Dynamic Safety-Stock Calculation

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***Abstract*—**In order to ensure a high service level industrial enterprises have to maintain safety-stock that directly influences the economic efficiency at the same time. This paper analyses established mathematical methods to calculate safety-stock. Therefore, the performance measured in stock and service level is appraised and the limits of several methods are depicted. Afterwards, a new dynamic approach is presented to gain an extensive method to calculate safety-stock that also takes the knowledge of future volatility into account.

***Keywords*—**Inventory dimensioning, material requirement planning, safety-stock calculation.

I. Introduction

THE logistic performance of industrial enterprises has become more and more important in the last two decades. Recent surveys show that "delivery time" and "delivery reliability" in particular are the main decision criteria besides price and quality of the product [1], [2]. However, in times of increasingly dynamic markets and growing interdependences of international enterprises a high service level can often just to be ensured by stocking sufficient inventories. In these cases the maintained safety-stock depend on volatile factors such as the demand, replenishment time and replenishment quantities [3].

On the one hand, industrial enterprises always try to improve their service level in order to reach customer requirements. As a result high safety-stock is required. On the other hand, inventory generates capital commitment that influences the economic efficiency. These competing logistic objectives "high delivery capability" and "low stock level" describe the so called *dilemma of inventory management* [2], [4]. Within this area of conflict many different methods were developed to dimension inventory.

Most of the existing approaches to calculate safety-stock based on statistical parameters (e.g. standard deviation of the demand, mean of the demand rate) and therefore just take historical data into consideration [5]-[7]. Some approaches deal with dynamic inventory control by applying statistical equations that are embedded in rolling planning [7]-[9]. Hence, they only can be characterized as quasi-dynamic [10].

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The goal of the paper is to introduce a dynamic approach to calculate safety-stock that is easy to implement for practitioners, as well.

II. Comparison of Safety-Stock Calculation Methods

A. Existing Approaches for Dimensioning Safety-Stock

In publications many different mathematical- stochastic methods for determining safety-stock are described and discussed. Previous simulation studies conducted at the IFA compared and benchmarked the most common basic approaches in industrial practice [11]. In the following, we will briefly characterize the approaches that perform best under the simulated conditions.

One of the frequently referred standard formula of safety-stock calculation (see a. o. [5], [12]) multiplies a safety factor which depends on the service level based on a normal distributed demand with the standard deviation of the demand during the replenishment time:

$$SSL = SF(SL) \cdot \sigma_D \tag{1}$$

where SSL: safety-stock level [units]; SF: safety factor depending on service level [-]; SL: service level [-]; σ_D: standard deviation of demand [units/SCD]; SCD: shop calendar day.

Where Method (1) the safety-stock is determined as a function of the service level (SL), which in turn is determined as the percentage of the total demands served punctually. The safety factor for the service level results via the inverse of the standard normal distribution [5], [13].

ALICKE provides a calculation rule for safety-stock that takes the replenishment time and a forecast error, derived from forecast data as a stochastic component, into consideration [5]:

$$SSL = SF(SL) \cdot \sigma_F \cdot \sqrt{TRP} \tag{2}$$

where σ_F: standard deviation of the forecast error for the demand during TRP [units/SCD]. The standard deviation of the prognosis error is calculated via historical data from the mean squared deviation of the forecasted demand from the actual. Thus, Method (2) applies independent of a specific statistical distribution of the demand [5].

The following concepts take up the preceding concepts and extend them with a stochastic replenishment time [5], [6]:

$$SSL = SF(SL) \cdot \sqrt{TRP \cdot \sigma_D^2 + D^2 \cdot \sigma_{TRP}^2} \tag{3}$$

where D: mean demand per period [units/SCD]; σ_{TRP}: standard deviation of replenishment time [SCD].

Method (4) by Herrmann also takes into account the so called "undershoot" that refers to the problem in which the stock might have already fallen below the order point immediately before an order is triggered [13]:

$$SSL = SF(SL) \cdot \sqrt{Var(U) + TRP \cdot \sigma_D^2 + D^2 \cdot \sigma_{TRP}^2} \qquad (4)$$

Gudehus amplifies Method (4) with an adaptive service level factor that takes into consideration that only disruptions during the replenishment cycle can lead to a lack of delivery capability (Method 5). If a mean delivery capability is to be attained over the period between the input of two orders, a value that is smaller than the required delivery capability suffices for the service level during the critical replenishment time phase [14]:

$$\begin{aligned} &SSL = SF\left(1 - \frac{(1-SL) \cdot QRP}{TRP \cdot D}\right) \cdot \sqrt{TRP \cdot \sigma_N^2 + D^2 \cdot \sigma_{TRP}^2} \\ &\forall \quad QRP > TRP \cdot D \end{aligned} \qquad (5)$$

where QRP: replenishment quantity [units].

All of the formulas described thus far have been developed – unless otherwise noted – under the condition of normally distributed parameters. The following Method (6) is a function for determining the safety-stock for a target service level that also takes into account extreme values beyond the mean and standard deviation and that refrains from a specific statistical distribution (for a detailed derivation cf. [15]):

$$\begin{aligned} &SSL = LSL_0 \cdot \left(SL^2 - 1\right) + SSL_{100\%} \cdot \sqrt[C]{1 - (1-SL)^C} \\ &with \quad LSL_0 = \frac{QRP}{2} \quad and \\ &SSL_{100\%} = \sqrt{\left(DV_{d,\max}^{+} \cdot D\right)^2 + \left((D_{\max} - D) \cdot TRP\right)^2 + \left(DV_{QRP,\max}^{-}\right)^2} \end{aligned} \qquad (6)$$

where LSL_0: lot stock level [units]; C: C-Norm parameter [-]; $DV_{d,max}^{+}$: max. positive deviation from due date [SCD]; D_{max}: maximum demand per period [units/SCD]; $DV_{QRP,max}^{-}$: max. negative deviation in replenishment quantity [units].

Method (6) is based on calculating a safety-stock for a target service level of 100% ($SSL_{100\%}$). This safety-stock can be adjusted via the C-Norm Function to a service level lower than 100% [15].

B. Underlying Structure of Simulation Study

The so called inventory replenishment policies provide the procurement quantities and time-points in order to control the stock in a storage echelon. According to approaches mainly used in industrial practice, the s, q- policy serves as a basis for the simulation of the stock [16]. When the order point s is reached an order of quantity q is triggered that restock the store after the replenishment time. For this purpose, the Economic Order Quantity approach according to Harris [17] is used for the optimal order quantity [11].

To simulate the necessity of safety-stock there are deviations from the planned values implemented. First, differences in the required quantities and the time intervals between the two demands constitute demand fluctuations from the output- side. Second, deviations are schedule or quantity related from the input- side [11].

For this simulation three different continual distribution forms are taken as a basis for the distribution of the demand since they can be seen in industrial practice. The normal distribution is suitable for displaying a large number of random variables [18]. The gamma distribution is particularly suited for depicting sporadic demands and does not take any negative value into account [6], [19]. The log-normal distribution is suitable for representing skewed distribution and also does not take any negative values into account [11].

Since three divisions are used for the classification of the demand, three classes are also used for the replenishment time. Though, these classes have stricter restrictions with regards to possible distribution parameters. First, an almost constant replenishment time is illustrated by a mean value of 3 SCD and a mean standard deviation of 1 SCD. Second, for an irregular replenishment time a mean of 5 SCD and a mean standard deviation of 3 SCD are assumed. Third, an uncertain replenishment time is characterized by a mean value of 20 SCD and a standard deviation of 16 SCD. Deviations from the planned supply quantities are not relevant from practical perspective. Thus, they are not taken into account [11].

Based on the total number of methods and parameter variations a total number of 3645 simulation runs were conducted. The resulting logistic performance (target service level 95%) and costs for a method are used as assessment factors [11].

C. Results of Simulation Study

The conducted simulation study illustrates that there is no superior approach among the various methods of safety stock calculation. Each of the presented methods has its respective strength depending on the particular conditions [11]. Fig. 1 depicts an overview of the gained results of the simulation study.

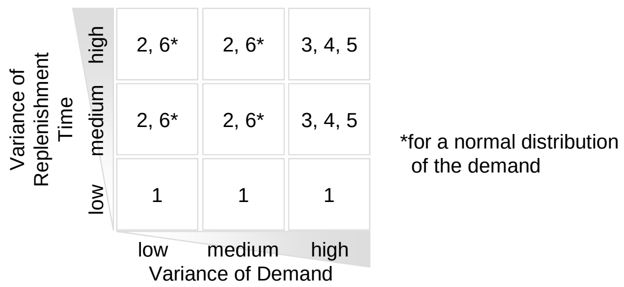

Fig. 1 Preferable application area Cluster of the calculation methods

It shows the application areas divided by the main influencing factors (variation of demand and replenishment time) and the approach which should be preferably used for each area.

Method (1) works properly as long as the replenishment time only has little variations. Although it shows high performance especially for large lot sizes with low variance of the replenishment time, it should not be used if the variance of the replenishment time is medium or high. Methods (2) and (6) provide excellent results concerning service and stock level for a medium or high variance of the replenishment time and a low or medium variance of the demand. For the cases of a high variance of the replenishment time and a high variance of the demand Methods (3), (4) or (5) achieve the best results [11].

Obviously, Method (6) should be preferred in four of nine cases. Therefore, it presents the basis for further research to develop a dynamic method to calculate safety-stock. Because of the presented lack of strengths in cases with a high variance of the demand the focus lies on an improvement of the approach for such circumstances.

III. Dynamic Safety- Inventory Calculation

With Method (6) Lutz provides a calculation rule for the safety stock that performs better than other concepts. However, this static approach based on the statistical analysis of date and does not take any knowledge of future volatility into account. Therefore, the paper presents a future- oriented dynamic approach to dimensioning stock.

In order to generate such a dynamic approach, forecast information is assumed to calculate an accurate demand for each shop calendar day. Previous research conducted at the IFA shows that cubic spline function are well suited to transform a monthly given forecast to a corresponding forecast quantity for each shop calendar day [10].

Moreover, since deviations from the planned supply quantities can be counter-balanced by pulling forward the next delivery quantity deviations are not taken into consideration.

Thus, two deviation types remain which have to be considered, the due- date deviations and the demand fluctuations.

A. Dynamic Safety-Stock Level for Due Date Deviations

First, due-date deviations are balanced in Method (6) by the product of the maximal positive deviation from due date and the mean demand. Is an accurate daily forecast given the integral of demand with respect to time can be used to calculate the required dynamic safety-stock. The lower limit of integration is the end of the replenishment time (t_2) because at this point the delay of the demand begins. The upper limit of integration is represented by the maximal positive deviation from due date (t_3).

$$SSL_{1,\min} = \int_{t_2}^{t_3} f(t)dt \qquad (7)$$

where $SSL_{1,min}$: Dynamic safety-stock level for due-date deviations; f(t): forecast information on a daily basis [units]; $DV_{d,max}^{+}$: max. pos. deviation from due date [SCD]. Fig. 2 depicts both the static and dynamic safety stock level for due-date fluctuations. The spotted area illustrates the static calculation whereas the sum of the spotted and the striped area represent the dynamic approach.

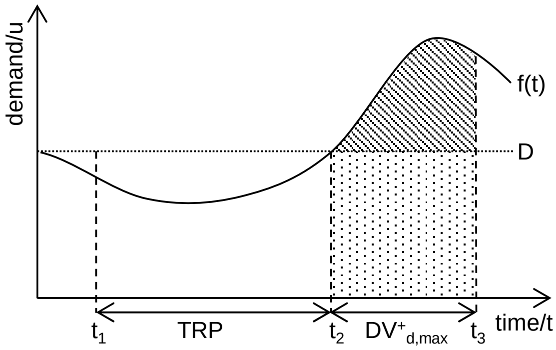

Fig. 2 Safety-stock level for due-date deviations

B. Dynamic Safety-Stock Level for Demand Fluctuation

Second, demand fluctuations come into play when the actual demand (D_{actual}) during the replenishment time is higher than the forecasted demand. In order to avoid lasting effects on the delivery capability a safety-stock level for demand fluctuations need to be maintained. Accordingly, a forecast error in regards to quantity effects should be utilized. This factor multiplied with the forecasted demand during the replenishment time can be determined as the required safety stock level.

$$SSL_{2,\min} = f_e \cdot \int_{t_1}^{t_2} f(t)dt \qquad (8)$$

where $SSL_{2,min}$: Safety stock level for demand fluctuations [unit]; f_e: forecast error of the demand [%].

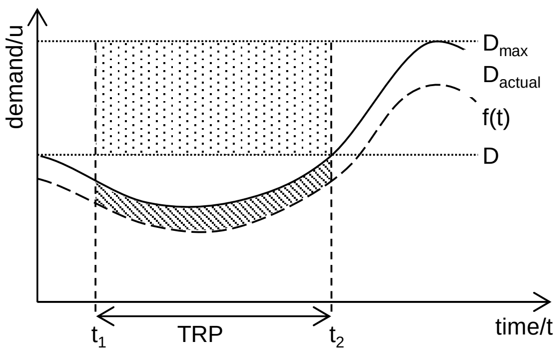

Fig. 3 Safety stock level for demand fluctuations

Fig. 3 illustrates the static calculation approach (spotted area) and the dynamic calculation approach (striped area) for demand fluctuations.

C. Dynamic Safety-Stock Level for a Target Service Level

According to the static method by Lutz both deviation types can occur individually or in combination. Moreover, a stochastic independence of due- date deviations and demand fluctuations are assumed [15]. Hence, both formulas combined constitute a safety-stock level for a target service level of 100% ($SSL_{100\%,dyn}$):

$$SSL_{100\%,dyn} = \sqrt{\left(SSL_{1,\min}\right)^2 + \left(SSL_{2,\min}\right)^2} \quad (9)$$

After substituting (6), (7) and (8):

$$SSL = LSL_0 \cdot \left(SL^2 - 1\right) + SSL_{100\%,dyn} \cdot \sqrt[C]{1 - \left(1 - SL\right)^C}$$
$$where \quad LSL_0 = \frac{QRP}{2} \quad and \quad (10)$$
$$SSL_{100\%,dyn} = \sqrt{\left(\int_{t_2}^{t_3} f(t)dt\right)^2 + \left(f_e \cdot \int_{t_1}^{t_2} f(t)dt\right)^2}$$

Finally, the safety-stock (10) can be adjusted via the C-Norm Function to a target service level lower than 100%.

IV. Conclusion

The paper introduces a new dynamic approach to calculate safety-stock. Based on previous simulation studies conducted by the IFA an appropriate static approach was selected to serve as the basis for the new method. Because the new approach includes forecast information regarding to quantity it should performer better than the presented static methods. In particular, a lower level of stock in times with less demand and a higher service level in boom times are expected.

However, further research is required to verify the advantages of the presented approach. Moreover, a suitable forecast error is to be examined. In order to consider high variations of the demand different kinds of calculations are possible, e.g. the absolute deviation, mean square deviation, variance of the demand, to name a few. All calculation should be based on the replenishment time, because only in this period deviations can cause lasting effects on the delivery capability.

After verifying the advantages of the presented dynamic method to calculate safety-stock and its final devising a demonstrator program is to be developed in order to bring the new approach into industrial practice.

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