows. The numerical solution for nonlinear differential equations is a major problem in computational mathematics, where the boundary value problem on an infinite interval is one of the sub-problems attracting many scientists.

We are to solve the following Falkner–Skan equation:

$$\frac{d^3 f}{d\eta^3} + \beta_0 \frac{d^2 f}{d\eta^2} + \beta\left[1 - \left(\frac{df}{d\eta}\right)^2\right] = 0, \quad 0 < \eta < \infty \tag{1}$$

with the boundary conditions:

$$\left.\begin{array}{l} f = 0 \quad at \quad \eta = 0 \\ \dfrac{df}{d\eta} = 0 \quad at \quad \eta = 0 \\ \dfrac{df}{d\eta} = 1 \quad at \; \eta \to \infty \end{array}\right\} \tag{2}$$

where β_0 and β are constants.

This is a nonlinear two-point boundary value problem for which no closed form solutions are available. The mathematical treatments of this problem by Rosenhead [5] and Weyl [6] have mainly focused on obtaining existence and uniqueness results. The numerical treatment of this problem was presented by many authors Cebeci and Keller [7], Na [8], Elgazery [9] and many more. These approaches have mainly used shooting or invariant imbedding.

The purpose of this paper is to develop an efficient method based on Genetic Algorithm which is much more straightforward and simpler than the other existing algorithms. The simplicity of the present method arises from the fact that it does not require to guess the first value near the solution and need not worry about singular points.

Genetic Algorithms were developed by John Holland in 1975 [10]. Genetic Algorithms are search algorithms based on the mechanics of the natural selection process (biological evolution). The most basic concept is that the strong tend to adapt and survive while the weak tend to die out. That is, optimization is based on evolution, and the "Survival of the fittest" concept. Genetic Algorithms have the ability to create an initial population of feasible solutions, and then recombine them in a way to guide their search to only the most promising areas of the state space. Each feasible solution is encoded as a chromosome (string) also called a genotype, and each chromosome is given a measure of fitness via a fitness (evaluation or objective) function. The fitness of a chromosome determines its ability to survive and produce offspring. A finite population of chromosomes is maintained. Genetic Algorithms use probabilistic rules to evolve a population from one generation to the next. The generations of the new solutions are developed by genetic recombination operators: 1) Crossover: combining parent chromosomes to produce children chromosomes. In other words Crossover combines the "fittest" chromosomes and passes superior genes to the next generation. 2) Mutation: altering some genes in a chromosome. Mutation ensures the entire state-space will be searched, (given enough time) and can lead the population out of a local minima.

II. Method of Solution

The non-linear differential equations (1) subject to the boundary conditions (2) constitute a two-point boundary value problem. In order to solve these equations numerically, we follow Runge–Kutta Fehlberg integration scheme with Genetic algorithm based shooting technique. In this method it is most important to choose the appropriate finite values of $\eta \to \infty$. The solution process is repeated with another large

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value of η_∞ until two successive values of $f''(0)$ differ only after a desired digit signifying the limit of the boundary along η. The last value of η_∞ is chosen as appropriate value of the limit $\eta \to \infty$ for that particular set of parameters. The ordinary differential equation (1) was first converted into a set of three first-order simultaneous equations. To solve this system we require three initial conditions but we have only two initial conditions, $f(0)$ and $f'(0)$ on $f(\eta)$. The initial condition $f''(0)$ is not prescribed. However the values of $f'(\eta)$ is known at $\eta = 0$. Now we employ the numerical shooting technique based on Genetic Algorithm where this ending boundary condition is utilized to produce unknown initial conditions at $\eta = 0$. Finally, the problem has been solved numerically using Runge-Kutta Fehlberg integration scheme.

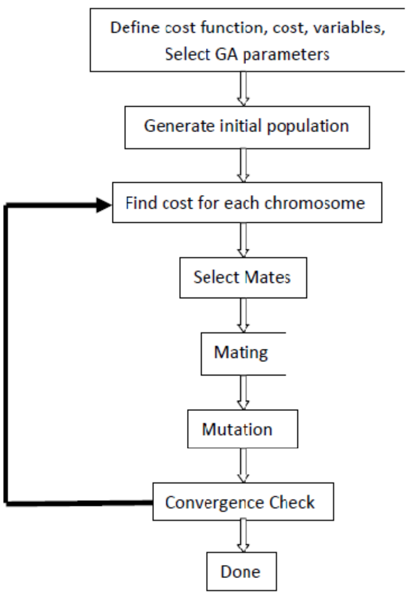

Fig. 1 Flowchart of GA [11]

III. Result and Discussion

In the absence of an analytical solution of a problem, a numerical solution is indeed an obvious and a natural choice. Thus, the nonlinear third-order Falkner–Skan equation (1) with boundary conditions (2), are solved using Runge-Kutta Fehlberg method with shooting technique based on Genetic Algorithm. To assess the accuracy of the present method, comparison with previously reported data available in the literature has been made via Table I.

TABLE I

Comparison of the Values of $f''(0)$ for $\beta_0 = 1/2$ and $\beta = 0$ from Different Authors

S. No.	Value of $f''(0)$	
1	Asaithambi [12],[13]	0.33205
2	Zhang and Chen [14]	0.33206
3	Present Result	0.33243

The The Falkner–Skan equation has two coefficients β_0 and β. The solutions corresponding to $\beta > 0$ have being known as accelerating flows (Fig. 2), those corresponding to $\beta = 0$ are called constant flows, and those corresponding to $\beta < 0$ are known as decelerating flows. Physically relevant solutions exist only for $-0.1988 < \beta \le 2$ (Figs. 3-5). If $\beta_0 = 1/2$ and $\beta = 0$, it is called Blasius flow; if $\beta_0 = 1$ and $\beta = 1/2$, it describes Homann axisymmetric stagnation flow; if $\beta_0 = 1$ and $\beta = 1$, it describes Hiemenz flow; if $\beta_0 = 0$ and $\beta = 1$, it is called Pohlhausen flow; if $\beta_0 = 2$ and $\beta = 1$ (Fig. 6), it represents the problem of Homann, describing the steady flow in the boundary layer along a surface of revolution near the stagnation point. Sometimes, the Falkner–Skan equation specifically refers to $\beta_0 = 1$. We give below the numerical results corresponding to different $\beta \in [-0.1988, \infty)$ when $\beta_0 = 1$ via Tables II and III.

TABLE II

Comparison of the Values of $f''(0)$ for $\beta_0 = 1$ and Different β from Different Authors

Value of β	Value of $f''(0)$ Zhang and Chen[14]	Zhu et al.[15]	Present Result
40	7.314785	7.314785	7.318525
30	6.338208	6.338208	6.338051
20	5.180718	5.180718	5.180599
15	4.491486	4.491487	4.491677
10	3.675234	3.675234	3.677009

TABLE III

Comparison of the Values of $f''(0)$ for $\beta_0 = 1$ and Different β from Different Authors

Value of β	Value of $f''(0)$ Zhang and Chen[14]	Zhu et al.[15]	Present Result
2	1.687218	1.687218	1.687014
1	1.232587	1.232588	1.232465
0.5	0.927680	0.927680	0.927570
0	0.469600	0.469600	0.469593
-0.1	0.319270	0.319270	0.319572
-0.15	0.216362	0.216362	0.217352
-0.18	0.128636	0.128637	0.129087
-0.1988	0.005222	0.005225	0.006019

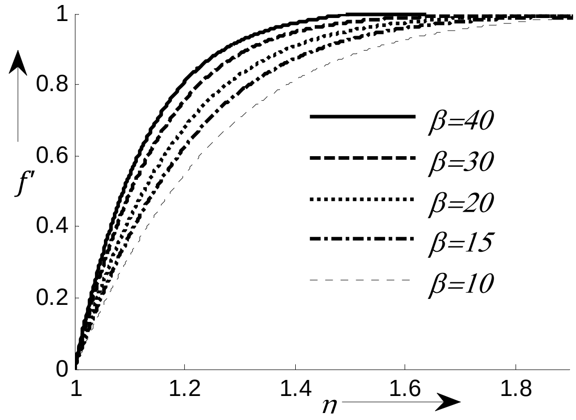

Fig. 2 The velocity profiles corresponding to different $\beta[10, \infty)$ when $\beta_0 = 1$

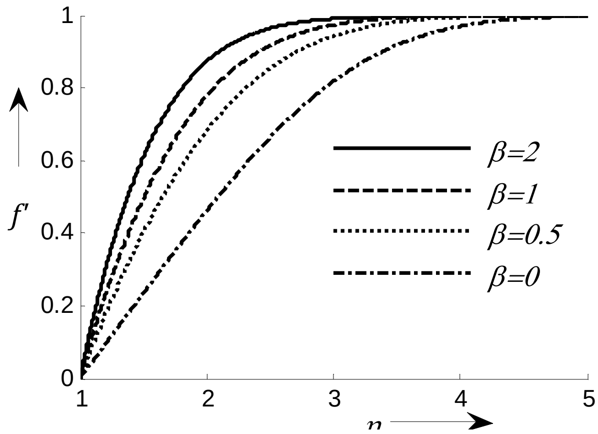

Fig. 3 The velocity profiles corresponding to different $\beta[0, 2]$ when $\beta_0 = 1$

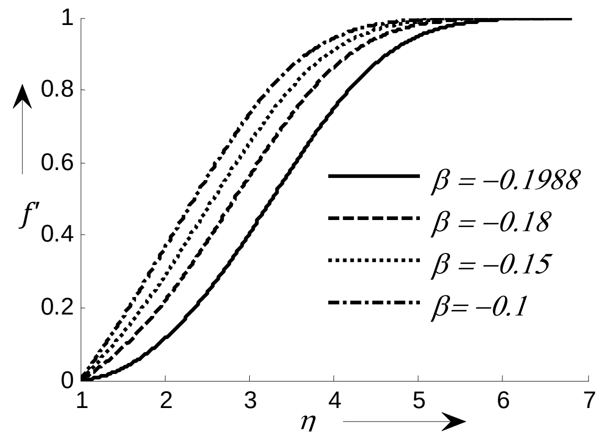

Fig. 4 The velocity profiles corresponding to different $\beta[-0.1988, 0)$ when $\beta_0 = 1$

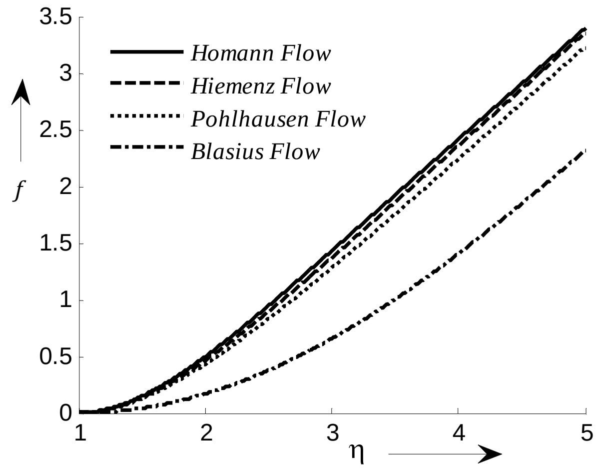

Fig. 5 Numerical solution $f(\eta)$ for several instances of Falkner-Skan equation

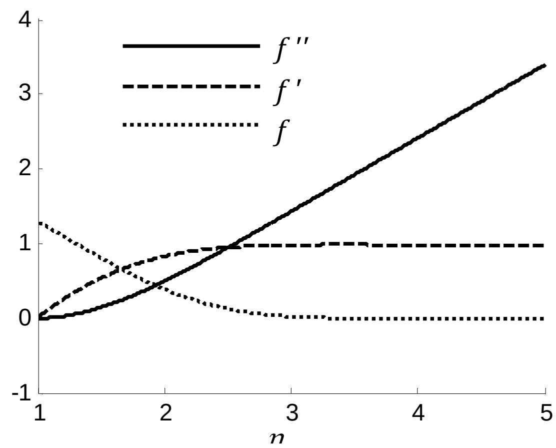

Fig. 6 Solution of the problem of Homann's Flow ($\beta = 1$ when $\beta_0 = 2$

IV. Conclusion

A new Genetic Algorithm based shooting technique has been presented for the boundary value problems of a class of nonlinear three-order differential equation on semi-infinite intervals. The initial value problems arisen during shooting are computed by Runge-Kutta Fehlberg method. We have successfully computed several instances of the Falkner–Skan equation. Our results are in excellent accordance with those already reported in literature.

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