On the Computation of a Common n-finger Robotic Grasp for a Set of Objects

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Abstract—Industrial robotic arms utilize multiple end-effectors, each for a specific part and for a specific task. We propose a novel algorithm which will define a single end-effector's configuration able to grasp a given set of objects with different geometries. The algorithm will have great benefit in production lines allowing a single robot to grasp various parts. Hence, reducing the number of end-effectors needed. Moreover, the algorithm will reduce end-effector design and manufacturing time and final product cost. The algorithm searches for a common grasp over the set of objects. The search algorithm maps all possible grasps for each object which satisfy a quality criterion and takes into account possible external wrenches (forces and torques) applied to the object. The mapped grasps are represented by high-dimensional feature vectors which describes the shape of the gripper. We generate a database of all possible grasps for each object in the feature space. Then we use a search and classification algorithm for intersecting all possible grasps over all parts and finding a single common grasp suitable for all objects. We present simulations of planar and spatial objects to validate the feasibility of the approach.

Keywords—Common Grasping, Search Algorithm, Robotic End-Effector.

I. INTRODUCTION

ROBOTIC arms equipped with end-effectors are widely used in industrial production lines for carrying out numerous tasks such as assembly and material handling. However, for each part and for each task an end-effector is specially designed and built. This makes them very inflexible and increase the production time and cost. Current automatic solutions to flexibility involve adding either active elements to the end-effectors in order to adjust to different part geometries or by simply adding an end-effector changer adjacent to the robot. Exchanging end-effectors requires more factory space and consumes time for connecting and calibrations and thus costly. The design, manufacture and testing phase of a typical end-effector consumes a considerable amount of engineering time and adds extra cost to the final product.

The aim of this work is the development of an algorithm which will find a configuration of a simple end-effector for grasping a given set of objects. Such algorithm could be used to reduce the number of end-effectors needed by using the same production line for multiple products and therefore reduce production time and costs. Given a set of CAD models of the objects, the goal is to design an end-effector that would be able to hold a wide set of components for multiple tasks. We propose a novel solution for designing a simple end-effector able to do so. The algorithm proposed computes the grasp configuration suitable for a set of objects. The grasp configuration defines the location of the contacts and the directions for applying forces. Hence, it implies for the design of the desired end-effector. In order to reduce costs, the final industrial end effector has to be simple and with minimal degrees of freedom. Moreover, the grasp has to be feasible, meaning, able to stably grasp the object even under the application of external forces or torques due to the task being done. That is, we require a force-closure grasp with an high-quality grasp measure.

This work uses the methods of force-closure and quality measure as criteria for determining and quantifying feasible grasps. Using the notion of wrenches (combining forces and torques), in [1]–[6] force-closure criterion is well defined and several algorithms for synthesis of a frictional and frictionless grasps were presented. Several grasp optimization methods using different grasp quality measures have been presented in the literature; Ferrari and Canny [7] and Li and Sastry [8] introduced a quality measure based on the external wrench to be resisted, where the first introduced a general measure based on the largest wrench that the grasp can resist; the second uses task oriented quality defined by the specific wrenches applied during execution. The first method is the common one and is embedded in this work. This paper is an extension of our previous work presented in [9].

To the best of our knowledge, no previous work has been done for searching common grasps or end-effectors design for a set of objects. However, the work of Rodriguez and Mason [10] has similarities to ours. In their work, a finger design that can hold an object invariant of its scale or pose is searched. Yet, in our work we focus on grasping objects of various geometries but of the same scale. Much work has been done in the area of 3D shape similarity comparison algorithms, such as [11] and [12]. Algorithms which are used for Internet and local storage search, face recognition, image processing or parts grasping in assembly lines. However, such methods deals with parameterization of the geometry of the objects and cannot be applied for grasping. The work of Li and Pollard [13] is based on shape matching for finding the best grasp for a set of objects. The best grasp is found by matching hand poses from a database to each object. This is done by using a predefined parameterization of the object surface and the hand poses, a method which inspired this work.

This paper presents an algorithm for parameterization of the force-closure grasps for each object and using it for classification of the objects with respect to these grasps. In the first stage of the algorithm a *Force Closure Grasp Set* (FCGS) is constructed for every object by sampling all possible grasps (up to mesh size), filtering out those with low grasp quality measure, and representing the possible grasps

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as feature vectors in the feature space. Each feature vector is constructed in a unique form to injectively define the grasp invariant to any reference frame. The feature vector implies for the end-effectors' design based on the common grasp. The next phase of the algorithm is the similarity join, for finding pairs of common feature vectors in the FCGS of all objects, followed by classification to find the common grasps. It is important to note that in this work we assume that the kinematics of the robotic arm could achieve all possible grasps of the objects. In the future, kinematic achievability criterion could be added to the algorithm. Moreover, we assume the grasped objects have uniform density.

The paper is organized as follows. The next section (II) gives an overview of grasping fundamentals used in this work. The basics of the algorithm are described in section III. Section IV presents simulations conducted on planar and spatial objects to validate the feasibility of the proposed algorithm. Finally, the last section contains a summary of the work and proposes future work.

II. OVERVIEW OF GRASPING CONCEPTS

In this section we shortly discuss grasping fundamentals which are used in this work. We present the grasp model used and discuss the notions of force-closure and quality measure.

A. Grasping Model

Forces and torques can be represented as a wrench vector in the wrench space. A wrench is a m-dimensional vector where m equals 3 for the planar case and 6 for a spatial object case. It is denoted as $w = \begin{pmatrix} \mathbf{f} & \tau \end{pmatrix}^T$ where $\mathbf{f}$ is a force vector and τ is a torque vector. Furthermore, a wrench applied at the contact point $\mathbf{p}_i$, can be described as $\mathbf{w}_i = \begin{pmatrix} \mathbf{f} & \mathbf{p}_i \times \mathbf{f}_i \end{pmatrix}^T$ where $\mathbf{p}_i$ is represented in the object coordinate frame. Friction exists at the contacts between the fingertips of the end-effector and the object's surface. Friction can be represented by the simple Coulomb friction model. In this model, forces exerted at the contact point must lie within a cone centered about the surface normal. The angle of the cone is derived from the coefficient of friction. This is known as the *Friction Cone* (FC). In the planar case, the FC can be defined by a linear combination of $\mathbf{f}_i^+$ and $\mathbf{f}_i^-$, which are unit vectors on the edges of the FC. The angle between them equals to $2\tan^{-1}\mu$, where μ is the coefficient of friction. If the contact force lies within the FC, the force can be represented as a linear combination given by

$$\mathbf{f_i} = \alpha_i^+ \mathbf{f_i^+} + \alpha_i^- \mathbf{f_i^-} \tag{1}$$

where α_i^+, α_i^- are nonnegative constants [5]. The associated wrenches can be expressed by the primitive forces as

$$\mathbf{w_i^+} = \begin{pmatrix} \mathbf{f_i^+} \\ \mathbf{p_i} \times \mathbf{f_i^+} \end{pmatrix}, \ \mathbf{w_i^-} = \begin{pmatrix} \mathbf{f_i^-} \\ \mathbf{p_i} \times \mathbf{f_i^-} \end{pmatrix}. \tag{2}$$

An n-finger grasp can be represented by the primitive wrenches $\mathcal{W} = (\mathbf{w_1^+}, \mathbf{w_1^-} ..., \mathbf{w_n^+}, \mathbf{w_n^-})$ applied at the contacts.

In the spatial case, the FC is non-linear and therefore can be approximated with an s-sided convex polytope and every force exerted within the FC can be represented by a linear combination of the unit vectors $\hat{\mathbf{f}}_{\mathbf{ik}} \in FC$ (primitive forces) constructing the linearized friction cone,

$$\mathbf{f_i} = \sum_{k=1}^{s} a_{ik} \hat{\mathbf{f}}_{\mathbf{ik}} \tag{3}$$

where a_{ik} are nonnegative coefficients. The ˆ sign denotes a unit vector. The associated wrenches can be expressed by the primitive forces as

$$\mathbf{w_i} = \sum_{k=1}^{s} a_{ik} \hat{\mathbf{w}}_{\mathbf{ik}} = \sum_{k=1}^{s} a_{ik} \begin{pmatrix} \hat{\mathbf{f}}_{\mathbf{ik}} \\ \mathbf{p_i} \times \hat{\mathbf{f}}_{\mathbf{ik}} \end{pmatrix} \tag{4}$$

where $\hat{\mathbf{w}}_{\mathbf{ik}}$ are the primitive wrenches associated with the primitive forces. Equivalent to the planar case, the wrench set formed by the frictional forces at the contact points can be expressed by the primitive wrenches $\mathcal{W} = (\mathbf{w_{11}}, \mathbf{w_{12}}, ...\mathbf{w_{1s}}, ..., \mathbf{w_{n1}}, \mathbf{w_{n2}}, ..., \mathbf{w_{ns}})$.

Based on the model of the grasp, we can now define feasibility of a grasp. Therefore, in the next subsection we present the notion of force closure which defines whether the grasp is feasible or not.

B. Force Closure

A grasp is said to be force-closure if it is possible to apply wrenches at the contacts such that any external forces and torques acting on the object can be counter-balanced by the contact forces. A system of wrenches can achieve force-closure when they positively span the entire wrench space. Hence, any external load can be balanced by a non-negative combination of the wrenches [1].

A necessary and sufficient condition for a system of wrenches $\mathcal{W}$ to be force-closure is that the origin of $\mathbb{R}^m$ lies in the interior of the convex-hull (CH) of the contact primitive wrenches [3], [14] (taken with reference to the center of gravity of the object). That is,

$$O \in \ interior(CH(\mathcal{W})) \tag{5}$$

C. Grasp Quality Measure

As was mentioned above, a grasp which is force closure can resist external loads, we now need to quantify the quality of the grasp. That is, how much external load it can resist or in other words, how much resources (in term of contact force) it needs to apply in order to resist the external load. The quality measure quantifies how much a grasp can resist an external wrench without the fingers loosing contact or starting to slip [15]. A higher quality measure reduces object deformations due to contact force and actuator resources.

The most common quality measure is the largest ball criterion which will be used in this work. This measure is used when there is no prior knowledge of the task forces. In this method, the grasp quality is equivalent to the radius of the largest ball centered at the origin and fully contained in the $CH(\mathcal{W})$ [2]. In other words, the grasp quality measure is defined as the distance from the origin to the closest facet of the $CH(\mathcal{W})$. Formally, we can say that the quality measure Q is defined as

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$$Q = \min_{w \in \partial CH(\mathcal{W})} \|w\| \qquad (6)$$

where $\partial CH(\mathcal{W})$ is the boundary of $CH(\mathcal{W})$ [7]. The quality measure in this method denotes the weakest net wrench that can be applied to counter-balance an external wrench in its direction.

III. OCOG Algorithm

Given q objects to be grasped with an n-finger frictional grasp. The OCOG (Object COmmon Grasp) search algorithm will output a feasible common grasp with the highest quality measure for the set of objects. The algorithm is presented in this section.

The algorithm for finding a common grasp of a set of objects is given in Algorithm 1. It receives as input a set of q CAD models of the query objects. The first step of the algorithm is the discretization of each CAD model to a mesh of k points. Each point in the mesh is characterized with its position vector $\mathbf{p_i}$ and the normal $\mathbf{n_i}$ (unit vector) to the surface at the point. Thus, the mesh of each object is defined with a set of points on the surface $\tilde{\mathcal{P}} = (\mathbf{p_1}, ..., \mathbf{p_k})$ and a set of normals at the points $\tilde{\mathcal{N}} = (\hat{\mathbf{n}}_1, ..., \hat{\mathbf{n}}_k)$. The next step of the algorithm is the marking of forbidden grasp regions on the surface of the objects. This is done by the user via graphical user interface according to operational demands due to the designated tasks. The next step of the algorithm is the generation of the *Force-*

Algorithm 1 Common grasp search

Input: CAD's of objects $B_1, ..., B_q$.

Output: A common grasp for all objects or common grasps for subsets of the objects.

1: **for** $\xi = 1 \rightarrow q$ **do**
2:    Mesh object B_ξ to form $\{\tilde{\mathcal{P}}_\xi, \tilde{\mathcal{N}}_\xi\}$.
3:    Manually label forbidden grasp regions on mesh of object B_ξ.
4:    Generate all possible grasps $\{\mathcal{P}_1, \mathcal{N}_1\}, ..., \{\mathcal{P}_\lambda, \mathcal{N}_\lambda\}$.
5:    **for** $j = 1 \rightarrow \lambda$ **do**
6:       **if** grasp $\{\mathcal{P}_j, \mathcal{N}_j\}$ if feasible **then**
7:          Map grasp $\{\mathcal{P}_j, \mathcal{N}_j\}$ to feature vector $\mathbf{e_j}$.
8:          Add $\mathbf{e_j}$ to set $\mathcal{E}_\xi$.
9:          Store pointer between $\mathbf{e_j}$ and $\{\mathcal{P}_j, \mathcal{N}_j\}$.
10:       **end if**
11:    **end for**
12: **end for**
13: $\mathcal{Z} = JoinFCGS(\mathcal{E}_1, ..., \mathcal{E}_q)$
14: $\mathcal{H} = Classification(\mathcal{Z})$
15: **if** $\mathcal{H} \neq \emptyset$ **then**
16:    **return** $\mathcal{H} = (\mathbf{u_1}, ..., \mathbf{u_\sigma})$
17: **else**
18:    Report: No common grasps.
19: **end if**

Closure Grasp set (FCGS) for each object. The FCGS is a set in a high-dimensional space mapping each feasible n-fingers grasp to a set of parameters termed *Feature Vector*. The FCGS

contains the parameterization of all possible and feasible n-fingers grasps up to mesh size. For example, a 3-finger grasp can be represented by a triangle formed by the contact points. This is illustrated in Figure 1 where all possible triangles are mapped for an ellipse object with mesh of 11 points. With more than 3-fingers, the contact points will form a polygon in the planar case and a polytope in the spatial case.
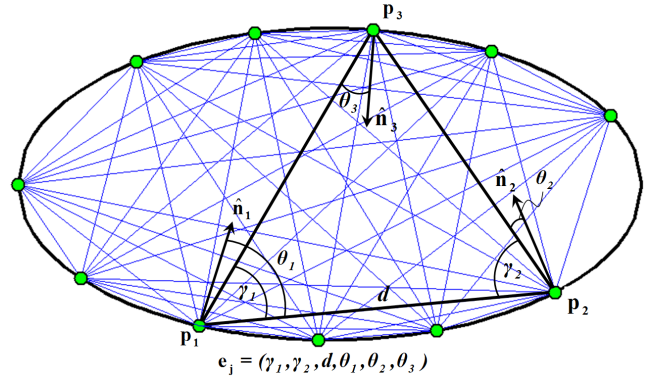

Fig. 1. All possible 3-finger grasps of an ellipse objects meshed to $k = 11$ points.

Of all possible grasps, we pick only the ones which are feasible. We define feasible grasps to be those which are force-closure and has a quality measure greater than a predefined lower bound Q_d. That is, a grasp that is defined by n contact points $\mathcal{P}_j = (\mathbf{p_1}, ..., \mathbf{p_n})$ and their normals $\mathcal{N}_j = (\hat{\mathbf{n}}_1, ..., \hat{\mathbf{n}}_n)$ is considered feasible if the condition

$$O \in interior(CH(\{\mathcal{P}_j, \mathcal{N}_j\})) \;\; and \;\; Q(\{\mathcal{P}_j, \mathcal{N}_j\}) \geq Q_d \quad (7)$$

is satisfied, where $CH(\{\mathcal{P}_j, \mathcal{N}_j\})$ is the convex-hull of the wrenches formed by the grasp contact points $\mathcal{P}_j$ and their normals $\mathcal{N}_j$. Next, the grasps which are feasible are parameterized to a feature vector in a feature space. That is, we define transformation map T to map grasp j represented with $\mathcal{P}_j$ and $\mathcal{N}_j$ into a d-dimensional feature vector $\mathbf{e_j}$:

$$T : \{\mathcal{P}_j, \mathcal{N}_j\} \rightarrow \mathbf{e_j} \in \mathbb{R}^d \quad . \qquad (8)$$

Transformation T forms a feature vector which injectively represents the grasp configuration invariant of any coordinate frame. The feature vector of a grasp is a set of parameters which constrain the size and shape of a polygon in the planar case and a polytope in the spatial case. Basically, these parameters are a set of angles and lengths that defines the polygon or polytope. Moreover, parameters in the feature vector constrain the normals directions at the contact points relative to the polygon or polytope itself. Example of such representation is illustrated in Figure 1 where a 3-finger grasp is represented by a 6-dimensional feature vector. The first three parameters define the triangle formed by the contact points and three angle parameters define the directions of the normals relative to the triangle. For consistent parameterization of the polygon we always choose the longest edge as the first parameter. For a polytope case we choose its largest area facet. The dimensionality of a feature vector is determined according to the number of contact points n which defines the number of vertices in the polygon or polytope.

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The feature vectors of object B_i which are considered to be feasible are added to the FCGS of the compatible object, denoted as $\mathcal{E}_i \in \mathbb{R}^d$. Once all feasible grasps of all objects are mapped to the FCGS sets $\mathcal{E}_1, ..., \mathcal{E}_q$, we would like to intersect the FCGS's to find common feature vectors which imply for common grasps. Therefore, we define function *joinFCGS* which is a similarity join algorithm to find common points over the sets. Hence, nearest-neighbor search is utilized to find pairs of common vectors among the sets. The nearest-neighbor search is done by constructing a kd-tree (see [16]) database representation from the cluster of vectors, enabling efficient search. Pairs of common vectors found are checked to satisfy tolerance demands derived from the friction cones angle, accuracy demands and hardware capabilities. Basically, two feature vectors over two sets are considered to be the same if they are both inside an hyper-rectangle with predefined edge lengths.

Two vectors which are considered to be the same are further added to a registry set $\mathcal{Z} \in \mathbb{R}^d$ of common vectors. The set $\mathcal{Z}$ is a d-dimensional database of vectors taken from $\mathcal{E}_1, ..., \mathcal{E}_q$. The vectors inserted to $\mathcal{Z}$ are the ones which exists in two or more sets of $\mathcal{E}_1, ..., \mathcal{E}_q$, i.e., those which are common to two or more sets. For each feature vector added to $\mathcal{Z}$, it is marked in which FCGS sets it exists in.

The final step of the algorithm is the classification of the vectors in $\mathcal{Z}$. After a set of vectors common to two or more of the sets $\mathcal{E}_1, ..., \mathcal{E}_q$ were acquired, classification is needed to find the minimal number of grasp configurations which can grasp subsets of the objects. We search for a minimum set $\mathcal{H} \subseteq \mathcal{Z}$ of vectors from the registry set which covers all of the FCGS sets. Basically, we search for a single feature vector from $\mathcal{Z}$ which exists in all of the sets $\mathcal{E}_1, ..., \mathcal{E}_q$. Such a vector represent a grasp which can grasp all of the objects. If a single vector is not to be found, we seek for a minimal number of grasps which can grasp subsets of the objects. That is, we divide the set of objects to subsets, where for each subset there is a compatible common grasp.

We scan all possible n-finger grasp combinations (up to mesh size). Therefore, the algorithm will certainly find a common grasp or a set of common grasps if such exist. Thus, if a single grasp for all objects or a set of grasps for subsets of the objects exist, the algorithm will find them. If failed to do so, the algorithm reports that no common grasps exist.

IV. SIMULATIONS

The following simulations of the proposed algorithm, were implemented in MATLAB[1] on an Intel-Core i7-2620M 2.7GHz laptop computer with 8GB of RAM. The operation of the algorithm was done using MATLAB parallel computing toolbox in order to reduce runtime. The following simulations present an example of the algorithm for grasps of planar and spatial objects. The parameters chosen for the simulations were chosen according to the conditions of future planned experiments.

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[1]Matlab® is a registered trademark of The Mathworks, Inc.

A. Simulations for Planar Objects

The performance of the proposed algorithm is illustrated using the four ($q = 4$) 2D shapes shown in Figure 2. The shapes are described with a mesh of $k = 210$ points uniformly distributed along their boundary. For such mesh there are $\frac{210!}{3!(210-3)!} = 1,521,520$ candidate grasps. We filter-out force-closure grasps which have quality measure smaller than $Q_d = 0.1$. The tolerances for the similarity join were chosen
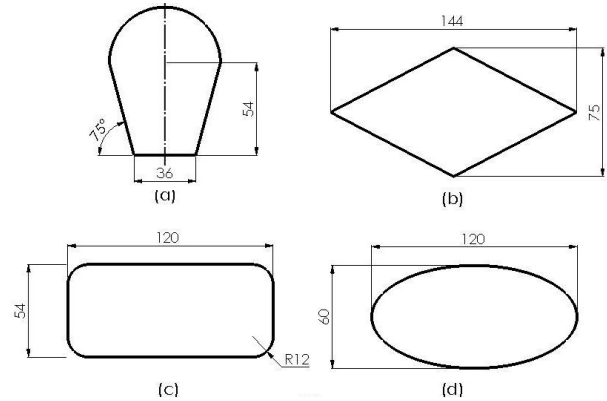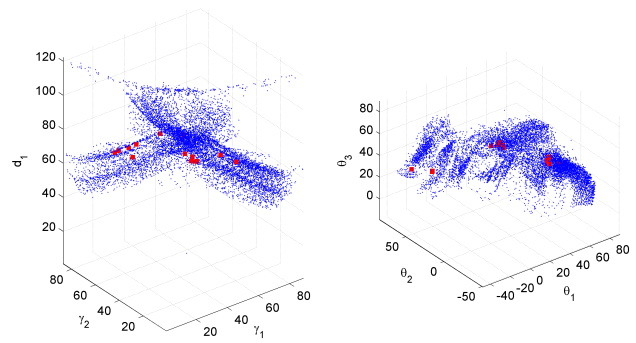

Fig. 2.   Four planar objects to be grasped.

Fig. 3.   Projections of the registry set $\mathcal{Z}$ for the four planar objects.

that the distance between the contact points will not extend or shorten by more than 5% of their original length. These tolerances are continuously computed during the simulation execution. Moreover, tolerances were defined to ensure that the normals at the contact points will be inside a friction cone where the friction coefficient is $\mu = 0.7$. Under these conditions, within 42 minutes of runtime, the algorithm's outputs were 15 solutions of common grasps for all shapes. Figure 3 shows 10,411 points/grasps which are common to two or more shapes (the blue dots). Due to the high-dimensionality of the space, for illustration, it is shown in two 3-dimensional spaces. The 15 solutions which are marked with red squares are common grasps for all shapes.

The highest quality common grasp solution ($Q = 0.53$) is shown in Figure 4. Moreover, simulations were performed to 4-fingers grasps as well and such solution is shown in Figure 5 with mesh size of k=150. For both simulations, slight differences can be seen between the grasps as the tolerances allowed so. However, the mean grasp of the four will be the

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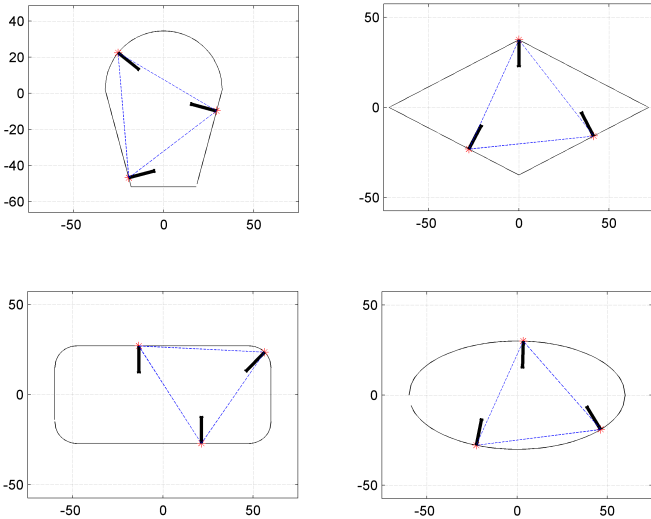International Journal of Mechanical and Mechatronics Engineering
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Fig. 4. One common 2D 3-fingers grasp solution ($k = 210$).

common grasp. With higher mesh resolution, more accurate solutions could be achieved; this of course is based on the grasp accuracy demands.
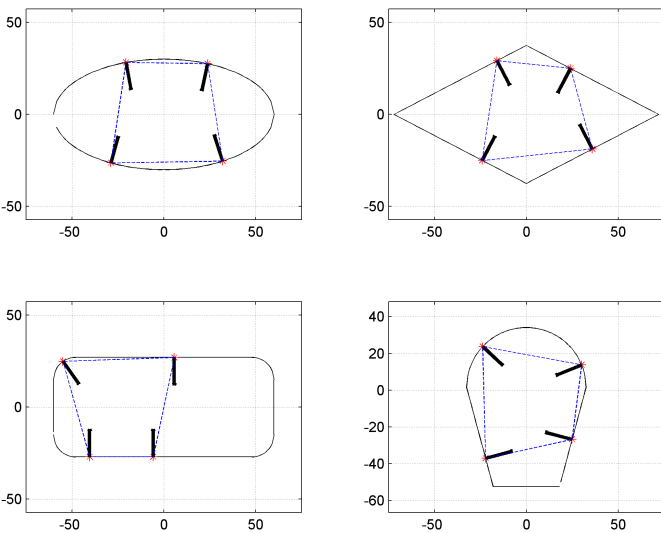

Fig. 5. Solution of a 4-finger common grasp ($k = 150$).

B. Simulations for Spatial Objects

Four objects ($q = 4$) were tested for the implementation of the algorithm and are shown in Figure 6. The objects were discretized using COMSOL Multiphysics[2] to a triangular mesh with approximated size of $k = 250$. For such mesh there are $\frac{250!}{3!(250-3)!} = 2,573,000$ candidate grasps. We implement the algorithm with a 3-finger frictional grasp, where the friction cones at each contact point are modeled as 5-sided convex cones ($s = 5$). The friction coefficient was chosen to be $\mu = 0.6$. We filter-out force-closure grasps which have quality measure smaller than $Q_d = 0.1$. The runtime of this

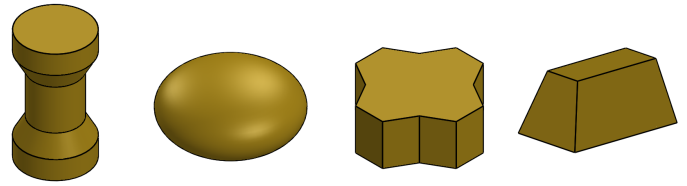[2]COMSOL Multiphysics is a registered trademark of COMSOL AB.

Fig. 6. Four objects to be grasped.

simulation was approximately 32 hours. Figure 7 shows one generated FCGS for the cylindrical object with 197,357 feature vectors (feasible grasps). In this case, the feature vector, which represent the polytope of the grasp, is in a 9-dimensional space and therefore, for illustration, the set is shown in three 3-dimensional projections.
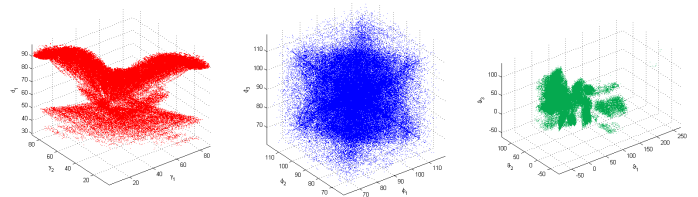

Fig. 7. Projections of the FCGS for the cylindrical object.

Classification of the registry set $\mathcal{Z}$ provides 40 grasps which are common to all objects. The output of the algorithm would be one grasp out of the 40 with the highest quality measure. Figure 8 presents a grasp with quality measure $Q = 0.47$. This grasp is a common one for all the four tested objects. Minor differences can be seen between the grasps as the tolerances mentioned allowed so. The differences can mostly be seen in the normals directions, however, due to the friction cone, such deviation between the normals is allowed. With demand of higher accuracy or low coefficient of friction, higher resolution computation is needed and should be done to achieve so. Figure 9 demonstrates the implementation of the algorithms for the 4-fingers grasp with mesh size of $k = 100$. However, it is hard to illustrate such grasp in a figure and deviations can be seen between objects as result of different orientation and allowed tolerances.

V. CONCLUSIONS

In this paper we introduced an algorithm for the search of a common grasp for a set of objects. Moreover, simulations of the proposed algorithm were presented. The algorithm is based on the map of all feasible grasps for each object and the parameterization of them to a feature vector in an high-dimensional space. Such representation of the grasps enables comparison between grasps and efficient similarity search based on nearest-neighbor search. Classification is done between feature vectors common to two or more sets to find the minimum set of grasps which is able to grasp the set of objects. Each common feature vector is the configuration of the end-effector able to grasp all objects. The search algorithm has an overall runtime of the order of $O(k^n)$. However, the objects FCGS's could be computed in parallel and therefore dramatically reduce runtime.

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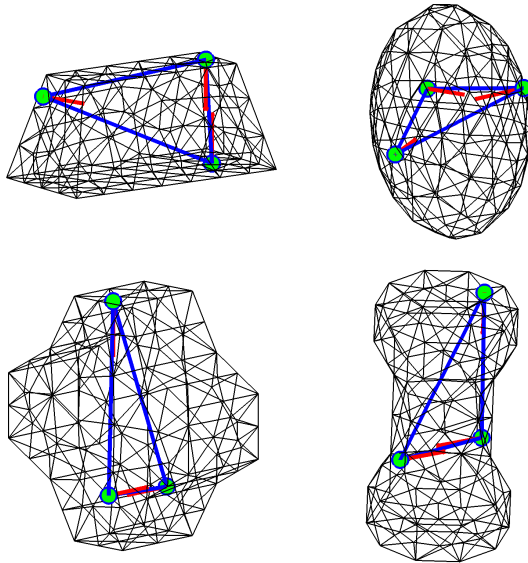Vol:7, No:11, 2013

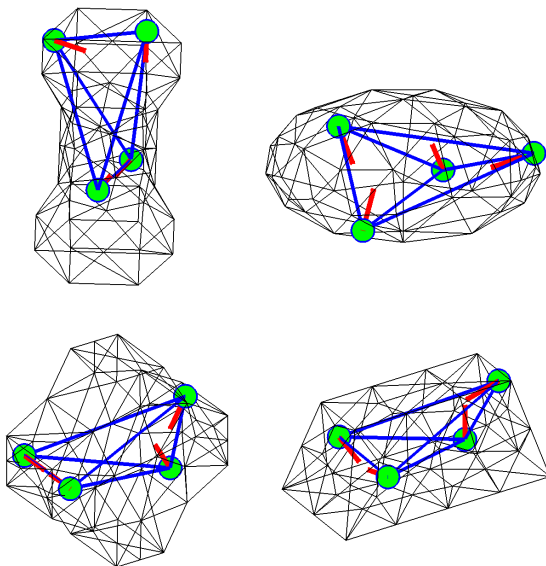Fig. 8.   A 3-fingers common grasp illustrated on the four objects (k=250).

Fig. 9.   A 4-fingers common grasp illustrated on the four objects (k=100).

Future work will be on adding conditions to filter out non-feasible grasps prior to the CH computation in order to reduce runtime. Work on adding minimal degrees of freedom to the end-effector will increase the number of solutions and deal with objects not of the same scale. Moreover, as we found a mean grasp which does not precisely overlap the original ones sampled on the objects, post-processing has to be done in order to correct positioning errors of the fingertips due to the tolerances and to accurately adjust the common grasp to the objects. This would be done by defining an optimization problem where its boundary constraints are contact regions which reserve force closure. Such feature will enable the run of the algorithm with low mesh resolution and the refinement will compensate on that.

REFERENCES

[1] J. Ponce and B. Faverjon, "On computing three-finger force-closure grasps of polygonal objects," *IEEE Transactions on Robotics and Automation*, vol. 11, no. 6, pp. 868 –881, dec 1995.

[2] M. Roa and R. Suarez, "Geometrical approach for grasp synthesis on discretized 3d objects," in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, Oct. 2007, pp. 3283–3288. [Online]. Available: http://dx.doi.org/10.1109/IROS.2007.4399440

[3] R. M. Murray, Z. Li, and S. S. Sastry, *A Mathematical Introduction to Robotic Manipulation*, 2nd ed. CRC Press, Oct. 2012. [Online]. Available: http://www.amazon.com/exec/obidos/redirect?tag=citeulike07-20&path=ASIN/0849379814

[4] N. Niparnan and A. Sudsang, "Computing all force-closure grasps of 2d objects from contact point set." in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2006, pp. 1599–1604.

[5] Y. H. Liu, "Computing n-Finger Form-Closure grasps on polygonal objects," *The International Journal of Robotics Research*, vol. 19, no. 2, pp. 149–158, Feb. 2000. [Online]. Available: http://dx.doi.org/10.1177/02783640022066798

[6] I.-M. Chen and J. Burdick, "A qualitative test for n-finger force-closure grasps on planar objects with applications to manipulation and finger gaits," in *Robotics and Automation, 1993. Proceedings., 1993 IEEE International Conference on*, 1993, pp. 814–820 vol.2.

[7] C. Ferrari and J. Canny, "Planning optimal grasps," in *Proceedings of the IEEE International Conference on Robotics and Automation*, May 1992, pp. 2290–2295. [Online]. Available: http://dx.doi.org/10.1109/ROBOT.1992.219918

[8] Z. Li and S. Sastry, "Task oriented optimal grasping by multifingered robot hands," in *Proceedings of the IEEE International Conference on Robotics and Automation*, vol. 4, Mar. 1987, pp. 389–394. [Online]. Available: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1087852

[9] A. Sintov, S. Raghothama, R. Menassa, and A. Shapiro, "A common 3-finger grasp search algorithm for a set of planar objects," in *Proceeding of the IEEE International Conference on Automation Science and Engineering (CASE)*, 2012, pp. 1091–1096.

[10] A. Rodriguez and M. T. Mason, "Grasp invariance," *Int. Journal of Robotic Research*, vol. 31, no. 2, pp. 236–248, 2012.

[11] R. Ohbuchi, T. Otagiri, M. Ibato, and T. Takei, "Shape-similarity search of three-dimensional models using parameterized statistics," in *Proceedings of the 10th Pacific Conference on Computer Graphics and Applications*, 2002, pp. 265–274. [Online]. Available: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1167870

[12] R. Osada, T. Funkhouser, B. Chazelle, and D. Dobkin, "Shape distributions," *ACM Transactions on Graphics*, vol. 21, no. 4, pp. 807–832, Oct. 2002. [Online]. Available: http://portal.acm.org/citation.cfm?id=571648

[13] Y. Li and N. S. Pollard, "A shape matching algorithm for synthesizing humanlike enveloping grasps," in *Proceedings of the 5th IEEE-RAS International Conference on Humanoid Robots*, 2005, pp. 442–449. [Online]. Available: http://dx.doi.org/10.1109/ICHR.2005.1573607

[14] B. Mishra, J. T. Schwartz, and M. Sharir, "On the existence and synthesis of multifinger positive grips," *Algorithmica*, vol. 2, pp. 541–558, 1987. [Online]. Available: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.38.7726

[15] C. Borst, M. Fischer, and G. Hirzinger, "A fast and robust grasp planner for arbitrary 3d objects," in *Proceedings of the IEEE International Conference on Robotics and Automation*, vol. 3, 1999, pp. 1890 –1896 vol.3.

[16] A. W. Moore, "Efficient memory-based learning for robot control," Ph.D. dissertation, Cambridge, UK, 1990. [Online]. Available: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.17.2654

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