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An Examination and Validation of the Theoretical Resistivity-Temperature Relationship for Conductors

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***Abstract*—**Electrical resistivity is a fundamental parameter of metals or electrical conductors. Since resistivity is a function of temperature, in order to completely understand the behavior of metals, a temperature dependent theoretical model is needed. A model based on physics principles has recently been developed to obtain an equation that relates electrical resistivity to temperature. This equation is dependent upon a parameter associated with the electron travel time before being scattered, and a parameter that relates the energy of the atoms and their separation distance. Analysis of the energy parameter reveals that the equation is optimized if the proportionality term in the equation is not constant but varies over the temperature range. Additional analysis reveals that the theoretical equation can be used to determine the mean free path of conduction electrons, the number of defects in the atomic lattice, and the 'equivalent' charge associated with the metallic bonding of the atoms. All of this analysis provides validation for the theoretical model and provides insight into the behavior of metals where performance is affected by temperatures (e.g., integrated circuits and temperature sensors).

***Keywords*—**Callendar–van Dusen, conductivity, mean free path, resistance temperature detector, temperature sensor.

I. Introduction

IT is understood that the electrical resistivity of metals or conductors is a fundamental property and that it is not always constant [1]-[2]. Electrical resistivity for conductors is a function of temperature and under certain conditions will be a function of the size of the material [1]-[2]. As the number of circuits and interconnections between circuits on a microchip steadily increases, it is important to understand how these interconnects or conductors could be adversely affected over a certain temperature range (e.g., due to overheating). Additionally, resistance temperature detectors (i.e., RTDs) are sensors that utilize this resistance-temperature property of metals to measure temperatures. Experimental evidence has shown that this temperature dependence is linear for metals when the resistivity is measured over a specific temperature range.

Until recently, there has not been a straightforward and reasonable model that elucidates the mechanisms that cause this resistivity-temperature relationship. A two dimensional model has been created and this model leads to an equation that produces data that is consistent with the aforementioned experimental results [2]. Thus, this theoretical model provides incredible insight into the underlying physics of the resistivity-temperature relationship.

The resistivity-temperature equation contains variables that depend upon the atomic diameter, atomic spacing, electron travel time before being scattered, and the proportionality constant that relates atomic force and separation distance. These variables or parameters can be selected to produce a resistivity-temperature response that is linear over a specified temperature range.

The resistivity-temperature relationships for platinum and nickel have been well characterized experimentally [3]-[5]. The general equation that represents metals is known as the Callendar-van Dusen equation and each metal has its own set of distinct coefficients for this equation. Parameter values can be selected for the theoretical model such that the theoretical and experimental equations are in very good agreement [2]. Although a very good match was obtained, an identical match between the experimental and theoretical was not achieved. It is believed that the choice of parameter values is responsible for this less than ideal response and that a more detailed analysis can be performed to select parameter values that will duplicate the experimental response.

The mean free path for conduction electrons represents the average distance that electrons travel before being scattered by another object associated with lattice structure of the material. The primary scattering mechanism for the conduction electrons is due to vibrating atoms, however, lattice defects and/or impurities can also contribute (and will significantly contribute at low temperatures) [5]. Nevertheless, a model that characterizes the resistivity-temperature relationship should incorporate the concepts of the mean free path and lattice defect scattering if it is going to be completely valid [6].

Additionally, electrostatic forces (involving positive ion cores and negative electrons) exist in materials to hold atoms together. These bonds exist between the atoms in the lattice to keep them at an equilibrium distance. If external energy is not supplied to the atoms, then this equilibrium distance between atoms minimizes the energy of the crystal [7]-[13]. As energy is supplied to the crystal, the atoms will move and this is demonstrated in the aforementioned theoretical model.

The bonds for conductors, known as metallic bonds, involve interactions between the positive ion cores and the 'sea of electrons' or the loosely bound conduction electrons [7]-[13]. Because the ion cores share these electrons, it is difficult to determine the charge distribution or the equivalent charge

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associated with the ions when they move from their equilibrium position. Although the crystal maintains its charge neutrality, there must be a redistribution of charge to force or move the atoms back to their equilibrium position. It is believed that some insight into this charge distribution can be obtained from the theoretical model.

Therefore, the theoretical equation parameters were evaluated to determine if (and how) they could be optimized to provide an identical match with the experimental equation. It was determined that modifications could be made to the value of the proportionality constant that relates atomic force and separation distance and as a result, the theoretical equation becomes identical to the experimental expression. Additionally, analysis was performed to demonstrate that the theoretical equation can be reduced to a standard equation that can be used to determine both the mean free path of conduction electrons and the number of defects in the lattice. Finally, based on analysis of the theoretical model and the energy parameter, it was determined that the 'equivalent' charge associated with the lattice atoms in this model could also be determined.

II. Summary of the Theoretical Model

The details of the theoretical model have been given elsewhere [2], but a brief summary of this model is provided here. A two-dimensional monatomic lattice with a face centered cubic (FCC) structure is used as the foundation for the theoretical model. The atoms are restricted to move in only one direction and conduction electrons are likewise restricted to travel in only one direction which is orthogonal to atomic motion.

Gaps exist between the atoms within the lattice and some electrons travel in gaps through the material while other electrons do not travel in the gaps. Additionally, atoms in the lattice are vibrating and in this process a minimum distance between the atoms is achieved. As the temperature is increased the atoms obtain more energy and thus are able to move closer together and reduce the gaps. The force (from Coulomb's Law) and energy associated with the atoms is given by $F = \frac{\beta}{r^{\varepsilon}}$ and $U = \frac{\gamma}{r^{\delta}}$ where β and γ are assumed to be proportionality constants, ε and δ are assumed to be integers ($\varepsilon = 2$ and $\delta = \varepsilon - 1 = 1$) and r is the distance between two atoms.

When the gaps are reduced or decrease in size, this increases the impedance of electron flow, and the electrical resistivity increases. With this information, an equation representing the resistivity-temperature relationship can be derived and is given by

$$\rho = \rho_0 \left[\frac{1}{\frac{\left[2\sqrt[\delta]{\gamma/kT} - b\right](\tau_1/\tau_2 - 1)}{2a + b} + 1} \right] \quad (1)$$

where a is the atomic radius, b is the size of the opening between atoms (when the atoms are stationary), τ_1/τ_2 is the ratio of travel time before scattering when an electron is in the gap to when an electron is not in a gap, k is Boltzmann's constant, and T is temperature (in K).

III. Assumptions Used in the Model

To create the resistivity-temperature model and obtain a straightforward equation as given in (1), several assumptions and simplifications were used to make analysis less difficult. The assumptions include using a two-dimensional model, approximating the atomic radius as a point source, restricting electron motion to one direction, as well as restricting atom vibration to one direction. Furthermore, it was assumed that the only factors contributing to the force and energy terms were from forces in the [010] direction within a lattice constant of an atom (this led to the assumption that β and γ are constant). These assumptions do not resemble a real material, nevertheless the results are remarkably precise and produce reasonably accurate Callendar-van Dusen equation coefficients for platinum and nickel.

IV. Modifications to the Model

Although the theoretical model has been simplified to make analysis easier, it produces very accurate results. However, to obtain results that match the Callendar-van Dusen equation exactly, the assumptions have to be modified. Since the force on atoms is clearly seen to be an oversimplification, it will be re-examined.

Fig. 1 shows atom A with forces acting on it due to repulsion from other atoms (atoms1, 2, and 3). Again, the original model only included the force $F_1 = \frac{\beta}{r^{\varepsilon}}$ which leads to an energy term $U_1 = \frac{\gamma}{r^{\delta}}$. However, by including the forces from atoms 2 and 3 (along with the force from atom 1) the result is seen to be $F = F_1 + F_2 + F_3$ or equivalently

$$F = \left[\frac{\beta}{r_1^2} + \frac{\beta}{r_2^2}\cos\theta_2 + \frac{\beta}{r_3^2}\cos\theta_3 \right] \quad (2)$$

Again, by assuming that the atoms will only vibrate or move in the [010] direction (i.e., the y-direction), the magnitude of the x-component for F_2 and F_3 must be equal. Thus, it is assumed that r_2 and r_3 are equal and that θ_2 and θ_3 are equal. As a result, (2) becomes

$$F = \beta\left[\frac{1}{r_1^2} + \frac{2}{r_2^2}\cos\theta_2 \right] \quad (3)$$

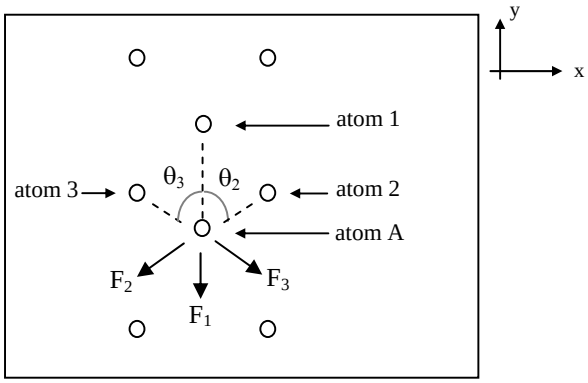

Fig. 1 Two dimensional model showing the forces on an atom
In the original model, only force F_1 was included, butthe modified model also includes forces F_2 and F_3.

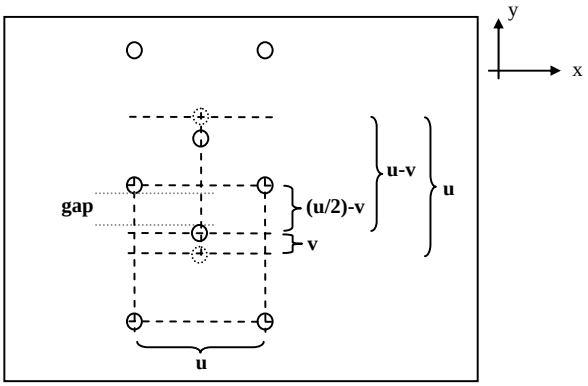

Fig. 2 Distances associated with the atoms in the theoretical model
The dotted circles represent the original location of the atoms and the solid circles closest to the dotted circles represent the current location of those atoms.

Now, Fig. 2 can be used to reduce (3). Using trigonometry and basic analysis, it is seen that $r_1 = u - 2v$ and $r_2 = \sqrt{\left(\frac{u}{2}\right)^2 + \left(\frac{u}{2} - v\right)^2}$ or $r_2 = \frac{1}{2}\sqrt{(u)^2 + (u-2v)^2}$ and thus $r_2 = \frac{1}{2}\sqrt{(u)^2 + (r_1)^2}$. Likewise, it is also seen that $\cos\theta_2 = \frac{\frac{u}{2} - v}{r_2}$ and equivalently $\frac{2\cos\theta_2}{r_2^2} = \frac{u-2v}{r_2^3}$. Substitution yields $\frac{2\cos\theta_2}{r_2^2} = \frac{8r_1}{\left[(u)^2 + (r_1)^2\right]^{3/2}}$. Therefore, (3) can be written as

$$F = \beta\left[\frac{1}{r_1^2} + \frac{8r_1}{\left[(u)^2 + (r_1)^2\right]^{3/2}}\right] \tag{4}$$

The energy is found by taking the integral of the force (i.e., $U = -\int F dx$). From an integral table, it is found that $\int \frac{x}{\left[a^2 + x^2\right]^{3/2}} dx = \frac{-1}{\sqrt{a^2 + x^2}}$. Thus, the energy will be

$$U = \gamma\left[\frac{1}{r_1} + \frac{8}{\sqrt{(u)^2 + (r_1)^2}}\right] \tag{5}$$

which has an additional term (compared to the original expression $U = \frac{\gamma}{r_1}$) due to forces F_2 and F_3. This equation can be rewritten to have the form

$$U = \frac{\gamma}{r_1}\left[1 + \frac{8}{\sqrt{\left(\frac{u}{r_1}\right)^2 + 1}}\right]$$

which can be written in the form

$$U = \frac{\gamma}{r_1}\left[\frac{8 + \sqrt{\left(\frac{u}{r_1}\right)^2 + 1}}{\sqrt{\left(\frac{u}{r_1}\right)^2 + 1}}\right] \tag{6}$$

Now, based on the form of expressions (5) or (6), it would be difficult to obtain a resistivity-temperature equation that is as compact and straightforward as the expression given in (1). Nevertheless, r_1 represents the distance between atom A and atom 1 and it varies with temperature T. Therefore, the term in the brackets in (6) will vary with temperature and thus the energy can be written as $U = \frac{\gamma}{r_1}\left[f_1(T)\right]$ where $f_1(T)$ is a function that varies with temperature T [and represents the term in brackets in (6)]. Since γ is a constant $\gamma[f_1(T)]$ can be written as $f(T)$ and thus, the energy is

$$U = \frac{f(T)}{r_1} \tag{7}$$

Therefore, since the energy was originally $U = \frac{\gamma}{r_1}$, as long as γ has a temperature dependency, the model will be more accurate.

Data generated from the theoretical model should match the Callendar-van Dusen equation since this equation provides experimentally verifiable data for the resistivity-temperature relationship. The Callendar-van Dusen equation has the general form $\rho = \rho_0\left[1 + AT + BT^2\right]$ (T is in °C) where each material has a different set of values for A and B. Platinum has values $A = 3.91*10^{-3}$ and $B = -5.78*10^{-7}$ for temperatures between 0°C to 650°C [3]-[5] and nickel has values A = $5.49*10^{-3}$ and $B = 6.65*10^{-6}$ for temperatures ranging from

0^{o}C to 250^{o}C [3]-[5].

V. Results and Discussions

A. Energy Coefficient

A computer program was written to evaluate (1) and determine values for τ_1/τ_2 as well as temperature dependent γ that would provide an exact match for the Callendar-van Dusen equation coefficients. Upon substituting platinum parameters $a=1x10^{-12}m$ and $b=3.92x10^{-10}m$ into (1), it is found that a multitude of values for τ_1/τ_2 and γ exists that will produce the Callendar-van Dusen coefficients for platinum. Fig. 3 shows a plot of γ varying with temperature for platinum with several values of τ_1/τ_2. The equations that represent these three curves are given by $\gamma = 1.23*10^{-37}T^2 + 1.97*10^{-33}T + 1.17*10^{-30}$ for τ_1/τ_2 = 5, $\gamma = 5.67*10^{-38}T^2 + 2.38*10^{-33}T + 0.932*10^{-30}$ for τ_1/τ_2 = 10, and $\gamma = 9.47*10^{-39}T^2 + 2.65*10^{-33}T + 0.775*10^{-30}$ for τ_1/τ_2 = 50. The correlation coefficient for all of these curves is $R^2 = 1$.

Likewise, substituting nickel coefficients $a=1x10^{-12}m$ and $b=3.52x10^{-10}m$ into (1) leads to a set of values for τ_1/τ_2 and γ that will produce the Callendar-van Dusen coefficients for nickel. Fig. 4 displays a graph of γ varying with temperature for nickel with several values of τ_1/τ_2. The equations that represent these three curves are given by $\gamma = 8.21*10^{-37}T^2 + 1.04*10^{-33}T + 0.959*10^{-30}$ for τ_1/τ_2 = 5, $\gamma = 3.86*10^{-37}T^2 + 1.80*10^{-33}T + 0.796*10^{-30}$ for τ_1/τ_2 = 10, and $\gamma = 6.94*10^{-38}T^2 + 2.31*10^{-33}T + 0.687*10^{-30}$ for τ_1/τ_2 = 50. As was the case for platinum, the correlation coefficient for each of the curves for nickel is $R^2 = 1$.

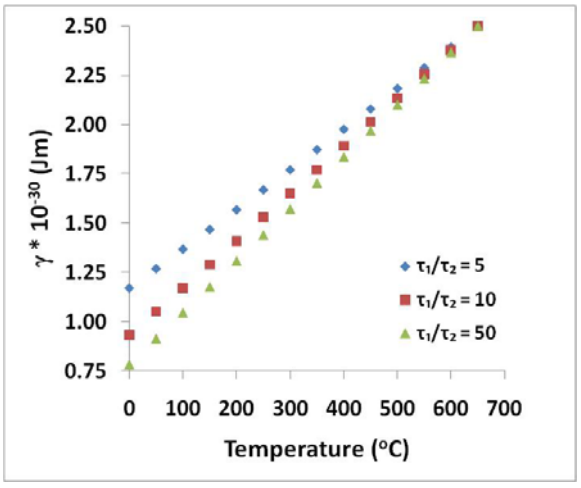

Fig. 3 Response of the energy coefficient γ as a function of temperature for platinum

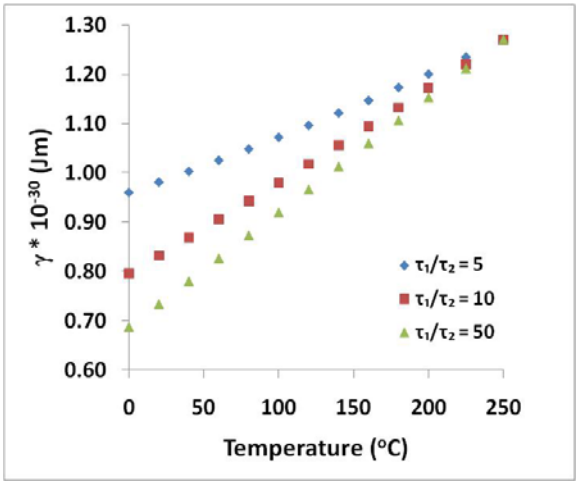

Fig. 4 Response of the energy coefficient γ as a function of temperature for nickel

It is well known from experimental evidence that conductors (such as platinum and nickel) have temperature regions where the electrical resistivity is linear. For platinum, this region is between 0^{o}C and 650^{o}C, and for nickel, this region is between 0^{o}C and 250^{o}C [3]-[5]. Based on the operation of this theoretical model, as the temperature becomes higher, the gap becomes smaller, and thus the resistivity becomes larger. Consequently, it is reasoned that at the upper end of the linear temperature region, the electrical resistivity becomes non-linear because the gap has vanished. Therefore, by examining the size of the gap, the value of γ can be selected to ensure that the gap becomes zero. This occurs for $\gamma = 2.5*10^{-30}$ Jm for platinum and $1.27*10^{-30}$ Jm for nickel and thus the curves in Figs. 3 and 4 converge to these values, respectively.

By examining the energy parameter of the theoretical model and then deriving a new expression for this parameter, this term is more accurate and thus the resistivity-temperature equation that is derived from the model is more accurate. The proportionality term for the force (and energy) was originally kept constant and thus could not sufficiently represent all of the forces on an atom that is in motion. Thus, this modification leads to an accurate atomic force and because of its general nature, it represents the true and total two-dimensional force on an atom. In other words, the force on an atom is not only composed of three terms as given in (2) [which was reduced to two terms and then to one term as given in (7)], but it is represented by a series of additional terms which can ultimately be represented by one term with a temperature dependent proportionality constant. As a result, the energy term is optimized, the theoretical model is enhanced and the output from the model provides an exact match for the Callendar-van Dusen equation.

B. Mean Free Path

By demonstrating an exact match of the Callendar-van Dusen coefficients, it has been shown that the theoretical model is very robust and provides tremendous insight into

experimental results. Additional validation for this theoretical model can be seen through the analysis of the mean free path for conduction electrons. To determine the electron mean free path, the general expression for electrical resistivity is typically used as a starting point [14], [15]. This is expression is given by

$$\rho = \frac{m}{ne^2\tau_{avg}} \tag{8}$$

where τ_{avg} is the average travel time by a conduction electron before it is scattered.

It is noted that this same expression given in (8) is obtained at the upper temperature when the resistivity ceases to be linear (i.e., when T = 650°C or $\gamma = 2.5*10^{-30}$ Jm for platinum and when T = 250°C or $\gamma = 1.27*10^{-30}$ Jm for nickel). In (1) the term $2\sqrt[\delta]{\gamma/kT} - b$, which is equal to the gap, becomes zero (after the appropriate selection of γ) and as a result (1) reduces to $\rho = \rho_0 = \frac{m}{ne^2\tau_2}$ where τ_2 is the scatter time when an electron is not in a gap (which is the case for all conduction electrons at the aforementioned temperatures). Thus, this resistivity equation is equivalent to (8).

For platinum, at T = 0°C, the resistivity is calculated to be $9.83*10^{-8}$ Ωm and thus τ_{avg} is found to be $5.4*10^{-15}$ s. Similarly, for nickel at T = 0°C, the resistivity is $6.31*10^{-8}$ Ωm and τ_{avg} is found to be $6.1*10^{-15}$ s. Quantum mechanical analysis has shown that conduction electrons exists at the Fermi surface and travel with the Fermi velocity. The Fermi velocity is calculated from $v_F = \left(\frac{\hbar}{m}\right)\left(3\pi^2 n\right)^{1/3}$ [16], [17] and for platinum is found to be $1.44*10^6$ m/s and for nickel is found to be $1.61*10^6$ m/s. Thus the mean free path l (= $v_F*\tau_{avg}$) for conduction electrons at T = 0°C for platinum is $7.78*10^{-9}$ m and for nickel is $9.82*10^{-8}$ m. Likewise, at T = 650°C, platinum has a resistivity of $34.76*10^{-8}$ Ωm and τ_2 is found to be $1.527*10^{-15}$ s while at T = 250°C nickel has a resistivity of $14.99*10^{-8}$ Ωm and τ_2 is found to be $2.57*10^{-15}$ s. Therefore, the mean free path for conduction electrons at T = 650°C for platinum is $l = 2.2*10^{-9}$ m and at T = 250°C for nickel is $l = 4.14*10^{-9}$ m. These mean free paths values are provided in Table I. These values are reasonable and comparable to those recorded in the literature [18]-[20].

TABLE I
MEAN FREE PATHS OF CONDUCTION ELECTRONS FOR PLATINUM AND NICKEL AT SPECIFIED TEMPERATURES

	T=0°C	T=250°C	T=650°C
Platinum	$7.78*10^{-9}$ m	---	$2.20*10^{-9}$ m
Nickel	$9.82*10^{-9}$ m	$4.14*10^{-9}$ m	---

C. Lattice Defects

The resistance or resistivity of a material is dependent not only upon electron interactions with phonons (in a temperature dependent manner), but also upon impurities, lattice imperfections and/or boundary misalignments [21]. Although resistivity is dominated by electron-phonon interaction at elevated temperatures, when the temperature is approximately equal to –273°C or 0K, the resistivity of the material will only depend upon the latter items because electron scattering due to temperature dependent lattice vibrations will be non-existent.

If a lattice were perfect and contained no defects, then at (or near) 0K the material would have zero electrical resistance and be able to sustain an electric current indefinitely. Thus measurements of a material's electrical resistance or resistivity at these very low temperatures will provide a direct indication of how pure the material is. In other words, the electrical resistivity will be proportional to the number of defects or it will give an indication of how often (on average) defects occur in the lattice (i.e., the lower the resistance is, the fewer impurities, misalignments, etc. there will be in the material). Thus, if the theoretical model is accurate, it should support or confirm these facts.

Again, it is understood that the mean travel time before a conduction electron is scattered is τ_{avg} and that the electrical resistivity of the material is inversely proportional to this time as given by (8). It is further understood that the atoms in the lattice or material will be practically stationary when the temperature is near 0K. Therefore, the gap between atoms will be $g = 2a + b$ and thus γ in the term $2\sqrt[\delta]{\gamma/kT} - b$ in (1) [again this term is equal to the gap] must be selected such that the gap equals $2a + b$. When this happens, (1) reduces to $\rho = \frac{m}{ne^2\tau_1}$ where τ_1 is the scatter time when an electron is in a gap (which will be the case for all conduction electrons when $T \approx 0K$). Thus, this electrical resistivity equation (containing τ_1 as the scatter time) is equivalent to (8) for $T \approx 0K$.

So for an ideal or perfect metal, at or near T=0K, the electrical resistivity value will be 0 and thus the electron scatter time and mean free path will be $\tau_1 = \infty$ and $l = \infty$, respectively. And thus an electron can travel a distance of $l = \infty$ before it encounters a defect (or equivalently there are no defects in the material). Now, as an example, consider the conductor with an electrical resistivity of 10^{-15} Ωm. Again, the Fermi velocity for platinum is $1.44*10^6$ m/s and for nickel is $1.61*10^6$ m/s and therefore, the electron scatter time and mean free path will be $\tau_1 = 5.31*10^{-7}$ s and $l = 0.765$ m, respectively for platinum, and $\tau_1 = 3.85*10^{-7}$ s and $l = 0.62$ m, respectively for nickel. This shows that an electron will travel 0.765 m in platinum before it is scattered or equivalently a defect occurs every 0.765m in the platinum sample (or there are 1.31 defects per meter); likewise, an electron will travel 0.62m in nickel before it is scattered or equivalently a defect occurs every 0.62m in the nickel sample (or there are 1.61 defects per meter).

It is noted that if the electrical resistivity of a sample of platinum or nickel is measured at $T \approx 0K$, then the number of defects in the sample will be known and the value of τ_1 can be determined. After this scattering time has been determined, the

value of the ratio τ_1/τ_2 will be known, and the energy coefficient γ can be found for a given temperature. Additionally, the size of the gap can be calculated as well as the average scatter time for the conduction electrons [2].

D.Equivalent Charge

Again atoms in the conductor will vibrate or oscillate and move from their equilibrium position because of thermal energy. Based upon specific aspects of the theoretical framework, this model can be used to determine the charge associated with the metal atoms to restore them to their original position. Fig. 1 shows that in general an atom will experience a force due to several of its nearest neighbor atoms. However, when the spacing or gap (as shown in Figs 1 and 2) between atom A and atoms 2 and 3 disappears, forces F_2 and F_3 will have no significant y-component and thus only act in the x-direction. Since forces F_2 and F_3 will have equal and opposite components in the x direction, these two forces will not contribute to the overall force on atom A when the atoms are in the position as shown in Fig. 5.

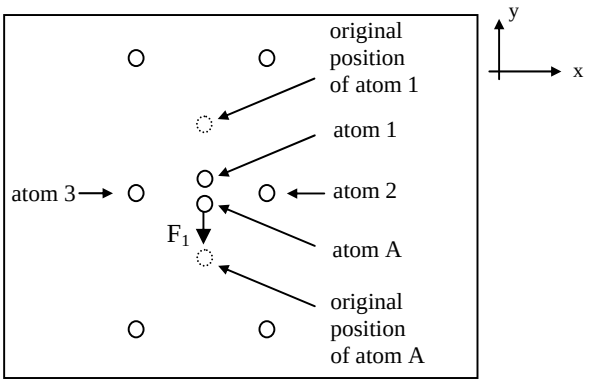

Fig. 5 Theoretical model showing the force on atom A when the 'gap' vanishes
In this state, only atom 1 contributes to the force. Forces from atoms 2 and 3 are negligible in the y-direction and thus make no contribution to the force on atom A.

As a result of the aforementioned analysis (2) can be reduced to

$$F_1 = \frac{\beta}{r_1^2} \tag{9}$$

where r_1 is the distance between atom A and atom 1 and β is the proportionality term. It is also known that Coulomb's Law provides a relationship between charges, distance, and the force on those charges. This relationship is given by

$$F = k\frac{q_1 q_2}{r^2} \tag{10}$$

where q_1 and q_2 are the charges, r is the distance between the charges, and k is the proportionality constant and has a value of $9*10^9$ Nm^2/C^2. Since the two forces in (8) and (9) are the same, they can be set equal to each other, and the result is $\beta = kq_1q_2$. It is assumed that the charge associated with each atom (i.e., atom 1 and atom A) is equal, so q_1 and q_2 will be equal. Also, because the force is equal to the negative gradient of the energy or equivalently because the energy is the negative integral of the force (i.e., $F = -\nabla U$ or $U = -\int F dy$), it is seen that the value of force and energy proportionality terms will equal (i.e., $\beta = \gamma$). Therefore, using the maximum values of γ as shown in Figs. 3 and 4, the equivalent charge for both platinum atoms and nickel atoms can be calculated. This equivalent charge is $1.67*10^{-20}$ C for platinum atoms and $1.19*10^{-20}$ C for nickel atoms. These results are given in Table II.

TABLE II
EQUIVALENT CHARGE FOR THE METALLIC BONDS ASSOCIATED WITH PLATINUM AND NICKEL ATOMS

	$\beta = \gamma_{max}$	$q\ (= q_1 = q_2)$
Platinum	$2.50*10^{-30}$ Nm^2	$1.67*10^{-20}$ C
Nickel	$1.27*10^{-30}$ Nm^2	$1.19*10^{-20}$ C

It is understood that metal atoms are surrounded by a 'sea of electrons' that are loosely bound conduction electrons shared by all of the atoms. The positively charged atoms (also known as the ion cores) share these negatively charged electrons and form a charge neutral material. Although as a whole, the metal is neutral in charge, at any particular instant in time, various alignments of ion cores can result in 'regions' where positive and negative charges exist.

As the ion cores move closer together (or farther apart), the atoms will undergo an 'equivalent redistribution' of charge. This charge will provide the force that moves the atoms back to their equilibrium position. Nevertheless, since the charge is only a 'redistribution', the overall charge neutrality remains intact. Thus, based on the previous analysis, when an atom moves into the position as shown by atom A in Fig. 5, atom 1 and atom A will acquire a net charge that will apply a force on the atoms and ultimately cause them to move back to their original positions.

It is seen that the two metals, platinum and nickel, have different equivalent charges associated with the atoms. Because the spacing between atoms is different (3.92nm for platinum and 3.52nm for nickel), the bonding strengths or forces are different, the maximum temperature for which the materials exhibit linear resistivity is different, it is reasonable that these two materials have different equivalent charges.

VI. CONCLUSION

A two dimensional theoretical model that produces an equation relating the electrical resistivity to temperature has been evaluated and optimized so that it conforms to experimental data. By varying the proportionality constant of the energy parameter as a function of temperature, the energy (and force) associated with the atoms simulates the actual energy (and force) and as a result provides an exact match for

the Callendar-van Dusen coefficients. To further validate this theoretical model, analysis was performed on the resistivity-temperature equation (that resulted from the theoretical model) which showed that the theoretical equation can be reduced to a well recognized or accepted equation that can then be used to determine the mean free path of conduction electrons. The mean free path values provide a good match to reported values. Similarly, the theoretical equation can be reduced to an equation that can be examined to provide the number of lattice defects in a material. Finally, this theoretical model has also been used to determine the electric charge associated with metallic bonds. This charge is coupled with the atoms when they oscillate or vibrate and serves to move the atoms back to their equilibrium positions in the lattice. As a result of this examination of the theoretical model, it is evident that the resulting equation is valid and provides significant insight into the mechanisms behind the temperature dependence of electrical resistivity for conductors.

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