

On the Steady-State Performance Characteristics of Finite Hydrodynamic Journal Bearing under Micro-Polar Lubrication with Turbulent Effect

Subrata Das, Sisir Kumar Guha

Abstract—The objective of the present paper is to theoretically investigate the steady-state performance characteristics of journal bearing of finite width, operating with micropolar lubricant in a turbulent regime. In this analysis, the turbulent shear stress coefficients are used based on the Constantinescu's turbulent model suggested by Taylor and Dowson with the assumption of parallel and inertia-less flow. The numerical solution of the modified Reynolds equation has yielded the distribution of film pressure which determines the static performance characteristics in terms of load capacity, attitude angle, end flow rate and frictional parameter at various values of eccentricity ratio, non-dimensional characteristics length, coupling number and Reynolds number.

Keywords—Hydrodynamic lubrication, steady-state, micropolar lubricant, turbulent.

I. INTRODUCTION

IN recent years, bearings are increasingly operated in turbulent flow regime in certain applications, such as large turbo machinery running at relatively high speeds with large diameters and in machine using process fluids with low viscosity as a lubricant. As a consequence, large research efforts are made following the early work of Wilcock [1] in 1950 in order to develop a reasonable engineering theory of turbulent lubrication for bearings. Later on, Constantinescu employed Prandtl mixing length concept for the representation of turbulent stresses in terms of the mean velocity gradient and his work has been well documented in [2]–[8]. Ng and Pan [9] and Elrod and Ng [10] used the concept of Reichardt's eddy diffusivity. Taylor and Dowson [11] suggested the application of the then existing lubrication theories developed by Ng, Pan and Elrod. Ghosh et al. [12] have analyzed the turbulent effect on the rotor dynamic characteristics of a four-lobe orifice-compensated hybrid bearing.

Of late, the most of the lubricants in practice are no longer Newtonian fluids since the use of the additives in lubricants has become a common practice in order to promote their performances. Therefore, the theory of micro-polar fluids [13] and [14] which are characterized by the presence of suspended

rigid micro-structure particles has been applied to solve the lubrication problems of such fluids. Theoretical investigations [15]–[21] on the theory of micro-polar lubrication in journal bearings under the steady-state condition have been initiated with the investigation of Allen and Klien [22].

Recently, Shenoy et al. [23] studied the effect of turbulence on the static performance of a misaligned externally adjustable fluid film bearing lubricated with coupled stress fluids and predicted the improvement of load capacity with reduced friction and end leakage flow. Gautam et al. [24] analyzed the steady-state characteristics of short journal bearings for turbulent micro-polar lubrication. The results of this work are generally valid for L/D value up to 0.1, which is the limitation of the analysis. However, so far no investigation is available, that addresses the effect of turbulent flow of micro-polar fluid on the performances of journal bearings of finite width. So, the thrust of the present article is to extend the turbulent theory under the micro-polar lubrication to predict the static performance characteristics in terms of load capacity, attitude angle end flow rate and frictional parameter of journal bearing of finite width at various parameters viz. eccentricity ratio, non-dimensional characteristics length, coupling number and Reynolds number. Although the present article has dealt with the results valid for a journal bearing of finite width ($L/D = 1.0$), but it is based on a more generalized approach to obtain the results of static performance characteristics for any L/D value, thus eliminating the limitation of the work [24].

II. ANALYSIS

A schematic diagram of a hydrodynamic journal bearing with the circumferential coordinate system used in the analysis is shown in Fig. 1. The journal operating with a steady-state eccentricity ratio, ε_0 rotates with a rotational speed, Ω about its axis. With the usual assumptions considered for the thin micro-polar lubrication film and the assumptions of the absence of the body and inertia forces and body couples, the modified Reynolds equation as mentioned in [24] and [25] for two-dimensional flow of micro-polar lubricant with turbulent effect is written as follows:

$$\frac{\partial}{\partial x} \left[\phi_x(\Lambda, N, h) \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[\phi_z(\Lambda, N, h) \frac{\partial p}{\partial z} \right] = \frac{U}{2} \frac{\partial h}{\partial x} \quad (1)$$

where,

Subrata Das is a research scholar at the Mechanical Engineering Department, Bengal Engineering and Science University, Howrah 711103, West Bengal, India (Corresponding author; e-mail: mechsubrata@gmail.com).

Sisir Kumar Guha is with the Mechanical Engineering Department, Bengal Engineering and Science University, Howrah 711103, West Bengal, India (e-mail: sk_guha@rediffmail.com).

$$\phi_{x,z} = \frac{h^3}{K_{x,z}} + \Lambda^2 h - \frac{N\Lambda h^2}{2} \coth\left(\frac{Nh}{2\Lambda}\right) N = \left(\frac{\chi}{2\mu + \chi}\right)^{\frac{1}{2}},$$

$$\Lambda = \left(\frac{\gamma}{4\mu}\right)^{\frac{1}{2}}, \mu_v = \mu + \frac{1}{2}\chi$$

Here, μ_v is the viscosity of the base fluid, μ is the viscosity of the Newtonian fluid, χ is the spin viscosity, γ is the material coefficient, N is the coupling number, Λ is the characteristics length of the micro-polar fluid.

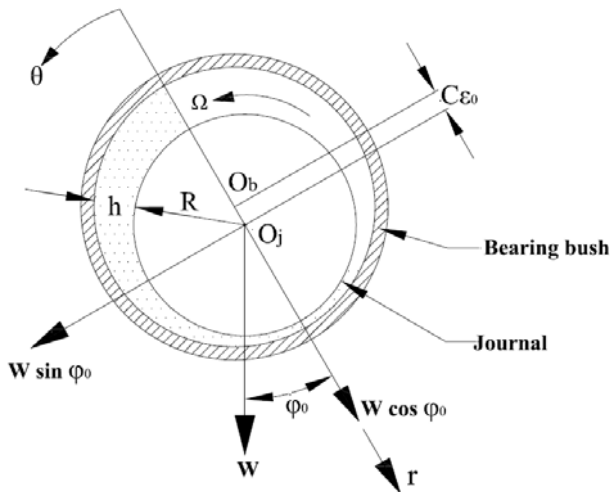


Fig. 1 Configuration of journal bearing geometry

The variation of the film thickness along the circumference direction is defined as

$$h = C(1 + \varepsilon_0 \cos \theta) \quad (2)$$

where, θ is the angular coordinate starting from the line of centers as shown in Fig. 1.

In the present analysis, for the effect of turbulent flow, the following expressions of the turbulent shear coefficients, K_x and K_z are obtained by the following expressions that emulate [11] and [25].

$$K_x = 12 + A_x (\text{Re}_h)^{B_x} \text{ and } K_z = 12 + C_z (\text{Re}_h)^{D_z} \quad (3)$$

where,

$$\text{Re}_h = \bar{h} \cdot \text{Re}, \bar{h} = \frac{h}{C}, \text{Re} = \frac{\rho \Omega R h_m}{\mu}$$

h_m = mean film thickness

For the case of journal bearing recalling (2), it can be written as:

$$h_{\max} = h|_{\theta=0} = C(1 + \varepsilon_0), h_{\min} = h|_{\theta=\pi} = C(1 - \varepsilon_0)$$

$$\therefore h_m = \frac{1}{2}(h_{\max} + h_{\min}) = C$$

$$\therefore \text{Re} = \frac{\rho \Omega R h_m}{\mu} = \frac{\rho \Omega R C}{\mu} = \text{Mean or average Reynolds}$$

number.

A_x, B_x, C_z and D_z are the constant parameters based on the range of Reynolds number [11].

Equation (1), when non-dimensionalised with the following substitutions

$$\theta = \frac{x}{R}, \bar{z} = \frac{2z}{L}, \bar{p} = \frac{pC^2}{\mu \Omega R^2}, l_m = \frac{C}{\Lambda}, \bar{h} = \frac{h}{C}$$

reduces to

$$\frac{\partial}{\partial \theta} \left[\phi_{\theta} (l_m, N, \bar{h}) \frac{\partial \bar{p}}{\partial \theta} \right] + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left[\phi_z (l_m, N, \bar{h}) \frac{\partial \bar{p}}{\partial \bar{z}} \right] = \frac{1}{2} \frac{\partial \bar{h}}{\partial \theta} \quad (4)$$

where

$$\phi_{\theta,z} (l_m, N, \bar{h}) = \frac{\bar{h}^3}{K_{\theta,z}} + \frac{\bar{h}}{l_m^2} - \frac{N\bar{h}^2}{2l_m} \coth\left(\frac{Nl_m \bar{h}}{2}\right)$$

III. NUMERICAL PROCEDURE

Equation (4) is discretised into the finite central difference form and solved by the Gauss-Seidel iterative procedure using the over-relaxation factor, satisfying the following boundary conditions:

- i. $\bar{p}(\theta, \pm 1) = 0$ (Ambient pressure at both bearing ends)
- ii. $\frac{\partial \bar{p}}{\partial \bar{z}} = 0$ (Symmetrical pressure at the midplane)
- iii. $\frac{\partial \bar{p}(\theta_c, \bar{z})}{\partial \theta} = 0, \bar{p}(\theta, \bar{z}) = 0$ for $\theta \geq \theta_c$ (Cavitation condition)

where, θ_c represents the angular coordinate at which film cavitates.

To implement the above numerical procedure, a uniform grid size is adopted in the circumferential (80 divisions) and axial direction (20 divisions). For calculating film pressure at each set of input parameters, the following convergence criterion is adopted.

$$\left| 1 - \frac{\sum \bar{p}_{old}}{\sum \bar{p}_{new}} \right| \leq 0.0001$$

IV. STEADY-STATE BEARING PERFORMANCE CHARACTERISTICS

With the pressure field known, the following bearing static

performance characteristics are carried out.

A. Load Carrying Capacity

The non-dimensional radial and transverse load components are obtained by

$$\bar{W}_r = -\int_0^1 \int_0^{\theta_c} \bar{p} \cdot \cos \theta \cdot d\theta d\bar{z} \quad (5a)$$

$$\bar{W}_\psi = \int_0^1 \int_0^{\theta_c} \bar{p} \cdot \sin \theta \cdot d\theta d\bar{z} \quad (5b)$$

where, $\bar{W}_r = \frac{W_r C}{\mu \Omega R^2 L}$ and $\bar{W}_\psi = \frac{W_\psi C}{\mu \Omega R^2 L}$

The load capacity is written as

$$\bar{W} = \sqrt{\bar{W}_r^2 + \bar{W}_\psi^2}, \quad \bar{W} = \frac{WC}{\mu \Omega R^2 L}$$

B. Attitude Angle

The attitude angle is obtained as follows:

$$\psi_0 = \tan^{-1} \left(\frac{\bar{W}_\psi}{\bar{W}_r} \right) \quad (6)$$

C. End Flow Rate

The volume flow rate in non-dimensional form from the clearance space is given by

$$\bar{Q}_z = -2 \int_0^{\theta_c} \phi_z(l_m, N, \bar{h}) \left(\frac{\partial \bar{p}}{\partial z} \right)_{\bar{z}=+1} d\theta \quad (7)$$

where, $\bar{Q}_z = \frac{Q_z L}{C \Omega R^3}$

The non-dimensional end flow rate is thus obtained first by finding numerically $\left[\frac{\partial \bar{p}}{\partial z} \right]_{\bar{z}=+1}$ following backward difference formula of order $\left[(\Delta z)^{-2} \right]$ and then by numerical integration using Simpson's one-third formula.

D. Frictional Parameter

The non-dimensional frictional force is given by [21]

$$\bar{F} = \int_0^1 \int_0^{\theta_c} A \cdot d\theta d\bar{z} + \int_0^1 \int_0^{\theta_c} A \cdot \frac{\bar{h}_{cav}}{h} \cdot d\theta d\bar{z} \quad (8)$$

where, $\bar{F} = \frac{FC}{\mu \Omega R^2 L}$, $\bar{h}_{cav} = 1 + \varepsilon_0 \cos \theta_c$

$$A = \frac{\bar{\tau}_c}{\bar{h} - \frac{2N}{l_m} \tanh\left(\frac{Nl_m \bar{h}}{2}\right)} + \frac{\bar{h}}{2} \cdot \frac{\partial \bar{p}}{\partial \theta}$$

The non-dimensional surface shear stress, $\bar{\tau}_c$ is obtained by considering the following expression as suggested by Taylor and Dowson [10] and [11] for dominant Couette flow and as referred in many research publications

$$\bar{\tau}_c = 1 + 0.00099(\text{Re}_h)^{0.96}$$

where $\bar{\tau}_c = \frac{\tau_c h}{\mu \Omega R}$

The frictional parameter is consequently obtained as follows:

$$f(R/C) = \frac{\bar{F}}{\bar{W}} \quad (9)$$

V. RESULTS AND DISCUSSION

It is evident from a modified Reynolds type equation denoted by (1) that the film pressure distribution depends on the parameters, namely, L/D, ε_0 , l_m , N^2 and Re. A parametric study has been carried out for all the above mentioned parameters excepting L/D which has been fixed at 1.0. A range of Re values (1000-10000) has been considered for the study on the turbulent effect.

In the present analysis, the inclusion of two non-dimensional parameters, i.e. l_m and N^2 imposes the condition of micropolar lubrication of journal bearing. l_m is considered as a characterization of the interaction of fluid with the bearing geometry and N^2 as the parameter coupling the linear momentum and the angular momentum equations arising out of the microrotational effect of the suspended particles in the lubricant. Furthermore, the Newtonian lubricating condition is achieved by setting $N^2 = 0$ or $l_m \rightarrow \infty$.

The validation of the results of the present study is not possible as of late the results of similar earlier works are not available in the literature. However, the trends of the results are similar to those reported in [24], but the values are apparently increased.

A. Load Carrying Capacity, \bar{W}

1) Effect of Coupling Number (N)

The combined effect of the micropolar parameters on the variation of the non-dimensional steady-state load, \bar{W} is shown as a function of l_m in Fig. 2 at L/D = 1.0, $\varepsilon_0 = 0.4$ and Re = 3000 for the parametric variation of N. The figure exhibits that the load capacity reduces expectedly with increase of l_m and approaches asymptotically to the Newtonian value as $l_m \rightarrow \infty$ and as $l_m \rightarrow 10$, the load capacity increases

considerably as the coupling number increases. The physical reason for the above observation is that as l_m tends to lower values, the velocity and the other flow characteristics will reduce to their equivalents in the Newtonian lubrication theory with μ everywhere replaced by $\left(\mu + \frac{1}{2}\chi\right)$, as the gradient of

microrotational velocity across the film thickness is very small. Hence effectively the viscosity has been enhanced. So, when the non-dimensional load has been referred to the Newtonian viscosity, it is increased by a factor $\left(\mu + \frac{1}{2}\chi\right)/\mu$ at

lower values of l_m . The decrease in the load capacity to the asymptotic value $l_m \rightarrow \infty$ is due to the fact that as $l_m \rightarrow \infty$, the characteristic length of the substructure is small and thus the micropolar characteristics is lost and it reduces to that of the classical hydrodynamic condition. $\left(\mu + \frac{1}{2}\chi\right)/\mu$ is equal to $1/(1-N^2)$ by virtue of definition of N^2 .

So, at lower values of l_m for higher values of N^2 , $1/(1-N^2)$ is more and so \bar{W} is more. Further, an increase in N^2 means a strong coupling effect between linear and angular momentum. This eventually gives enhanced effective viscosity and hence non-dimensional load carrying capacity increases. This has been reported by references [20] and [21] while analyzing the performances of plane journal bearings under laminar micropolar lubrication.

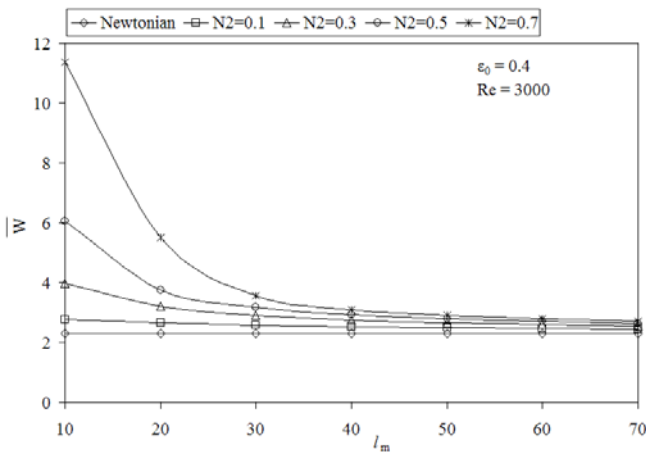


Fig. 2 \bar{W} vs. l_m for different values of N^2

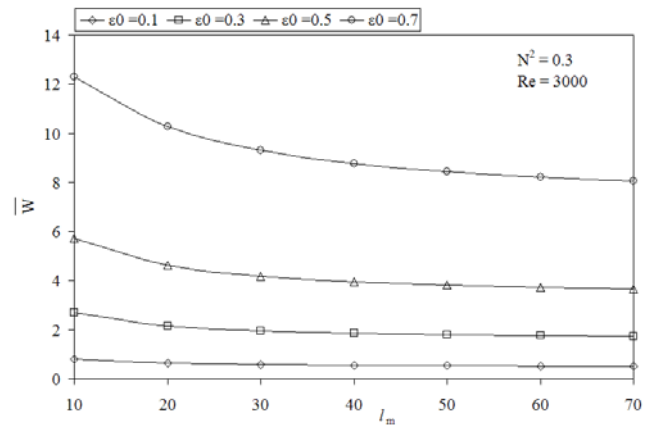


Fig. 3 \bar{W} vs. l_m for different values of ϵ_0

2) Effect of Eccentricity Ratio (ϵ_0)

Eccentricity always plays an important role in contributing the load capacity of a bearing. This is exhibited in Fig. 3, which shows that an increase of l_m reduces the load capacity at a particular value of ϵ_0 . The rate of decrease of \bar{W} with l_m is more pronounced at higher values of ϵ_0 . The effect of ϵ_0 is to improve the load capacity at any value of l_m . This is because, the increase of ϵ_0 results in higher film pressure and consequently higher load carrying capacity.

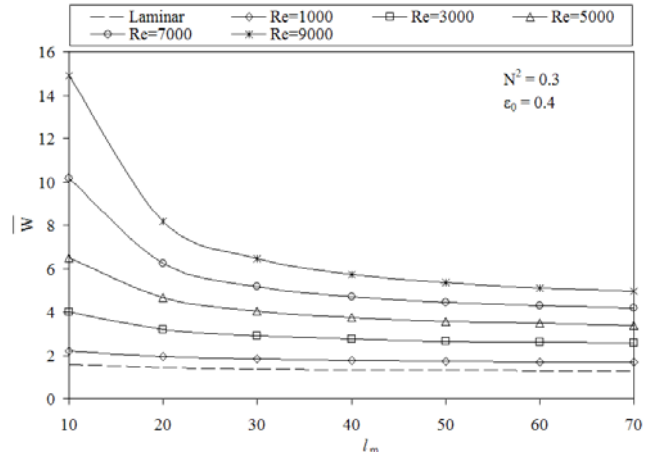


Fig. 4 \bar{W} vs. l_m for different values of Re

3) Effect of Average Reynolds Number (Re)

The effect of l_m on the load capacity is shown in Fig. 4 when the average Reynolds number is considered as a parameter. It is observed that load capacity shows the minimum value for laminar flow. At any value of average Reynolds number, an increase of l_m reduces the load capacity of bearing. The declining trend of the family of curves becomes more conspicuous at higher values of average Reynolds number, Re. At any value of l_m , the effect of average Reynolds number is to improve the load capacity. The

physical reason for this observation is that as the average Reynolds number increases, the turbulence coefficients, K_x and K_z increase (as the Reynolds number approaches zero, the laminar coefficient of 12.0 is achieved). As these turbulent coefficients increase, the film thickness terms $\left(\frac{\bar{h}^3}{K_x}$ and $\frac{\bar{h}^3}{K_z}\right)$

decrease. This causes an increase in the film pressure which, over the same area, results in higher load carrying capacity. A similar observation has been dealt with in [1].

B. Attitude Angle, ψ_0

1) Effect of Coupling Number (N)

Fig. 5 shows the variation of attitude angle with micropolar parameter, l_m for $L/D = 1.0$, $\varepsilon_0 = 0.4$ and $Re = 3000$ when the coupling number N is treated as a parameter. It can be observed from the figure that for a particular value of l_m , attitude angle decreases as N is increased. Furthermore, as l_m increases, the values of the attitude angle converge asymptotically to that for the Newtonian fluid. For any coupling number, attitude angle initially decreases with increase of l_m reaching a minimum value and then reversing the trend as l_m is further increased. It is also observed that the optimum value of l_m at which Φ_0 becomes a minimum value increases with a decrease in N . It can be demonstrated that to the left of the optimum value of l_m , micropolar effect becomes significant and to the right of this value, the micropolar effect diminishes.

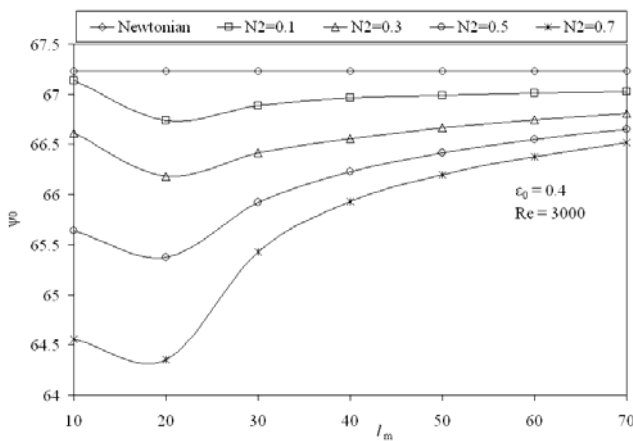


Fig. 5 ψ_0 vs. l_m for different values of N^2

2) Effect of Eccentricity Ratio (ε_0)

Attitude angle is shown as a function of l_m for various values of ε_0 as indicated in Fig. 6. Attitude angle more or less remains constant throughout the l_m values for any value of ε_0 . But it decreases with increase of ε_0 for any value of l_m .

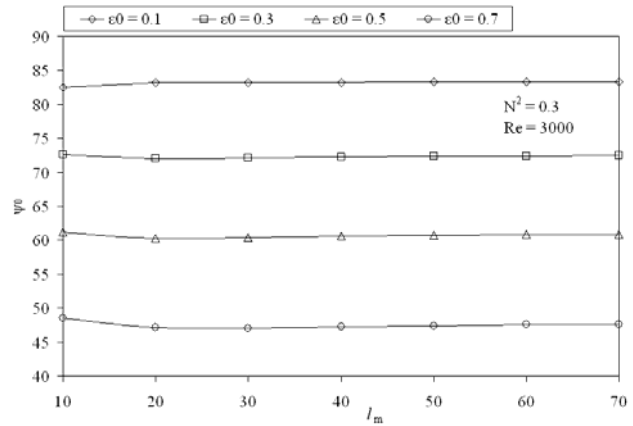


Fig. 6 ψ_0 vs. l_m for different values of ε_0

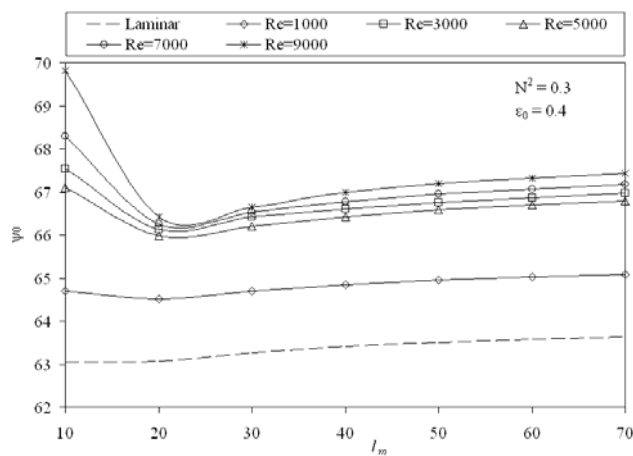


Fig. 7 ψ_0 vs. l_m for different values of Re

3) Effect of Average Reynolds Number (Re)

Fig. 7 depicts the variation of attitude angle as a function of l_m for various values of Re . It can be discerned from the figure that for any Re value, an increase in l_m initially reduces the attitude angle to a minimum value and then increases the attitude angle as l_m is increased more and more. The attitude angle has the minimum value for the laminar flow condition. The drooping tendency of the curves at lower values of l_m becomes more predominant as Re is increased. For a particular value of l_m , the effect of Re is to increase attitude angle.

C. End flow rate, \bar{Q}_z

1) Effect of Coupling Number (N)

Variation of end flow (from clearance space) of the bearing with l_m for various values of N is presented in Fig. 8. Non-dimensional end flow is found to decrease with increase of l_m for any value of N . Here too, the family of the curves shows the declining trend which becomes more significant at lower values of l_m as N is increased. Moreover, the micropolar effect is predominant at lower values of l_m and at higher values of l_m

all the curves irrespective of coupling number converges to that for Newtonian fluid. Further, an increase of N tends to increase the end flow for any l_m value.

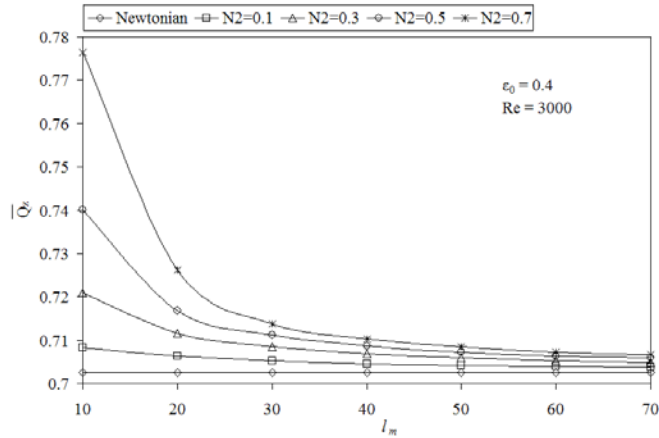


Fig. 8 \bar{Q}_z vs. l_m for different values of N^2

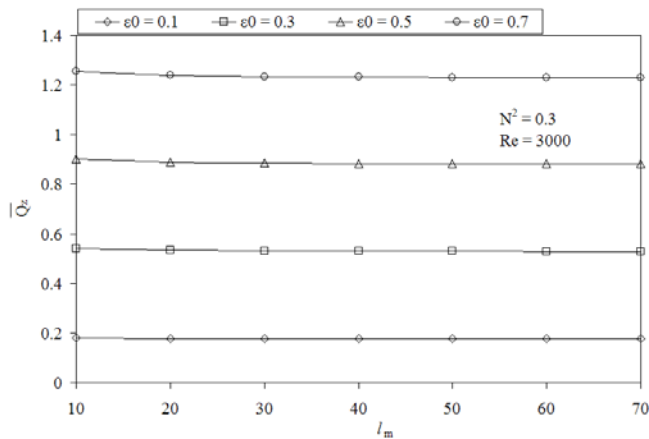


Fig. 9 \bar{Q}_z vs. l_m for different values of ϵ_0

2) Effect of Eccentricity Ratio (ϵ_0)

Effect of l_m on the end flow of bearing when ϵ_0 is taken as a parameter can be studied from Fig. 9. For any value of ϵ_0 , end flow decreases with increase of l_m , but the change in end flow with l_m is insignificant. For any value of l_m , an increase of ϵ_0 tends to increase the end flow. This is due to the fact that an increase of ϵ_0 results in higher film pressure gradient in the axial direction of bearing and consequently higher end flow of bearing.

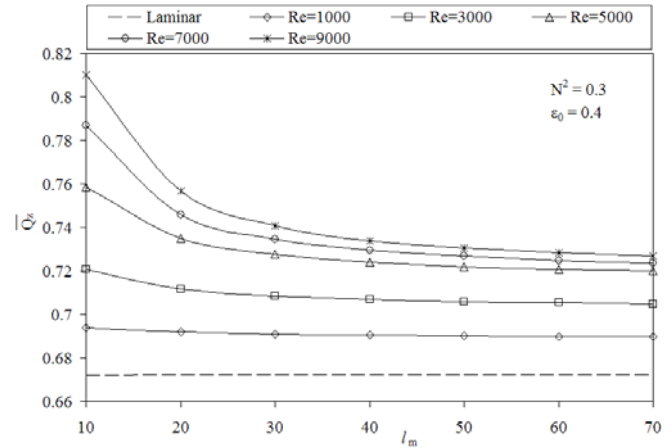


Fig. 10 \bar{Q}_z vs. l_m for different values of Re

3) Effect of Average Reynolds Number (Re)

Fig. 10 reveals that an increase of l_m tends to reduce the end flow for any value of Re. The drooping tendency of curves especially at lower values of l_m becomes more predominant as Re is increased. It is also found that the end flow rate shows minimum value at laminar flow conditions and it remains constant for all values of l_m .

For any value of l_m , an increase of Re tends to increase the end flow. The physical reason for this observation is that an increase of Re improves the values of turbulence coefficients, K_x and K_z which enhances the film pressure and consequently axial pressure gradient causing higher end flow of bearing.

D. Frictional parameter, $f(R/C)$

1) Effect of Coupling Number (N)

Frictional parameter, $f(R/C)$ is shown in Fig. 11 as a function of l_m for different values of coupling number, N . The figure reveals that the frictional parameter decreases with an increase of coupling number when the other parameters are held fixed. The increase in frictional parameter with l_m is more prominent at lower values of N . Also, it is observed that it increases and converges to that of Newtonian fluid when l_m assumes a very large value. This is because at $l_m \rightarrow \infty$, the fluid becomes Newtonian fluid and so the frictional parameter converges to that of Newtonian value.

2) Effect of Eccentricity Ratio (ϵ_0)

Fig. 12 shows the plot of frictional parameter as a function of l_m for different values of ϵ_0 . The frictional variable has the increasing trend with increase of l_m when ϵ_0 is considered as a parameter. This increasing trend is more predominant as ϵ_0 decreases more and more. Moreover, at any value of l_m , the frictional parameter decreases with increase of ϵ_0 . The difference between the respective values of $f(R/C)$ for the micropolar fluid and the Newtonian fluid decreases with

increase of ε_0 .

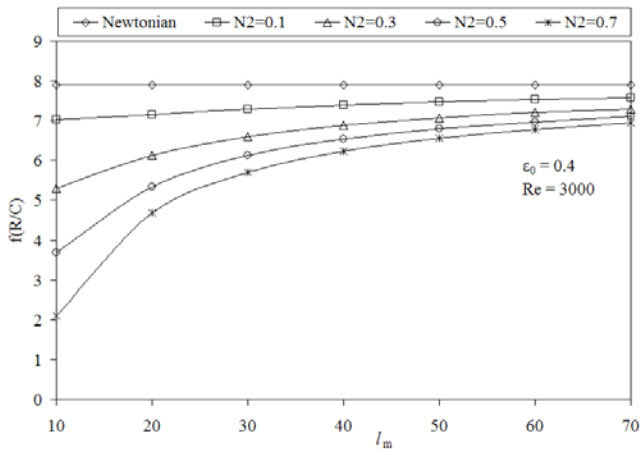


Fig. 11 $f(R/C)$ vs. l_m for different values of N^2

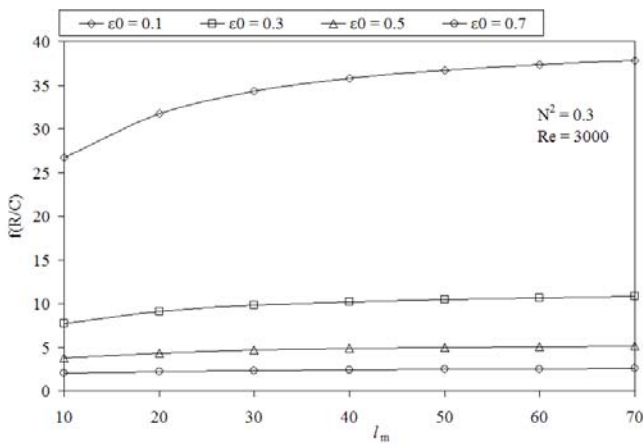


Fig. 12 $f(R/C)$ vs. l_m for different values of ε_0

3) Effect of Average Reynolds Number (Re)

Frictional parameter is shown as a function of l_m for different values of Re in Fig. 13. It is found that, in general, the frictional parameter increases with l_m for a particular value of Re, when all other parametric conditions remain unaltered. The effect of Re is to reduce the frictional parameter upto the l_m value around 35. Beyond $l_m = 35$, the trend is reversed. Such effect is not observed in the case of laminar flow and in case of Re = 1000. However, at higher values of l_m (>15) the frictional parameter at laminar flow conditions has the lowest value.

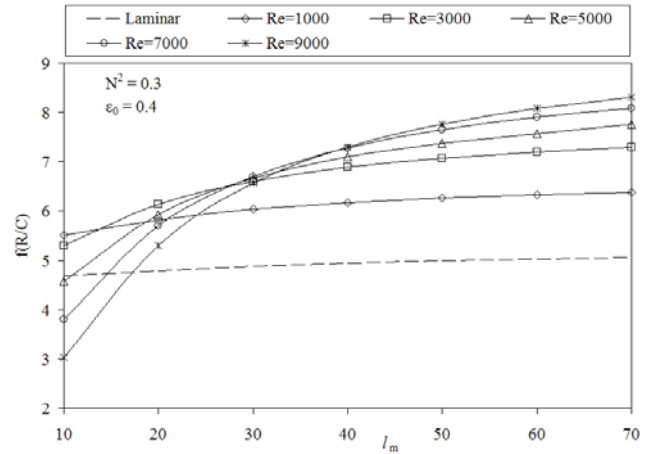


Fig. 13 $f(R/C)$ vs. l_m for different values of Re

VI. CONCLUSION

A study of the lubricating effectiveness of micro-polar fluids in the turbulent regime in case of journal bearings of finite width is presented. From the results of this study, the following conclusions can be drawn:

1. According to the results obtained for a particular value of average Reynolds number, the influences of various parameters on the static performance characteristics can be highlighted as follows.
 - (a) Load capacity reduces with increase of non-dimensional micro-polar characteristics length and approaches asymptotically to the Newtonian value as the micro-polar characteristics length tends to infinity at any value of coupling number. On the other hand, the effect of coupling number is to improve the load capacity when the non-dimensional characteristics length is taken as a parameter. In general, the micro-polar fluids exhibit a better load capacity than a Newtonian fluid under the condition of turbulent lubrication. Moreover, the effect of eccentricity ratio is to improve the load capacity at any value of non-dimensional micro-polar characteristics length.
 - (b) At any value of characteristics length, the effect of increase in coupling number is to reduce the attitude angle. In case of coupling number, taken as a parameter, attitude angle initially decreases with increase of characteristics length, achieving a minimum value followed by a reverse trend of variation. Moreover, the value of the characteristics length corresponding to the minimum value of attitude angle increases with decrease of coupling number. The effect of eccentricity ratio is to reduce the attitude angle at any value of characteristics length.
 - (c) The variation of end flow of the bearing with the non-dimensional characteristics length follows the similar trend of that of load with the characteristics length when the coupling number is taken as a parameter. However, an increase of characteristics length causes an insignificant

reduction in end flow at any value of eccentricity ratio. But the effect of increase of eccentricity ratio is to increase the end flow at any value of characteristics length.

- (d) The effect of increase in non-dimensional characteristics length is to enhance the frictional parameter when the coupling number is taken as a parameter. On the other hand, frictional parameter reduces with increase of coupling number at any value of characteristics length. Like the situation of laminar lubrication [20], the micropolar fluids also exhibit a beneficial effect in that the frictional parameter is less than that of Newtonian lubricant under turbulent flow condition. The effect of increase of eccentricity ratio is to reduce the frictional parameter at any value of characteristics length. At lower values of eccentricity ratio, the improvement of frictional parameter with the characteristics length is more pronounced.
- The effect of increase of average Reynolds number is to improve the load capacity at any value of non-dimensional characteristics length. At lower values of characteristics length, this increase of load capacity is more significant.
 - At any value of characteristics length, an increase of average Reynolds number favors the enhancement in attitude angle.
 - An increase of average Reynolds number tends to increase the end flow of bearing at any value of characteristics length.
 - At higher values of characteristics length (exceeding the value of around 30), the frictional parameter increases with increase of average Reynolds number. But at lower values of characteristics length, the trend of variation is reversed, exhibiting a beneficial effect in terms of reducing frictional parameter at higher values of average Reynolds number.

NOMENCLATURE

A_x	Constant parameter of turbulent shear coefficient for circumferential flow
B_x	Exponential constant parameter of turbulent shear coefficient for circumferential flow
C	Radial clearance, m
C_z	Constant parameter of turbulent shear coefficient for axial flow
D	Journal diameter, m
D_z	Exponential constant parameter of turbulent shear coefficient for axial flow
e_0	Steady-state eccentricity, m
ε_0	Steady-state eccentricity ratio, $\varepsilon_0 = \frac{e_0}{C}$
$f(R/C)$	Frictional parameter, $f(R/C) = \frac{\bar{F}}{\bar{W}}$
F	Frictional force, N
\bar{F}	Non-dimensional frictional force, $\bar{F} = \frac{FC}{\mu \Omega R^2 L}$

h	Local film thickness, m
\bar{h}	Non-dimensional film thickness, $\bar{h} = \frac{h}{C}$
h_{cav}	Film thickness at the point of cavitation, m
\bar{h}_{cav}	Non-dimensional film thickness at the point of cavitation, $\bar{h}_{cav} = \frac{h_{cav}}{C}$
h_m	Mean or average film thickness, m
h_{max}	Maximum film thickness, m
h_{min}	Minimum film thickness, m
K_x, K_z	Turbulent shear coefficients in x and z directions respectively
l_m	Non-dimensional characteristics length of micropolar fluid, $l_m = \frac{C}{\Lambda}$
L	Bearing length, m
N	Coupling number, $N = \left[\frac{\chi}{(2\mu + \chi)} \right]^{\frac{1}{2}}$
p	Steady-state film pressure, Pa
\bar{p}	Non-dimensional steady-state film pressure $\bar{p} = \frac{pC^2}{\mu \Omega R^2}$
Q_z	End flow rate of lubricant, m ³ /s
\bar{Q}_z	Non-dimensional end flow rate of lubricant, $\bar{Q}_z = \frac{Q_z L}{C \Omega R^3}$
R	Journal radius, m
Re	Mean or average Reynolds number defined by radial clearance, $C, Re = \frac{\rho \Omega R C}{\mu}$
Re_h	Local Reynolds number defined by the local film thickness, $h, Re_h = \frac{\rho \Omega R h}{\mu}$
U	Velocity of journal 'm/s', $U = \Omega R$
W	Steady-state load on bearing, N
\bar{W}	Non-dimensional steady-state load on bearing, $\bar{W} = \frac{WC}{\mu \Omega R^2 L}$
W_r	Radial component of the steady-state load on bearing, N
\bar{W}_r	Non-dimensional radial component of the steady-state load on bearing, $\bar{W}_r = \frac{W_r C}{\mu \Omega R^2 L}$
W_ψ	Transverse component of the steady-state load on bearing, N
\bar{W}_ψ	Non-dimensional transverse component of the steady-state load on bearing, $\bar{W}_\psi = \frac{W_\psi C}{\mu \Omega R^2 L}$
\bar{x}, \bar{x}	Cartesian coordinate axis in the circumferential direction, $x = R\theta$
\bar{z}, \bar{z}	Cartesian coordinate axis along the bearing axis, $\bar{z} = \frac{2z}{L}$

γ, χ viscosity coefficients for the micropolar fluid, Pa.s
 $\phi_{x,z}$ micropolar functions for turbulent flow along x and z directions
 ψ_0 angle between the eccentricity vector and the line of action of load(W), rad
 μ Newtonian viscosity coefficient, Pa.s
 μ_v Effective viscosity coefficient of micropolar fluid, Pa.s
 $\mu_v = \frac{(2\mu + \chi)}{2}$
 Λ characteristics length of the micropolar fluid, $\Lambda = \left(\frac{\gamma}{4\mu}\right)^{1/2}$
 θ Angular coordinate in the bearing circumferential direction
 θ_c Angular coordinate where the film cavitates
 ρ Mass density of lubricant, kg/m³
 Ω Angular velocity of journal, rad/s
 τ_c Surface shear stress for Couette's flow, N/m²
 $\bar{\tau}_c$ Non-dimensional surface shear stress for Couette's flow,
 $\bar{\tau}_c = \frac{\tau_c h}{\mu \Omega R}$.

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