World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:7, No:2, 2013

Performance Evaluation of Faculties of Islamic Azad University of Zahedan Branch Based-On Two-Component DEA

Ali Payan

Abstract—The aim of this paper is to evaluate the performance of the faculties of Islamic Azad University of Zahedan Branch based on two-component (teaching and research) decision making units (DMUs) in data envelopment analysis (DEA). Nowadays it is obvious that most of the systems as DMUs do not act as a simple input-output structure. Instead, if they have been studied more delicately, they include network structure. University is such a network in which different sections i.e. teaching, research, students and office work as a parallel structure. They consume some inputs of university commonly and some others individually. Then, they produce both dependent and independent outputs. These DMUs are called two-component DMUs with network structure. In this paper, performance of the faculties of Zahedan branch is calculated by using relative efficiency model and also, a formula to compute relative efficiencies teaching and research components based on DEA are offered.

Keywords—Data envelopment analysis, faculties of Islamic Azad University of Zahedan branch, two-component DMUs.

I. Introduction

PERFORMANCE evaluation is one of the most significant factors in decision making. To determine the most appropriate approach for performance evaluation is one of the most important duties of researchers and managers of organizations [1]. Also, as the lack of sources is a very significant concern of managers in today's competitive environment, so, performance evaluation and extracting the weak points is very considerable for systems.

Different researchers offer various approaches for performance evaluation. One of these approaches is to use statistical concepts. One more approach is to develop and shift economical concepts to mathematical models which are used mostly by the researchers in recent years. The very first approach was offered by Farrell [2] and continued by Charnes et al. [3]. This approach is called Data Envelopment Analysis (DEA). DEA is a technique for mathematical programming to compute efficiency and evaluate performance of decision making units (DMUs). DMUs are such systems which act similarly and use some inputs to produce some indicators which are called outputs. In DEA model, relative efficiency can be maximized by selecting appropriate weights for inputs and outputs. Based on this approach, DMUs is divided into two groups, efficient and inefficient. Efficient units obtain the same score in efficiency which equals to number *one*. Inefficient units obtain scores in efficiency *less than one*. This approach was offered by Charnes et al. [3] in a paper called CCR, for the first time. Banker et al. [4] expanded CCR model and named that BCC. Since then various approaches were suggested based on different perspectives for performance evaluation according to main DEA models. A review of all these models can be studied on Cook and Seiford's paper [5].

A. Payan is with the Department of Mathematics, Zahedan Branch, Islamic Azad University, Zahedan, Iran (e-mail: payan_iauz@yahoo.com, a.payan@iauzah.ac.ir).

In simple structures DMUs are independent and each unit produces s output by receiving m input. But it is possible for the structure of DMUs to be in a way that each unit includes two or more components. These components may receive some inputs of the whole system and they may produce some outputs of it; they may also have the same inputs or outputs. The issue of the DMUs with such a structure is called multi-component DEA. This can be widely applied on evaluation systems and different industries. For instance, bank industry, education centers etc. can be mentioned. One of the most obvious examples of multi-component DEA is the university. They include several components such as teaching, research, financial and students' fields.

Universities or their branches such as faculties or faculties can be evaluated and compared based on each of the components. By using multi-component DEA models besides determining the overall efficiency score of the DMUs, the weak points of each component can be extracted.

Cook et al. [6] offered a model to evaluate the efficiency of multi-component DMUs in a way that they get the same inputs. They applied this model on different branches of Canadian banks. Their components include sails and services in bank branches. Jahanshahloo et al. [7] offered a model by using DEA, which compute the efficiency of DMUs in a way that their components also receive and produce the common inputs and outputs. In another study, they [8] firstly, evaluated the efficiency of multi-component DMUs. Then, they computed efficiency by dividing banks' branches according to their organizational role.

One of problems of multi-component DEA models till that time was their inability in calculating the relative efficiency of multi-component DMUs, which is one of the very first duties of the DEA. This problem was solved by Noora et al. [9]. They modified the previous models to calculate the relative efficiency in multi-component DEA. According the above mentioned points, in this paper the faculties of Islamic Azad University of Zahedan branch are studied as two- component DMUs. Therefore, considering that all the active fields of a university work in the service of teaching and research fields, which are the outputs of a university; in this paper, these two fields are studied as the components. After determining the indicators of performance evaluation and a combination of them, which is mentioned in the second section of the present paper, two-component structures are offered for the faculties. In section three a model is presented to compute relative efficiency of the faculties. Then, according to the gained weights of the offered model, a formula is suggested to calculate the relative efficiency of the components. The next section belongs to calculating the relative efficiency of the faculties and their components. The results and conclusions are mentioned in the last section.

II. The Structure of the Faculties as DMUs

The indicators in performance evaluation is various among universities. A couple of these indicators are listed as: board of examiners, employers, students, budget, atmosphere of university, educated people, essays, books, research projects, and being accepted in the next degree.

As it was mentioned, the above indicators do not express all the performance evaluation factors, but it is tried to mention those factors that are usually used. Among ten mentioned indicators above, the first five ones are inputs and the last five ones are outputs. These indicators can be divided into two main groups of teaching and research, as well. Indicators like students, educated people, being accepted for the next degrees are in teaching group. Essays, books and research projects belong to output indicators of research field.

Other indicators are some factors that do not belong to one of the two groups of teaching and research because they influence both fields simultaneously. For instance, the board of examiners of a university plays roles in producing both teaching field which determines the number of educated people and those who are accepted in the above degree, and in producing research outputs, as well. According to the above mentioned issues, university structure as a DMU is more complicated than normal DMUs. In other words, it has got network structure. In network structure decision making components act parallel with each other and with some indicators; so, it offers the same inputs to some components. Some outputs are developed by the cooperation of some components. So, according to the concerns of this paper, faculties of Islamic Azad University of Zahedan branch are determined as two-component DMUs. A general picture of such units is in Fig. 1.

The above structure is used for performance evaluation of the faculties of Zahedan branch. According to the gathered data, inputs and outputs of teaching and research fields can be classified and computed to evaluate the performance of the faculties as mentioned in Table I.

Based on input-output indicators the structure of the faculties can be explained in Fig. 2.

The number of the students is a quantitative number which indicates the number of the students of each faculty to the end of the second semester of the years 90-91. Professors' equivalence is a number which is obtained based on the number of full time, part time and invited professors and it is computed according to the following formula:

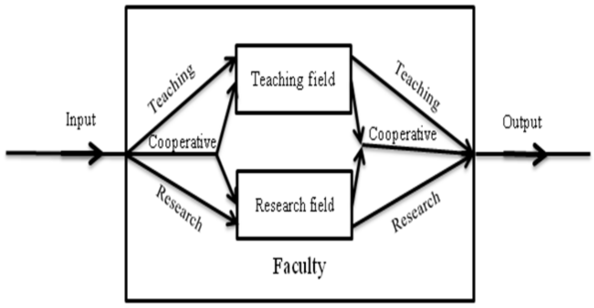

Fig. 1. The Presentation of the Faculties as Two-component DMUs

TABLE I
Input-outputs of Faculties

Teaching factors		Research factors	
Inputs	Outputs	Inputs	Outputs
1. Students 2. Professors 3. Facilities	1. Educated students	1. Facilities 2. Professor	1. Research grade

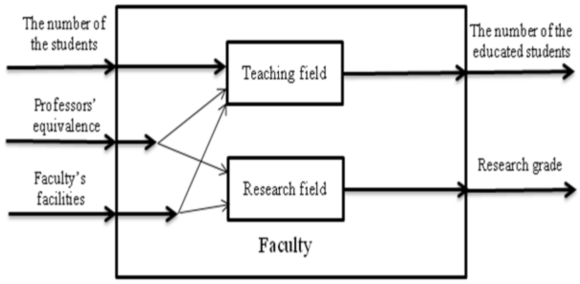

Fig. 2. The faculties of Zahedan branch as two-component DMUs

Professors equivalence $\equiv$ the number of full time professors $\times$5+the number of part time professors$\times$2+the number of invited professors$\times$1.

Teaching facility is an indicator which is obtained based on three criteria, i.e. the number of employers, educational departments and also the classes of the faculty. It is calculated according to the following formula:

Teaching facilities $\equiv$ the number of employers$\times$5+the number of educational departments$\times$2+the number of the classes$\times$2.

The number of educated students includes a quantitative number which shows their number to the end of the first of the years 90-91.

Research grade is an indicator which explains research activities of each faculty, and it is computed with an appropriate rate based on the number of published papers in journals, books and research projects.

Research grade $\equiv$ the number of research projects$\times$5+the number of papers$\times$10+the number of books$\times$15.

III. Evaluation of the Faculties by DEA

In the mentioned issues in the previous sections, suppose x^1 shows the input of the first component, x^{s1} and x^{s2} were the same inputs of both components. Also, consider that y^1 and y^2 are sequentially the outputs of the first and the second component. In this way, the efficiency of the jth faculty can be explained as follows:

$$e_j^a = \frac{\mu^1 y_j^1 + \mu^2 y_j^2}{v^1 x_j^1 + \sum_{i=1}^{2} v_1^{si} \alpha^i x_j^{si} + \sum_{i=1}^{2} v_2^{si}(1-\alpha^i) x_j^{si}} \tag{1}$$

In which μ^1 and μ^2 are sequentially the weights of the outputs first and second components. v^1 is the weight of the independent input of the first component, v_1^{s1} and v_1^{s2} are the weights of the two common inputs for the first component and v_2^{s1} and v_2^{s2} are the weights of the two common inputs for the second component. Here, α^1 indicates a portion of the first common input which is used in the first component. We have $0 \le \alpha^1 \le 1$. So, $1-\alpha^1$ is the remaining portion of the first common input that is used in second component. α^2 also shows a portion of the second common input which is used in the first component. So then, $0 \le \alpha^2 \le 1$. Therefore, $1-\alpha^2$ is the remaining portion of the second common input which is used in the second component. Based on these issues the efficiency of the first and the second components of the jth faculty which is shown as e_j^1 and e_j^2 is offered as follows:

$$e_j^1 = \frac{\mu^1 y_j^1}{v^1 x_j^1 + \sum_{i=1}^{2} v_1^{si} \alpha^i x_j^{si}} \tag{2}$$

$$e_j^2 = \frac{\mu^2 y_j^2}{\sum_{i=1}^{2} v_2^{si}(1-\alpha^i) x_j^{si}} \tag{3}$$

As formula (1) calculates the absolute efficiency and in contrast, the very first duty of DEA models is to compute relative efficiency, so, we should use the following formula:

$$\frac{e_j^a}{\max\limits_{k=1,\ldots,K}\{e_k^a\}} \tag{4}$$

Also, considering that in DEA models [10] maximum relative efficiency is obtained, therefore, the following model is suggested to compute relative efficiency the faculties.

$$\begin{aligned}
\max \quad & \frac{e_j^a}{\max\limits_{k=1,\ldots,K}\{e_k^a\}}, \\
s.t \quad & e_k^1 \le 1, \quad k=1,..,K, \\
& e_k^2 \le 1, \quad k=1,..,K, \\
& \mu^1, \mu^2, v_1^{s1}, v_1^{s2}, v_2^{s1}, v_2^{s2}, v^1 \ge 0
\end{aligned} \tag{5}$$

As it is important to compute component efficiency of each unit based on overall efficiency and it expresses the dependency of overall efficiency and components' efficiency, therefore, constraints $e_k^1 \le 1$ and $e_k^2 \le 1$ $(k=1,..,K)$ are added to the model. The right number in this group guarantees that components' efficiency is not more than one. The above model can be rewritten as follows:

$$\max \frac{\dfrac{\mu^1 y_j^1 + \mu^2 y_j^2}{v^1 x_j^1 + \sum_{i=1}^{2} v_1^{si} \alpha^i x_j^{si} + \sum_{i=1}^{2} v_2^{si}(1-\alpha^i) x_j^{si}}}{\max\limits_{k=1,\ldots,K}\left\{\dfrac{\mu^1 y_k^1 + \mu^2 y_k^2}{v^1 x_k^1 + \sum_{i=1}^{2} v_1^{si} \alpha^i x_k^{si} + \sum_{i=1}^{2} v_2^{si}(1-\alpha^i) x_k^{si}}\right\}},$$

$$\begin{aligned}
s.t \quad & \frac{\mu^1 y_k^1}{v^1 x_k^1 + \sum_{i=1}^{2} v_1^{si} \alpha^i x_k^{si}} \le 1, \quad k=1,..,K, \\
& \frac{\mu^2 y_k^2}{\sum_{i=1}^{2} v_2^{si}(1-\alpha^i) x_k^{si}} \le 1, \quad k=1,..,K, \\
& \mu^1, \mu^2, v_1^{s1}, v_1^{s2}, v_2^{s1}, v_2^{s2}, v^1 \ge 0
\end{aligned} \tag{6}$$

The above model by variable transformation [11] as:

$$t = \max_{k=1,\ldots,K}\left\{\frac{\mu^1 y_k^1 + \mu^2 y_k^2}{v^1 x_k^1 + \sum_{i=1}^{2} v_1^{si} \alpha^i x_k^{si} + \sum_{i=1}^{2} v_2^{si}(1-\alpha^i) x_k^{si}}\right\}$$

will become the following fractional model:

$$\begin{aligned}
\max \quad & \frac{\mu^1 y_j^1 + \mu^2 y_j^2}{\bar{v}^1 x_j^1 + \sum_{i=1}^{2} \bar{v}_1^{si} \alpha^i x_j^{si} + \sum_{i=1}^{2} \bar{v}_2^{si}(1-\alpha^i) x_j^{si}} \\
s.t \quad & \frac{\mu^1 y_k^1 + \mu^2 y_k^2}{\bar{v}^1 x_k^1 + \sum_{i=1}^{2} \bar{v}_1^{si} \alpha^i x_k^{si} + \sum_{i=1}^{2} \bar{v}_2^{si}(1-\alpha^i) x_k^{si}} \le 1, \\
& k=1,\ldots,K, \\
& \frac{\bar{\mu}^1 y_k^1}{\bar{v}^1 x_k^1 + \sum_{i=1}^{2} \bar{v}_1^{si} \alpha^i x_k^{si}} \le 1, \quad k=1,..,K, \\
& \frac{\bar{\mu}^2 y_k^2}{\sum_{i=1}^{2} \bar{v}_2^{si}(1-\alpha^i) x_k^{si}} \le 1, \quad k=1,..,K, \\
& \mu^1, \mu^2, \bar{\mu}^1, \bar{\mu}^2, \bar{v}_1^{s1}, \bar{v}_1^{s2}, \bar{v}_2^{s1}, \bar{v}_2^{s2}, \bar{v}^1 \ge 0
\end{aligned} \tag{7}$$

In which

$$\bar{\mu}^1 = \mu^1 t, \bar{\mu}^2 = \mu^2 t, \bar{v}_1^{s1} = v_1^{s1} t, \bar{v}_2^{s1} = v_2^{s1} t,$$
$$\bar{v}_1^{s2} = v_1^{s2} t, \bar{v}_2^{s2} = v_2^{s2} t, \bar{v}^1 = v^1 t$$

Furthermore, model (7) by variable transformation [11] is transformed to the following non-linear model:

$$\begin{aligned}
\max \quad & \mu^1 y_j^1 + \mu^2 y_j^2 \\
s.t \quad & \bar{v}^1 x_j^1 + \sum_{i=1}^{2} \bar{v}_1^{si} \alpha^i x_j^{si} + \sum_{i=1}^{2} \bar{v}_2^{si}(1-\alpha^i) x_j^{si} = 1, \\
& \mu^1 y_k^1 + \mu^2 y_k^2 - \bar{v}^1 x_k^1 - \sum_{i=1}^{2} \bar{v}_1^{si} \alpha^i x_k^{si} \\
& - \sum_{i=1}^{2} \bar{v}_2^{si}(1-\alpha^i) x_k^{si} \le 0, \ k = 1, \ldots, K, \\
& \bar{\mu}^1 y_k^1 - \bar{v}^1 x_k^1 - \sum_{i=1}^{2} \bar{v}_1^{si} \alpha^i x_k^{si} \le 0, \ k = 1, .., K, \\
& \bar{\mu}^2 y_k^2 - \sum_{i=1}^{2} \bar{v}_2^{si}(1-\alpha^i) x_k^{si} \le 0, \quad k = 1, .., K, \\
& \mu^1, \mu^2, \bar{\mu}^1, \bar{\mu}^2, \bar{v}_1^{s1}, \bar{v}_1^{s2}, \bar{v}_2^{s1}, \bar{v}_2^{s2}, \bar{v}^1 \ge 0 \qquad (8)
\end{aligned}$$

The above model by changing variables $\bar{v}_1^{s1} = \bar{v}_1^{s1}\alpha^1, \bar{v}_1^{s2} = \bar{v}_1^{s2}\alpha^2, \bar{v}_2^{s1} = \bar{v}_2^{s1}(1-\alpha^1), \bar{v}_2^{s2} = \bar{v}_2^{s2}(1-\alpha^2)$ can be converted to an equivalent linear model:

$$\begin{aligned}
\max \quad & \mu^1 y_j^1 + \mu^2 y_j^2 \\
s.t \quad & \bar{v}^1 x_j^1 + \sum_{i=1}^{2} \bar{v}_1^{si} x_j^{si} + \sum_{i=1}^{2} \bar{v}_2^{si} x_j^{si} = 1, \\
& \mu^1 y_k^1 + \mu^2 y_k^2 - \bar{v}^1 x_k^1 - \sum_{i=1}^{2} \bar{v}_1^{si} x_k^{si} \\
& - \sum_{i=1}^{2} \bar{v}_2^{si} x_k^{si} \le 0, \ k = 1, \ldots, K, \\
& \bar{\mu}^1 y_k^1 - \bar{v}^1 x_k^1 - \sum_{i=1}^{2} \bar{v}_1^{si} x_k^{si} \le 0, \quad k = 1, .., K, \\
& \bar{\mu}^2 y_k^2 - \sum_{i=1}^{2} \bar{v}_2^{si} x_k^{si} \le 0, \quad k = 1, .., K, \\
& \mu^1, \mu^2, \bar{\mu}^1, \bar{\mu}^2, \bar{v}_1^{s1}, \bar{v}_1^{s2}, \bar{v}_2^{s1}, \bar{v}_2^{s2}, \bar{v}^1 \ge 0 \qquad (9)
\end{aligned}$$

The optimal value of the above model can compute relative efficiency of evaluating units. Suppose $\mu^{1*}, \mu^{2*}, \bar{\mu}^{1*}, \bar{\mu}^{2*}, \bar{v}_1^{s1*}, \bar{v}_1^{s2*}, \bar{v}_2^{s1*}, \bar{v}_2^{s2*}, \bar{v}^{1*}$ are the optimal multipliers of the above model. So, relative efficiency of the first and the second components can be calculated as follows:

$$Re_j^1 = \frac{\dfrac{\bar{\mu}^{1*} y_j^1}{\bar{v}^{1*} x_j^1 + \sum_{i=1}^{2} \bar{v}_1^{si*} x_j^{si}}}{\max\limits_{k=1,\ldots,K} \left\{ \dfrac{\bar{\mu}^{1*} y_k^1}{\bar{v}^{1*} x_k^1 + \sum_{i=1}^{2} \bar{v}_1^{si*} x_k^{si}} \right\}} \qquad (10)$$

TABLE II
INPUT-OUTPUTS DATA OF THE FACULTIES

Faculties	x^1	x^{s1}	x^{s2}	y^1	y^2
Humanities	3672	313	126	923	190
Basic science	965	213	69	371	345
Medical science	874	144	77	136	55
Technical-engineering	3810	361	164	743	225
Teacher training	880	155	79	419	50

TABLE III
OVERALL EFFICIENCY AND COMPONENTS' EFFICIENCY

Faculties	Re_k^a	Re_k^1	Re_k^2
Humanities	1.00	0.97	0.34
Basic science	1.00	0.79	1.00
Medical science	0.40	0.34	0.20
Technical- engineering	0.69	0.61	0.35
Teacher training	1.00	1.00	0.17

$$Re_j^2 = \frac{\dfrac{\bar{\mu}^{2*} y_j^2}{\sum_{i=1}^{2} \bar{v}_2^{si*} x_j^{si}}}{\max\limits_{k=1,\ldots,K} \left\{ \dfrac{\bar{\mu}^{2*} y_k^2}{\sum_{i=1}^{2} \bar{v}_2^{si*} x_k^{si}} \right\}} \qquad (11)$$

IV. EXAMPLE

In this section, performance evaluation of the faculties of Islamic Azad University of Zahedan is studied. In this paper, faculties are considered as two-component DMUs which include teaching and research fields. Finally, the efficiency of each faculty and their components are determined. Based on them the weak points of the faculty in two fields of research and teaching, can be extracted and analyzed separately.

In this example, five faculties (i.e. humanities, basic science, medical science, technical-engineering and teacher training) are analyzed. The data and the information of these faculties are summarized in Table II.

Based on model (9), formulas (10) and (11), relative efficiency of the faculties and relative efficiency of the components are reported in Table III.

As it can be seen in Table III, faculties of humanities, basic science and teacher- training are efficient. Although humanities faculty is overall efficient, it is not efficient in teaching or research fields. However, basic science faculty is efficient in research field and it is inefficient in teaching field. The rate of this inefficiency is 21%. In contrast, teacher training faculty acts effectively in teaching field and obtained efficiency 1. But in research field it obtained a weak performance which equals 0.17. Technical-engineering faculty is generally and individually in components better than medical science. In both recent faculties teaching efficiency is higher than research efficiency.

V. Conclusion

In this paper, a mathematical efficiency model is analyzed based on the DEA in order to obtain relative efficiency of the faculties of Islamic Azad University of Zahedan as two-component DMUs. Then the obtained solution of the solved model was used to compute relative efficiency of teaching and research components. It should be mentioned that the previous models in the literature of computing components' efficiency were not able to compute the relative efficiency of them. This deficiency was met in this paper. The present approach can be expanded to multi-component DMUs with fuzzy and random data. Further studies may concern computing relative efficiency and estimating the return to scale of two-component DMUs in the situation of variable return to scale.

References

[1] W. W. Cooper, L. M. Seiford, and K. Tone, *Introduction to data envelopment analysis and its uses with DEA-solver software and references*, Springer, New York, 2006.

[2] M. L. Farrell, "The measurement of productive efficiency," *Journal of the Royal Statistical Society A*, vol. 120, pp. 253–290, 1957.

[3] A. Charnes, W. W. Cooper, and E. Rhodes, "Measuring the efficiency of decision making units," *European Journal of Operational Research*, vol 2, pp. 429–444, 1978.

[4] R. D. Banker, A. Charnes, and W. W. Cooper, "Some models for estimating technical and scale efficiencies in data envelopment analysis," *Management Science*, vol. 30, pp. 1078–1092, 1984.

[5] W. D. Cook, and L.M. Seiford, "Data envelopment analysis (DEA) Thirty years on," *European Journal of Operational Research*, vol. 192, pp. 1-17, 2009.

[6] W. D. Cook, M. Hababou, and H. J. H. Tuenter, "Multicomponent efficiency measurement and shared inputs in DEA: an application to sales and service performance in bank branches," *Journal of Productivity Analysis*, vol. 14, pp. 209-224, 2000.

[7] G. R. Jahanshahloo, A. R. Amirteimoori, and S. Kordrostami, Measuring the multi-component efficiency with shared inputs and outputs in data envelopment analysis, *Applied Mathematics and Computation*, vol. 155, pp. 283-293, 2004.

[8] G. R. Jahanshahloo, A. R. Amirteimoori, and S. Kordrostami, "Multi-component performance, progress and regress measurement and shared-inputs and outputs in DEA for panel data:an application in commercial bank branches," *Applied Mathematics and Computation*, vol. 151, pp. 1-16, 2004.

[9] A. A. Noora, F. Hosseinzadeh Lotfi, and A. Payan, "Measuring the relative efficiency in multi-component decision making units and its application to bank branches," *Journal of mathematical extension*, vol. 5, pp. 101-119, 2011.

[10] R. G. Thompson, P. S. Dharmapala, and M. R. Thral, "Importance for DEA of zeroes in data multipliers and solutions," *Journal of Productivity Analysis*, vol. 4, pp. 379-390, 1993.

[11] A. Charnes, and W. W. Cooper, "Programming with linear fractional functional," *Noval Research Logistics Quarterly*, vol.9, pp. 181-185, 1962.