World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:7, No:5, 2013

Improved robust stability criteria of a class of neutral Lur'e systems with interval time-varying delays

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Abstract—This paper addresses the robust stability problem of a class of delayed neutral Lur'e systems. Combined with the property of convex function and double integral Jensen inequality, a new tripe integral Lyapunov functional is constructed to derive some new stability criteria. Compared with some related results, the new criteria established in this paper are less conservative. Finally, two numerical examples are presented to illustrate the validity of the main results.

Keywords—Lur'e system; Convex function; Jensen integral inequality; Triple-integral method; Exponential stability.

I. Introduction

As one of the most important classes of nonlinear systems, Lur'e system received many researcher's attention, because it includes many nonlinear physical systems such as Lorenz, Chua, and L$\ddot{u}$ system as special cases.

Since the notion of absolute stability was first time introduced by Lur'e in [1], the stability analysis for Lur'e system has been extensively researched (see [2]-[6]) in the past decades. However, because of the existence of time delays, stochastic disturbances, parameter uncertainties and so on, the convergence of Lur'e system may often be destroyed. This makes the design or performance for the corresponding closed-loop systems become difficult. Therefore, the stability analysis of delayed Lur'e system becomes very important. Up to now, various stability conditions have been obtained, and many excellent papers and monographs have been available (see [7]-[12]). Generally speaking, these so-far obtained stability results for delayed Lur'e system can be mainly classified into two types: that is, delay-independent and delay-dependent. Since sufficiently considered the information of time delays, delay-dependent criteria may be less conservative than delay-independent ones when the size of time delay is small. For delay-dependent type, the size of the allowable upper bound of delay is always regarded as an important criterion to discriminate the quality between different criteria.

Recently, a great deal of effort has been done to the stability analysis of delayed Lur'e system with sector and slope restricted nonlinearities. To enlarge the feasibility region of the stability criteria, by introducing variables in cross-term, P.G. Park researched a new bounding technique in [13]. Concerning the descriptor method for delayed system, an extensive work was developed by Fridman in [14]. By employing linear matrix inequality and matrix decomposing technique, J.W. Cao et al [6] researched the absolute stability problem of Lur'e control systems with multiple time delays and nonlinearities, and established some improved delay-dependent criteria. In [15], Souza et al dealt with the robust stability problem by extending the existing technique. By using genetic algorithm, Yu and Lien studied the stability and stabilization problem in [16]. For reducing the conservatism, free-weighting technique is also introduced to research the stability problem of Lur'e system, and some new improved criteria were established by Y. Chun in [18].

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On the other hand, it can be see that the Jensen inequality used in these references only focused on the relationship between $\int_{t-\tau}^{t} x^T(s)Qx(s)ds$ and $(\int_{t-\tau}^{t} x(s))^T Q(\int_{t-\tau}^{t} x(s)ds)$ or between $\int_{-\tau}^{0}\int_{t+\theta}^{t} x^T(s)Qx(s)dsd\theta$ and $(\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)dsd\theta)^T Q(\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)dsd\theta)$. One nature question is whether there exists a relationship between $\int_{t-\tau}^{t} x^T(s)Qx(s)ds$ and $(\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)dsd\theta)^T Q(\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)dsd\theta)$. This idea motivates this study. By using the property of convex function, we establish the relationship between $\int_{t-\tau}^{t} x^T(s)Qx(s)ds$ and $(\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s) dsd\theta)^T Q(\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)\ dsd\theta)$. On the basis of this new established inequality relationship, a class of new Lyapunov functional including triple-integral is proposed, and some less conservative delay-dependent stability criteria are derived. Finally, two numerical examples are presented to illustrate the validity of the main results.

II. Preliminaries

Consider the following delayed neutral Lur'e system:

$$\begin{cases} \dot{y}(t) - C\dot{y}(t-h(t)) = (A+\Delta A(t))y(t) \\ \quad + (B+\Delta B(t))y(t-\tau(t)) + (D+\Delta D(t))f(\sigma(t)), \\ \sigma(t) = H^T y(t),\ \forall t \geq 0, \\ y(s) = \varphi(s),\ s\in[-max\{h_u,\tau_u\},0], \end{cases} \quad (1)$$

where $y(t) \in R^n$ denotes the state vector; $\sigma(t) \in R^m$ is the output vector; $H = (h_1, h_2, \cdots, h_m)_{n\times m} \in R^{n\times m}$; $f(\sigma(t)) \in R^m$ denotes the nonlinear function in feedback path, which has the following form:

$$\begin{cases} f(\sigma(t)) = [f_1(\sigma_1(t)), f_2(\sigma_2(t)), \cdots, f_m(\sigma_m(t))]^T \\ \sigma(t) = [\sigma_1(t), \sigma_2(t), \cdots, \sigma_m(t)]^T \\ \qquad \triangleq [h_1^T y(t), h_2^T y(t), \cdots, h_2^T y(t)]^T, \end{cases} \quad (2)$$

which satisfies a sector condition with $f_i(\cdot)$, $(i = 1, 2, \cdots, m)$ belong to sector $[l_i^-, l_i^+]$, where l_i^-, l_i^+ are known constant scalars. C, A, B, D are constant know matrices of appropriate dimensions. $\Delta A, \Delta B, \Delta D$ denote the time-varying uncertainties, which are assumed to satisfy $[\Delta A, \Delta B, \Delta D] = WF(t)[E_1, E_2, E_3]$, where W, E_1, E_2, E_3 are know real constant matrices, and $F(t)$ satisfies $\| F(t) \| \leq 1, \forall t \geq 0$. The delay $\tau(t)$ is assumed to satisfy: $0 < \tau_l \leq \tau(t) \leq \tau_u$, $\dot{\tau}(t) \leq \tau < 1$. Note $L^- = diag(l_1^-, l_2^-, \cdots, l_n^-)$, $L^+ = diag(l_1^+, l_2^+, \cdots, l_n^+)$. For further discussion, we can make a translation for system (1) as follows. For any matrices N_1, N_2 of appropriate dimensions, by using Newton-Leibnitz formula, we can obtain the following zero equations:

$$N_1[\tau_u y(t) - \int_{t-\tau_u}^{t} y(s)ds - \int_{-\tau_u}^{0}\int_{t+\theta}^{t} \dot{y}(s)dsd\theta] = 0,$$

$$N_2[\tau_l y(t) - \int_{t-\tau_l}^{t} y(s)ds - \int_{-\tau_l}^{0}\int_{t+\theta}^{t} \dot{y}(s)dsd\theta] = 0.$$

By using above two zero equations, system (1) can be rewritten as

$$\begin{cases} \dot{y}(t) = C\dot{y}(t-h(t)) + (A + \tau_u N_1 + \tau_l N_2)y(t) \\ \quad + By(t-\tau(t)) + Df(\sigma(t)) + Wp(t) \\ \quad - N_1(\int_{t-\tau_u}^{t} y(s)ds + \int_{-\tau_u}^{0}\int_{t+\theta}^{t} \dot{y}(s)dsd\theta) \\ \quad - N_2(\int_{t-\tau_l}^{t} y(s)ds + \int_{-\tau_l}^{0}\int_{t+\theta}^{t} \dot{y}(s)dsd\theta), \\ p(t) = F(t)q(t), \\ q(t) = E_1 y(t) + E_2 y(t-\tau(t)) + E_3 f(\sigma(t)). \end{cases} \quad (3)$$

Before deriving the main results, the following lemmas are needed.

Lemma 2.1: [18] For any positive definite symmetric constant matrix Q and scalar $\tau > 0$, such that the following integrations are well defined, then

$$-\int_{-\tau}^{0}\int_{t+\theta}^{t} y^T(s)Qy(s)dsd\theta \leq$$

$$-\frac{1}{\tau^2}(\int_{-\tau}^{0}\int_{t+\theta}^{t} y(s)dsd\theta)^T Q(\int_{-\tau}^{0}\int_{t+\theta}^{t} y(s)dsd\theta).$$

Additional, the result on integral inequality obtained by J.H. Park in [19] can be redescribed as

Lemma 2.2: [19] For any positive definite symmetric constant matrix Q, any matrix F of appropriate dimensions, and scalar $\tau > 0$, such that the following integration are well defined, then

$$-\int_{t-\tau}^{0} \dot{y}^T(s)Q\dot{y}(s)ds \leq \xi^T(t)F\xi(t) + \tau\xi^T(t)F^TQ^{-1}F\xi(t),$$

where $\xi(t)$ is an augmented vector of appropriate dimensions including $\int_{t-\tau}^{0} \dot{y}(s)ds$ as a sub-vector.

III. Main Results

In this section, we attempt to establish some new practically computable stability criteria for system (1). By constructing a new Lyapunov functional including tripe-integral item, we obtain the following stability result.

Theorem 3.1: For given scalars $\tau_l > 0, \tau_u > 0, \tau > 0$, $L^- = diag(l_1^-, l_2^-, \cdots, l_n^-)$, $L^+ = diag(l_1^+, l_2^+, \cdots, l_n^+)$, system (1) is globally exponentially stable if $\rho(C) < 1$ and there exist positive definite diagonal matrices $\widetilde{D} = diag\{d_1, d_2, \cdots, d_n\}$, $\Lambda_1 = diag\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$, $\Lambda_2 = diag\{\alpha_1, \alpha_2, \cdots, \alpha_n\}$, $\Gamma = diag\{\gamma_1, \gamma_2, \cdots, \gamma_n\}$, symmetric positive definite matrices $P, Q_1, \widetilde{Q}_1, Q_2, Q_3, Q_4$, and arbitrary matrices $F_1, F_2, N_1, N_2, M_1, M_2, M_3, M_4$ of appropriate dimensions such that the following condition holds:

$$\begin{bmatrix} \Xi & F^T & \widetilde{F}^T \\ * & -\frac{\tau_l^2}{4}Q_2 & 0 \\ * & * & -\frac{\tau_u^2}{4}Q_3 \end{bmatrix} < 0,$$

where $\Xi = (\Xi_{ij})\ i, j = 1, 2, \cdots, 12$, and

$$\Xi_{11} = (P - L^-HH^T\Lambda_1 + L^+HH^T\widetilde{D})(A + \tau_u N_1 + \tau_l N_2)$$
$$+(A + \tau_u N_1 + \tau_l N_2)^T(P - L^-HH^T\Lambda_1 + L^+HH^T\widetilde{D})^T$$
$$+Q_4 + (\tau_l^2 + \tau_u^2)H\Lambda_2(L^+ - L^-)H^T$$
$$+2\tau_l Q_1 + 2\tau_u\widetilde{Q}_1 - 2HL^-\Gamma L^+H^T,$$
$$\Xi_{12} = A^TM_3^T,$$
$$\Xi_{13} = PD - L^-HH^T\Lambda_1 D + L^+HH^T\widetilde{D}D$$
$$+(A + \tau_u N_1 + \tau_l N_2)^T(\Lambda_1 - \widetilde{D})H$$
$$+H\Gamma(L^+ + L^-) + A_TM_1^T,$$
$$\Xi_{14} = PB - L^-HH^T\Lambda_1 B + L^+HH^T\widetilde{D}B,$$
$$\Xi_{15} = PC - L^-HH^T\Lambda_1 C + L^+HH^T\widetilde{D}C + A^TM_2^T,$$
$$\Xi_{16} = PN_2 - L^-HH^T\Lambda_1 N_2 + L^+HH^T\widetilde{D}N_2 = \Xi_{1,10},$$
$$\Xi_{17} = PN_1 - L^-HH^T\Lambda_1 N_1 + L^+HH^T\widetilde{D}N_1 = \Xi_{1,11},$$
$$\Xi_{1,12} = PW - L^-HH^T\Lambda_1 W$$
$$+L^+HH^T\widetilde{D}W + A^TM_4^T,$$
$$\Xi_{22} = \frac{\tau_l^2}{2}Q_2 + \frac{\tau_u^2}{2}Q_3 - M_3 - M_3^T,$$
$$\Xi_{23} = -M_1 + M_3 D,$$
$$\Xi_{24} = M_3 B,\ \Xi_{25} = -M_2^T + M_3 C,$$
$$\Xi_{2,12} = M_3 W - M_4^T,$$
$$\Xi_{33} = H^T(\Lambda_1 D - \widetilde{D}D)$$
$$+(\Lambda_1 D - \widetilde{D}D)^TH - 2\Gamma + M_1 D + M_1^TD^T,$$
$$\Xi_{34} = H^T(\Lambda_1 B - \widetilde{D}B) + M_1 B,$$
$$\Xi_{35} = H^T(\Lambda_1 C - \widetilde{D}C) + M_1 C + D^TM_2^T,$$
$$\Xi_{36} = H^T(-\Lambda_1 N_2 + \widetilde{D}N_2) = \Xi_{3,10},$$
$$\Xi_{37} = H^T(-\Lambda_1 N_1 + \widetilde{D}N_1) = \Xi_{3,11},$$

$$\Xi_{3,12} = H^T(\Lambda_1 W + \widetilde{D}W) + M_1 W + D^T M_4^T,$$

$$\Xi_{44} = -(1-\tau)Q_4,\ \Xi_{45} = B^T M_2^T,\ \Xi_{4,12} = B^T M_4^T,$$

$$\Xi_{55} = M_2 C + C^T M_2^T,\ \Xi_{5,12} = M_2 W + C^T M_4^T,$$

$$\Xi_{66} = -\frac{1}{\tau_l}Q_1,\ \Xi_{77} = -\frac{1}{\tau_u}\widetilde{Q}_1,$$

$$\Xi_{88} = -[\frac{2}{\tau_l^2}H\Lambda_2(L^+ - L^-)H^T + \frac{2}{\tau_l^3}Q_1],$$

$$\Xi_{99} = -[\frac{2}{\tau_u^2}H\Lambda_2(L^+ - L^-)H^T + \frac{2}{\tau_u^3}\widetilde{Q}_1],$$

$$\Xi_{10,10} = -\frac{1}{2\tau_l^2}Q_2 - F_1$$

$$\Xi_{11,11} = -\frac{1}{2\tau_u^2}Q_3 - F_2,\ \Xi_{12,12} = M_4 W + W^T M_4^T,$$

$$F = [0,0,0,0,0,0,0,0,0,F_1,0,0],$$

$$\widetilde{F} = [0,0,0,0,0,0,0,0,0,0,F_2,0].$$

Proof: Choose a new class of Lyapunov functional candidate as follows:

$$V(y(t)) = V_1(y(t)) + V_2(y(t)) + V_3(y(t)) + V_4(y(t)),$$

where

$$V_1(y(t)) = y^T(t)Py(t) + 2\sum_{i=1}^{n}\{\int_0^{h_i^T y(t)} \lambda_i(f_i(s) - l_i^- s)ds$$
$$+ \int_0^{h_i^T y(t)} d_i(l_i^+ s - f_i(s))ds\}$$

$$V_2(y(t)) = 2\sum_{i=1}^{n}\{\int_{-\tau_l}^0 \int_\theta^0 \int_{t+\mu}^t \alpha_i\sigma_i(s)[f_i(\sigma_i(s))$$
$$- l_i^-\sigma_i(s)]dsd\mu d\theta\}$$
$$+ 2\sum_{i=1}^{n}\{\int_{-\tau_l}^0 \int_\theta^0 \int_{t+\mu}^t \alpha_i\sigma_i(s)[l_i^+\sigma_i(s) - f_i(\sigma_i(s))]dsd\mu d\theta\}$$
$$+ 2\sum_{i=1}^{n}\{\int_{-\tau_u}^0 \int_\theta^0 \int_{t+\mu}^t \alpha_i\sigma_i(s)[f_i(\sigma_i(s)) - l_i^-\sigma_i(s)]dsd\mu d\theta\}$$
$$+ 2\sum_{i=1}^{n}\{\int_{-\tau_u}^0 \int_\theta^0 \int_{t+\mu}^t \alpha_i\sigma_i(s)[l_i^+\sigma_i(s) - f_i(\sigma_i(s))]dsd\mu d\theta\}$$
$$+ 2\int_{-\tau_l}^0 \int_{t+\theta}^t y^T(s)Q_1 y(s)dsd\theta$$
$$+ 2\int_{-\tau_u}^0 \int_{t+\theta}^t y^T(s)\widetilde{Q}_1 y(s)dsd\theta,$$

$$V_3(y(t)) = \int_{-\tau_l}^0 \int_\theta^0 \int_{t+\mu}^t \dot{y}^T(s)Q_2\dot{y}(s)dsd\mu d\theta$$
$$+ \int_{-\tau_u}^0 \int_\theta^0 \int_{t+\mu}^t \dot{y}^T(s)Q_3\dot{y}(s)dsd\mu d\theta,$$

$$V_4(y(t)) = \int_{t-\tau(t)}^t y^T(s)Q_4 y(s)ds.$$

The time derivative of $V(y(t))$ along the trajectory of system (1) is given as $\dot{V}(y(t)) = \dot{V}_1(y(t)) + \dot{V}_2(y(t)) + \dot{V}_3(y(t)) + \dot{V}_4(y(t))$, where

$$\begin{aligned}
&\dot{V}_1(y(t)) = \\
&2y^T(t)P\dot{y}(t) + 2[f^T(\sigma(t))H^T - y^T(t)L^-HH^T]\times \\
&\Lambda_1\dot{y}(t) + 2[y^T(t)L^+HH^T - f^T(\sigma(t))H^T]\widetilde{D}\dot{y}(t), \\
&= 2[y^T(t)(P - L^-HH^T\Lambda_1 + L^+HH^T\widetilde{D}) \\
&+ f^T(\sigma(t))H^T(\Lambda_1 - \widetilde{D})]\dot{y}(t) \\
&= 2y^T(t)(P - L^-HH^T\Lambda_1 + L^+HH^T\widetilde{D})\times \\
&(A + \tau_u N_1 + \tau_l N_2)y(t) \\
&+ 2y^T(t)(PB - L^-HH^T\Lambda_1 B + L^+HH^T\widetilde{D}B)y(t-\tau(t)) \\
&+ 2y^T(t)(PC - L^-HH^T\Lambda_1 C + L^+HH^T\widetilde{D}C)\dot{y}(t-h(t)) \\
&+ 2y^T(t)(PD - L^-HH^T\Lambda_1 D + L^+HH^T\widetilde{D}D)f(\sigma(t)) \\
&+ 2y^T(t)(-PN_1 + L^-HH^T\Lambda_1 N_1 \\
&- L^+HH^T\widetilde{D}N_1)\int_{t-\tau_u}^t y(s)ds \\
&+ 2y^T(t)(-PN_2 + L^-HH^T\Lambda_1 N_2 \\
&- L^+HH^T\widetilde{D}N_2)\int_{t-\tau_l}^t y(s)ds \\
&+ 2y^T(t)(-PN_1 + L^-HH^T\Lambda_1 N_1 \\
&- L^+HH^T\widetilde{D}N_1)\int_{-\tau_u}^0\int_{t+\theta}^t \dot{y}(s)dsd\theta \\
&+ 2y^T(t)(-PN_2 + L^-HH^T\Lambda_1 N_2 \\
&- L^+HH^T\widetilde{D}N_2)\int_{-\tau_l}^0\int_{t+\theta}^t \dot{y}(s)dsd\theta \\
&+ 2y^T(t)(PW - L^-HH^T\Lambda_1 W + L^+HH^T\widetilde{D}W)p(t) \\
&+ 2f^T(\sigma(t))H^T(\Lambda_1 - \widetilde{D})(A + \tau_u N_1 + \tau_l N_2)y(t) \\
&+ 2f^T(\sigma(t))H^T(\Lambda_1 B - \widetilde{D}B)y(t-\tau(t)) \\
&+ 2f^T(\sigma(t))H^T(\Lambda C - \widetilde{D}C)\dot{y}(t-h(t)) \\
&+ 2f^T(\sigma(t))H^T(\Lambda_1 D - \widetilde{D}D)f(\sigma(t)) \\
&+ 2f^T(\sigma(t))H^T(\Lambda_1 - \widetilde{D})Wp(t). \\
&+ 2f^T(\sigma(t))H^T(-\Lambda_1 N_1 + \widetilde{D}N_1)\int_{t-\tau_u}^t y(s)ds \\
&+ 2f^T(\sigma(t))H^T(-\Lambda_1 N_2 + \widetilde{D}N_2)\int_{t-\tau_l}^t y(s)ds \\
&+ 2f^T(\sigma(t))H^T(-\Lambda_1 N_1 + \widetilde{D}N_1)\int_{-\tau_u}^0\int_{t+\theta}^t \dot{y}(s)dsd\theta \\
&+ 2f^T(\sigma(t))H^T(-\Lambda_1 N_2 + \widetilde{D}N_2)\int_{-\tau_l}^0\int_{t+\theta}^t \dot{y}(s)dsd\theta.
\end{aligned} \quad (4)$$

$$\begin{aligned}
&\dot{V}_2(y(t)) = \\
&2\sum_{i=1}^{n}\{\int_{-\tau_l}^0 \int_\theta^0 \alpha_i\sigma_i(t)[f_i(\sigma_i(t)) - l_i^-\sigma_i(t)]d\mu d\theta \\
&- \int_{-\tau_l}^0 \int_\theta^0 \alpha_i\sigma_i(t+\mu)[f_i(\sigma_i(t+\mu)) - l_i^-\sigma_i(t+\mu)]d\mu d\theta\} \\
&+ 2\sum_{i=1}^{n}\{\int_{-\tau_l}^0 \int_\theta^0 \alpha_i\sigma_i(t)[l_i^+\sigma_i(t) - f_i(\sigma_i(t))]d\mu d\theta \\
&+ 2\sum_{i=1}^{n}\{\int_{-\tau_u}^0 \int_\theta^0 \alpha_i\sigma_i(t)[f_i(\sigma_i(t)) - l_i^-\sigma_i(t)]d\mu d\theta
\end{aligned}$$

$$
\begin{aligned}
&- \int_{-\tau_u}^{0} \int_{\theta}^{0} \alpha_i \sigma_i(t+\mu)[f_i(\sigma_i(t+\mu)) \\
&- l_i^- \sigma_i(t+\mu)] d\mu d\theta\} \\
&+ 2 \sum_{i=1}^{n} \{ \int_{-\tau_u}^{0} \int_{\theta}^{0} \alpha_i \sigma_i(t)[l_i^+ \sigma_i(t) - f_i(\sigma_i(t))] d\mu d\theta \\
&- \int_{-\tau_u}^{0} \int_{\theta}^{0} \alpha_i \sigma_i(t+\mu)[l_i^+ \sigma_i(t+\mu) \\
&- f_i(\sigma_i(t+\mu))] d\mu d\theta\} \\
&+ 2\tau_l y^T(t) Q_1 y(t) - 2\int_{t-\tau_l}^{t} y^T(s) Q_1 y(s) ds \\
&+ 2\tau_u y^T(t) \widetilde{Q}_1 y(t) - 2\int_{t-\tau_u}^{t} y^T(s) \widetilde{Q}_1 y(s) ds \\
&= 2 \sum_{i=1}^{n} \{ \alpha_i \sigma_i(t)[f_i(\sigma_i(t)) - l_i^- \sigma_i(t)] \frac{\tau_l^2}{2} \\
&- \int_{-\tau_l}^{0} \int_{t+\theta}^{t} \alpha_i \sigma_i(s)[f_i(\sigma_i(s)) - l_i^- \sigma_i(s)] ds d\theta\} \\
&+ 2 \sum_{i=1}^{n} \{ \alpha_i \sigma_i(t)[l_i^+ \sigma_i(t) - f_i(\sigma_i(t))] \frac{\tau_l^2}{2} \\
&- \int_{-\tau_l}^{0} \int_{t+\theta}^{t} \alpha_i y_i(s)[l_i^+ y_i(s) - f_i(\sigma_i(s))] ds d\theta\} \\
&+ 2 \sum_{i=1}^{n} \{ \alpha_i \sigma_i(t)[f_i(\sigma_i(t)) - l_i^- \sigma_i(t)] \frac{\tau_u^2}{2} \\
&- \int_{-\tau_u}^{0} \int_{t+\theta}^{t} \alpha_i \sigma_i(s)[f_i(\sigma_i(s)) - l_i^- \sigma_i(s)] ds d\theta\} \\
&+ 2 \sum_{i=1}^{n} \{ \alpha_i \sigma_i(t)[l_i^+ \sigma_i(t) - f_i(\sigma_i(t))] \frac{\tau_u^2}{2} \\
&- \int_{-\tau_u}^{0} \int_{t+\theta}^{t} \alpha_i \sigma_i(s)[l_i^+ \sigma_i(s) \\
&- f_i(\sigma_i(s))] ds d\theta\} + 2\tau_l y^T(t) Q_1 y(t) \\
&- 2\int_{t-\tau_l}^{t} y^T(s) Q_1 y(s) ds + 2\tau_u y^T(t) \widetilde{Q}_1 y(t) \\
&- 2\int_{t-\tau_u}^{t} y^T(s) \widetilde{Q}_1 y(s) ds \\
&= 2 \sum_{i=1}^{n} \{ \frac{\alpha_i \tau_l^2}{2} \sigma_i(t)[l_i^+ - l_i^-] \sigma_i(t) \\
&+ \sum_{i=1}^{n} \frac{\alpha_i \tau_u^2}{2} \sigma_i(t)[l_i^+ - l_i^-] \sigma_i(t) \\
&- \int_{-\tau_l}^{0} \int_{t+\theta}^{t} \alpha_i \sigma_i(s)[l_i^+ - l_i^-] \sigma_i(s) ds d\theta \\
&- \int_{-\tau_u}^{0} \int_{t+\theta}^{t} \alpha_i \sigma_i(s)[l_i^+ - l_i^-] \sigma_i(s) ds d\theta\} \\
&+ 2\tau_l y^T(t) Q_1 y(t) - 2\int_{t-\tau_l}^{t} y^T(s) Q_1 y(s) ds \\
&+ 2\tau_u y^T(t) \widetilde{Q}_1 y(t) - 2\int_{t-\tau_u}^{t} y^T(s) \widetilde{Q}_1 y(s) ds.
\end{aligned} \tag{5}
$$

Notice that

$$
\sum_{i=1}^{n} \alpha_i \tau_l^2 \sigma_i(t)[l_i^+ - l_i^-] \sigma_i(t) = \tau_l^2 y^T(t) H \Lambda_2 [L^+ - L^-] H^T y(t),
$$

$$
\sum_{i=1}^{n} \alpha_i \tau_u^2 \sigma_i(t)[l_i^+ - l_i^-] \sigma_i(t) = \tau_u^2 y^T(t) H \Lambda_2 [L^+ - L^-] H^T y(t).
$$

From Lemma 2.1, we have

$$
\begin{aligned}
&2 \sum_{i=1}^{n} \{ - \int_{-\tau_l}^{0} \int_{t+\theta}^{t} \alpha_i \sigma_i(s)[l_i^+ - l_i^-] \sigma_i(s) ds d\theta\} \\
&= -2 \int_{-\tau_l}^{0} \int_{t+\theta}^{t} y^T(s) H \Lambda_2 [L^+ - L^-] H^T y(s) ds d\theta \\
&\le -\frac{2}{\tau_l^2} (\int_{-\tau_l}^{0} \int_{t+\theta}^{t} y(s) ds d\theta)^T H \Lambda_2 [L^+ \\
&- L^-] H^T (\int_{-\tau_l}^{0} \int_{t+\theta}^{t} y(s) ds d\theta).
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
&2 \sum_{i=1}^{n} \{ - \int_{-\tau_u}^{0} \int_{t+\theta}^{t} \alpha_i \sigma_i(s)[l_i^+ - l_i^-] \sigma_i(s) ds d\theta\} \\
&\le -\frac{2}{\tau_u^2} (\int_{-\tau_u}^{0} \int_{t+\theta}^{t} y(s) ds d\theta)^T H \Lambda_2 [L^+ \\
&- L^-] H^T (\int_{-\tau_u}^{0} \int_{t+\theta}^{t} y(s) ds d\theta).
\end{aligned}
$$

Additional, let $f(x) = x^T Q x$, where $Q > 0$, then $f''(x) = Q > 0$. Set $x = \frac{1}{\tau} \int_{-\tau}^{0} \int_{t+\theta}^{t} x(s) ds d\theta$, $\tau > 0$, from Jensen integral inequality, we have

$$
\begin{aligned}
f(x) &= \frac{1}{\tau^2} (\int_{-\tau}^{0} \int_{t+\theta}^{t} x(s) ds d\theta)^T Q (\int_{-\tau}^{0} \int_{t+\theta}^{t} x(s) ds d\theta) \\
&= f(\frac{1}{\tau} \int_{-\tau}^{0} \int_{t+\theta}^{t} x(s) ds d\theta) \\
&\le \frac{1}{\tau} \int_{-\tau}^{0} \{ (\int_{t+\theta}^{t} x(s) ds)^T Q (\int_{t+\theta}^{t} x(s) ds) \} d\theta \\
&= \frac{1}{\tau} \int_{-\tau}^{0} \theta^2 \{ (-\frac{1}{\theta} \int_{t+\theta}^{t} x(s) ds)^T Q (-\frac{1}{\theta} \int_{t+\theta}^{t} x(s) ds) \} d\theta \\
&= \frac{1}{\tau} \int_{-\tau}^{0} \theta^2 f(-\frac{1}{\theta} \int_{t+\theta}^{t} x(s) ds) d\theta \\
&\le \frac{1}{\tau} \int_{-\tau}^{0} \int_{t+\theta}^{t} -\theta f(x(s)) ds d\theta \\
&= \frac{1}{\tau} \int_{-\tau}^{0} \int_{t+\theta}^{t} -\theta x^T(s) Q x(s) ds d\theta \\
&= \frac{1}{\tau} \int_{t-\tau}^{t} \int_{-\tau}^{s-t} -\theta x^T(s) Q x(s) d\theta ds \\
&= \frac{1}{\tau} \int_{t-\tau}^{t} x^T(s) Q x(s) (\frac{\tau^2}{2} - \frac{(s-t)^2}{2}) ds \qquad (6) \\
&\le \frac{\tau}{2} \int_{t-\tau}^{t} x^T(s) Q x(s) ds.
\end{aligned}
$$

Thus, we have

$$
- \int_{t-\tau}^{t} x^T(s) Q x(s) ds \le
$$

$$-\frac{2}{\tau^3}(\int_{-\tau}^{0}\int_{t+\theta}^{t}x(s)dsd\theta)^TQ(\int_{-\tau}^{0}\int_{t+\theta}^{t}x(s)dsd\theta). \quad (7)$$

This means that

$$\begin{aligned}
&-2\int_{t-\tau_l}^{t}y^T(s)Q_1y(s)ds \le \\
&-\frac{2}{\tau_l^3}(\int_{-\tau_l}^{0}\int_{t+\theta}^{t}y(s)dsd\theta)^TQ_1(\int_{-\tau_l}^{0}\int_{t+\theta}^{t}y(s)dsd\theta)\\
&-\frac{1}{\tau_l}(\int_{t-\tau_l}^{t}y(s)ds)^TQ_1(\int_{t-\tau_l}^{t}y(s)ds).\\
&-2\int_{t-\tau_u}^{t}y^T(s)\widetilde{Q}_1y(s)ds \le \\
&-\frac{2}{\tau_u^3}(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y(s)dsd\theta)^T\widetilde{Q}_1(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y(s)dsd\theta)\\
&-\frac{1}{\tau_u}(\int_{t-\tau_u}^{t}y(s)ds)^T\widetilde{Q}_1(\int_{t-\tau_u}^{t}y(s)ds).
\end{aligned}$$

Namely,

$$\begin{aligned}
&\dot{V}_2(y(t)) \le y^T(t)[(\tau_l^2+\tau_u^2)H\Lambda_2(L^+-L^-)H^T\\
&+2\tau_lQ_1+2\tau_u\widetilde{Q}_1]y(t)\\
&-(\int_{-\tau_l}^{0}\int_{t+\theta}^{t}y(s)dsd\theta)^T[\frac{2}{\tau_l^2}H\Lambda_2(L^+-L^-)H^T\\
&+\frac{2}{\tau_l^3}Q_1]\int_{-\tau_l}^{0}\int_{t+\theta}^{t}y(s)dsd\theta\\
&-(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y(s)dsd\theta)^T[\frac{2}{\tau_u^2}H\Lambda_2(L^+-L^-)H^T\\
&+\frac{2}{\tau_u^3}\widetilde{Q}_1]\int_{-\tau_u}^{0}\int_{t+\theta}^{t}y(s)dsd\theta\\
&-\frac{1}{\tau_l}(\int_{t-\tau_l}^{t}y(s)ds)^TQ_1(\int_{t-\tau_l}^{t}y(s)ds)\\
&-\frac{1}{\tau_u}(\int_{t-\tau_u}^{t}y(s)ds)^T\widetilde{Q}_1(\int_{t-\tau_u}^{t}y(s)ds);
\end{aligned}$$

$$\begin{aligned}
\dot{V}_3(y(t)) &= \int_{-\tau_l}^{0}\int_{\theta}^{0}[\dot{y}^T(t)Q_2\dot{y}(t)-\dot{y}^T(t+\mu)Q_2\dot{y}(t+\mu)]d\mu d\theta\\
&+\int_{-\tau_u}^{0}\int_{\theta}^{0}[\dot{y}^T(t)Q_3\dot{y}(t)-\dot{y}^T(t+\mu)Q_3\dot{y}(t+\mu)]d\mu d\theta\\
&=\frac{\tau_l^2}{2}\dot{y}^T(t)Q_2\dot{y}(t)+\frac{\tau_u^2}{2}\dot{y}^T(t)Q_3\dot{y}(t)\\
&-\int_{-\tau_l}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)Q_2\dot{y}(s)dsd\theta\\
&-\int_{-\tau_u}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)Q_2\dot{y}(s)dsd\theta\\
&\le\frac{\tau_l^2}{2}\dot{y}^T(t)Q_2\dot{y}(t)+\frac{\tau_u^2}{2}\dot{y}^T(t)Q_3\dot{y}(t)\\
&-\frac{1}{2\tau_l^2}(\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{y}(s)dsd\theta)^TQ_2(\int_{-\tau_l}^{0}\int_{t+\theta}^{t}\dot{y}(s)dsd\theta)\\
&-\frac{1}{2\tau_u^2}(\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{y}(s)dsd\theta)^TQ_3(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}\dot{y}(s)dsd\theta)\\
&-\frac{1}{2}\int_{-\tau_l}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)Q_2\dot{y}(s)dsd\theta\\
&-\frac{1}{2}\int_{-\tau_u}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)Q_2\dot{y}(s)dsd\theta.
\end{aligned}$$

From Lemma 2.2, for arbitrary matrix F_1 of appropriate dimensions,

$$\begin{aligned}
&-\int_{t+\theta}^{t}\dot{y}^T(s)Q_2\dot{y}(s)ds\\
&\le -2(\int_{t+\theta}^{t}\dot{y}(s)ds)^TF\xi(t)+\int_{t+\theta}^{t}\xi^T(t)F^TQ_2^{-1}F\xi(t)ds \quad (8)\\
&=-2(\int_{t+\theta}^{t}\dot{y}(s)ds)^TF\xi(t)-\theta\xi^T(t)F^TQ_2^{-1}F\xi(t),
\end{aligned}$$

where$\xi^T(t)=[y^T(t),\ \dot{y}^T(t),\ f^T(\sigma(t)),\ y(t-\tau(t)),\ \dot{y}(t-h(t)),$ $\int_{t-\tau_l}^{t}y^T(s)ds,\ \int_{t-\tau_u}^{t}y^T(s)ds,\ \int_{-\tau_l}^{0}\int_{t+\theta}^{t}y^T(s)dsd\theta,\ \int_{-\tau_u}^{0}\int_{t+\theta}^{t}$ $y^T(s)dsd\theta,\int_{-\tau_l}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)dsd\theta,\int_{-\tau_u}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)dsd\theta,p(t)^T]$.

Thus, we have

$$\begin{aligned}
&-\frac{1}{2}\int_{-\tau_l}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)Q_2\dot{y}(s)dsd\theta\\
&\le-\int_{-\tau_l}^{0}[\int_{t+\theta}^{t}\dot{y}^T(s)dsF\xi(t)]d\theta\\
&-\frac{1}{2}\int_{-\tau_l}^{0}\theta\xi^T(t)F^TQ_2^{-1}F\xi(t)d\theta, \quad (9)\\
&=(-\int_{-\tau_l}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)dsd\theta)F\xi(t)\\
&+\frac{\tau_l^2}{4}\xi^T(t)F^TQ_2^{-1}F\xi(t).
\end{aligned}$$

Similarly, for arbitrary matrix F_2 of appropriate dimensions, we have

$$\begin{aligned}
&-\frac{1}{2}\int_{-\tau_u}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)Q_3\dot{y}(s)dsd\theta\\
&\le(-\int_{-\tau_u}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)dsd\theta)\widetilde{F}\xi(t) \quad (10)\\
&+\frac{\tau_u^2}{4}\xi^T(t)\widetilde{F}^TQ_3^{-1}\widetilde{F}\xi(t).
\end{aligned}$$

Combined with inequalities (7)-(10), we have

$$\begin{aligned}
&\dot{V}_3(y(t))\le\frac{\tau_l^2}{2}\dot{y}^T(t)Q_2\dot{y}(t)+\frac{\tau_u^2}{2}\dot{y}^T(t)Q_3\dot{y}(t)\\
&-\frac{1}{2\tau_l^2}(\int_{-\tau_l}^{0}\int_{t+\theta}^{t}\dot{y}(s)dsd\theta)^TQ_2(\int_{-\tau_l}^{0}\int_{t+\theta}^{t}\dot{y}(s)dsd\theta)\\
&-\frac{1}{2\tau_u^2}(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}\dot{y}(s)dsd\theta)^TQ_3(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}\dot{y}(s)dsd\theta) \quad (11)\\
&-(\int_{-\tau_l}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)dsd\theta)F\xi(t)\\
&-(\int_{-\tau_u}^{0}\int_{t+\theta}^{t}\dot{y}^T(s)dsd\theta)\widetilde{F}\xi(t)\\
&+\frac{\tau_l^2}{4}\xi^T(t)F^TQ_2^{-1}F\xi(t)+\frac{\tau_u^2}{4}\xi^T(t)\widetilde{F}^TQ_3^{-1}\widetilde{F}\xi(t).
\end{aligned}$$

$$\begin{aligned}
&\dot{V}_4(y(t))=y^T(t)Q_4y(t)\\
&-(1-\dot{\tau}(t))y^T(t-\tau(t))Q_4y(t-\tau(t)) \quad (12)\\
&\le y^T(t)Q_4y(t)-(1-u)y^T(t-\tau(t))Q_4y(t-\tau(t)).
\end{aligned}$$

Furthermore, for positive definite diagonal matrix Γ, arbitrary matrices M_1, M_2, M_3, M_4 of appropriate dimensions, we have

$$-2f^T(\sigma(t))\Gamma f(\sigma(t))+2y^T(t)H\Gamma(L^++L^-)f(y(t))$$

$$-2y^T(t)HL^-\Gamma L^+H^Ty(t) \geq 0,$$
$$2f^T(\sigma(t))M_1[-\dot{y}(t) + C\dot{y}(t-h(t))$$
$$+Ay(t) + By(t-\tau(t)) + Df(\sigma(t)) + Wp(t)] = 0,$$
$$2\dot{y}(t-h(t))^TM_2[-\dot{y}(t) + C\dot{y}(t-h(t))$$
$$+Ay(t) + By(t-\tau(t)) + Df(\sigma(t)) + Wp(t)] = 0,$$
$$2\dot{y}^T(t)M_3[-\dot{y}(t) + C\dot{y}(t-h(t))$$
$$+Ay(t) + By(t-\tau(t)) + Df(\sigma(t)) + Wp(t)] = 0,$$
$$2p^T(t)M_4[-\dot{y}(t) + C\dot{y}(t-h(t))$$
$$+Ay(t) + By(t-\tau(t)) + Df(\sigma(t)) + Wp(t)] = 0.$$

Hence, combined with Schur complement [22], we can obtain

$$\dot{V}(y(t)) \leq \xi^T(t)\Xi\xi(t) + \frac{\tau_u^2}{4}\xi^T(t)\widetilde{F}^TQ_3^{-1}\widetilde{F}\xi(t)$$
$$+\frac{\tau_l^2}{4}\xi^T(t)F^TQ_2^{-1}F\xi(t) < 0.$$

This means that system (1) is asymptotically stable. If $\rho(C) < 1$, then system (1) is exponentially stable, which completes the proof. ■

Remark 3.1: Different from [10]-[14], in the proof of Theorem 3.1, we establish the relationship between $-\int_{t-\tau}^{t} x^T(s)Qx(s)ds$ and $(\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)dsd\theta)^TQ(\int_{-\tau}^{0}\int_{t+\theta}^{t} x(s)dsd\theta)$. On the basis of this new relationship, items $\int_{-\tau_l}^{0}\int_{t+\theta}^{t} y^T(s) \quad dsd\theta$, $\int_{-\tau_u}^{0}\int_{t+\theta}^{t} y^T(s)dsd\theta$, $\int_{-\tau_l}^{0}\int_{t+\theta}^{t} \dot{y}^T(s)dsd\theta$, $\int_{-\tau_u}^{0}\int_{t+\theta}^{t} \dot{y}^T(s)dsd\theta$ are introduced as the state of $\xi(t)$, this may reduce criterion's conservatism

Remark 3.2: In [16], [17], time-varying delay $h(t)$ is requested to satisfy $0 \leq h_l \leq h(t) \leq h_u$, and $\dot{h}(t) \leq \mu$. However, in this paper, we have no registration for $h(t)$. This means that $h(t)$ can be no bound, even non derivable.

Remark 3.3: As mentioned in [20], Theorem 3.1 gives a stability criterion for system (1) with $0 < \tau_l \leq \tau(t) \leq \tau_u$, $\dot{\tau}(t) \leq \tau$, where τ is a given constant. In many cases, τ is unknown. Considering this case, a rate-independent criteria for a delay satisfying $0 < \tau_l \leq \tau(t) \leq \tau_u$ is derived as follows.

Theorem 3.2: For given scalars $\tau_l > 0, \tau_u > 0$, $L^- = diag(l_1^-, l_2^-, \cdots, l_n^-)$, $L^+ = diag(l_1^+, l_2^+, \cdots, l_n^+)$, system (1) is globally exponentially stable if $\rho(C) < 1$ and there exist positive definite diagonal matrices $\widetilde{D} = diag\{d_1, d_2, \cdots, d_n\}$, $\Lambda_1 = diag\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$, $\Lambda_2 = diag\{\alpha_1, \alpha_2, \cdots, \alpha_n\}$, $\Gamma = diag\{\gamma_1, \gamma_2, \cdots, \gamma_n\}$, symmetric positive definite matrices $P, Q_1, \widetilde{Q}_1, Q_2, Q_3$, and arbitrary matrices $F_1, F_2, N_1, N_2, M_1, M_2, M_3, M_4, M_5$ of appropriate dimensions such that the following condition holds:

$$\begin{bmatrix} \widetilde{\Xi} & F^T & \widetilde{F}^T \\ * & -\frac{\tau_l^2}{4}Q_2 & 0 \\ * & * & -\frac{\tau_u^2}{4}Q_3 \end{bmatrix} < 0,$$

where $\widetilde{\Xi} = (\Xi_{ij})\ i, j = 1, 2, \cdots, 12$,

$$\Xi_{11} = (P - L^-HH^T\Lambda_1 + L^+HH^T\widetilde{D})(A + \tau_uN_1$$
$$+\tau_lN_2) + (A+\tau_uN_1+\tau_lN_2)^T(P - L^-HH^T\Lambda_1 + L^+HH^T\widetilde{D})^T,$$
$$+(\tau_l^2+\tau_u^2)H\Lambda_2(L^+ - L^-)H^T + 2\tau_lQ_1 + 2\tau_u\widetilde{Q}_1 - 2HL^-\Gamma L^+H^T,$$
$$\Xi_{14} = PB - L^-HH^T\Lambda_1B + L^+HH^T\widetilde{D}B + A^TM_5^T,$$
$$\Xi_{24} = M_3B - M_5^T$$
$$\Xi_{34} = H^T(\Lambda_1B - \widetilde{D}B) + M_1B + D^TM_5^T,$$
$$\Xi_{44} = M_5 + M_5^T, \Xi_{45} = B^TM_2^T + C^TM_5^T,$$
$$\Xi_{4,12} = W^TM_5^T,$$

the rests sub-matrices of $\widetilde{\Xi}$ are the same as Ξ.

Proof: Choosing $Q_4 = 0$ in Theorem 3.1, and notice that

$$2y(t-\tau(t))^T(t)M_5[-\dot{y}(t) + C\dot{y}(t-h(t))$$
$$+Ay(t) + By(t-\tau(t)) + Df(\sigma(t)) + Wp(t)] = 0,$$

one can easily obtains this result, which completes the proof. ■

Remark 3.4: In many cases, $\tau_l = 0$, which researched in [6], [7], [16], [17]. In this time, similar to the proof of Theorem 3.1, a simplified criterion can be derived as follows.

Corollary 3.1: For given scalars $\tau_u > 0, \tau > 0$, $L^- = diag(l_1^-, l_2^-, \cdots, l_n^-)$, $L^+ = diag(l_1^+, l_2^+, \cdots, l_n^+)$, system (1) is globally exponentially stable if $\rho(C) < 1$ and there exist positive definite diagonal matrices $\widetilde{D} = diag\{d_1, d_2, \cdots, d_n\}$, $\Lambda_1 = diag\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$, $\Lambda_2 = diag\{\alpha_1, \alpha_2, \cdots, \alpha_n\}$, $\Gamma = diag\{\gamma_1, \gamma_2, \cdots, \gamma_n\}$, symmetric positive definite matrices $P, \widetilde{Q}_1, Q_3, Q_4$, and arbitrary matrices $F_2, N_1, M_1, M_2, M_3, M_4$ of appropriate dimensions such that the following condition holds:

$$\begin{bmatrix} \Xi & \widetilde{F}^T \\ * & -\frac{\tau_u^2}{4}Q_3 \end{bmatrix} < 0,$$

where $\Xi = (\Xi_{ij})\ i, j = 1, 2, \cdots, 9$, and

$$\Xi_{11} = (P - L^-HH^T\Lambda_1 + L^+HH^T\widetilde{D})(A + \tau_uN_1)$$
$$+(A + \tau_uN_1)^T(P - L^-HH^T\Lambda_1 + L^+HH^T\widetilde{D})^T + Q_4,$$
$$+\tau_u^2H\Lambda_2(L^+ - L^-)H^T + 2\tau_u\widetilde{Q}_1 - 2HL^-\Gamma L^+H^T,$$
$$\Xi_{12} = A^TM_3^T,$$
$$\Xi_{13} = PD - L^-HH^T\Lambda_1D + L^+HH^T\widetilde{D}D$$
$$+(A + \tau_uN_1)^T(\Lambda_1 - \widetilde{D})H + H\Gamma(L^+ + L^-) + A_TM_1^T,$$
$$\Xi_{14} = PB - L^-HH^T\Lambda_1B + L^+HH^T\widetilde{D}B,$$
$$\Xi_{15} = PC - L^-HH^T\Lambda_1C + L^+HH^T\widetilde{D}C + A^TM_2^T,$$
$$\Xi_{16} = PN_1 - L^-HH^T\Lambda_1N_1 + L^+HH^T\widetilde{D}N_1 = \Xi_{1,8},$$
$$\Xi_{1,9} = PW - L^-HH^T\Lambda_1W$$
$$+L^+HH^T\widetilde{D}W + A^TM_4^T,$$
$$\Xi_{22} = \frac{\tau_u^2}{2}Q_3 - M_3 - M_3^T,\ \Xi_{23} = -M_1 + M_3D,$$
$$\Xi_{24} = M_3B,\ \Xi_{25} = -M_2^T + M_3C,$$
$$\Xi_{2,9} = M_3W - M_4^T,$$
$$\Xi_{33} = H^T(\Lambda_1D - \widetilde{D}D) + (\Lambda_1D - \widetilde{D}D)^TH$$
$$-2\Gamma + M_1D + M_1^TD^T,$$

$$\Xi_{34} = H^T(\Lambda_1 B - \widetilde{D}B) + M_1 B,$$
$$\Xi_{35} = H^T(\Lambda_1 C - \widetilde{D}C) + M_1 C + D^T M_2^T,$$
$$\Xi_{36} = H^T(-\Lambda_1 N_1 + \widetilde{D}N_1) = \Xi_{3,8},$$
$$\Xi_{3,9} = H^T(\Lambda_1 W + \widetilde{D}W) + M_1 W + D^T M_4^T,$$
$$\Xi_{44} = -(1-\tau)Q_4,\ \Xi_{45} = B^T M_2^T,\ \Xi_{4,9} = B^T M_4^T,$$
$$\Xi_{55} = M_2 C + C^T M_2^T,\ \Xi_{5,12} = M_2 W + C^T M_4^T,$$
$$\Xi_{66} = -\frac{1}{\tau_u}\widetilde{Q}_1,$$
$$\Xi_{77} = -[\frac{2}{\tau_u^2} H\Lambda_2(L^+ - L^-)H^T + \frac{2}{\tau_u^3}\widetilde{Q}_1],$$
$$\Xi_{8,8} = -\frac{1}{2\tau_u^2}Q_3 + F_2,\ \Xi_{9,9} = M_4 W + W^T M_4^T,$$
$$\widetilde{F} = [0,0,0,0,0,0,0,F_2,0].$$

IV. Numerical examples

In this section, two numerical examples will be presented to show the validity of the main result derived in this paper.

Example 4.1: In order to compare with previews results easily, consider the delayed system (1) with parameters given by [16], [17]

$$C = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}, A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix},$$
$$B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \Delta A = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix},$$
$$\Delta B = \begin{bmatrix} \partial_1 & 0 \\ 0 & \partial_2 \end{bmatrix}, D = \Delta D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Time-varying delay $h(t) = h$ is constant, ω, α_i and $\partial_i, (i = 1, 2)$ denote the uncertainties which satisfy $-1 < \omega < 1$, $-1.6 < \alpha_1 < 1$, $-0.05 < \alpha_2 < 0.05$, $-0.1 < \partial_1 < 0.1$, $-0.3 < \partial_2 < 0.3$. For $\tau_l = 0.5, \omega = \pm 0.1$, Yu and Lie ones [16] gave out the upper bounds of τ_u for different values of $\dot{h}(t)$. At the same time, they also gave out the upper bounds of τ_u for different ω when $\tau_l = 0, \dot{h}(t) = 0.1$. In Ref [17], Yin and Zhong improved the criteria obtained in [16], and gave out different comparisons results. However, these results obtained in [16], [17] require $h(t)$ must be differentiable and $\dot{h}(t)$ has upper bound. Compared with these previews results, our criteria established in this paper are $h(t)$ and $\dot{h}(t)$ independent. Additional, the maximum values of τ_u obtained in [17] are 1.172 and 0.991 when $\dot{h}(t) = 0$ and $\omega = 0$ respectively. By Theorem 3.1, for $\tau_l = 0.5, \omega = \pm 0.1$, we can get the maximum value of τ_u is 1.101. By Corollary 3.1, the maximum value of τ_u is 1.203 when $\tau_l = 0, \omega = 0$, this means that our method is less conservative and more effective than the existing ones.

Example 4.2: Consider the delayed system (1) with parameters given by [17], [21]

$$C = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, A = \begin{bmatrix} -2 & 0.5 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0.4 \\ 0.4 & -1 \end{bmatrix},$$
$$D = \begin{bmatrix} -0.5 \\ -0.75 \end{bmatrix} = 0, H = \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix} = 0,$$
$$\Delta A = \Delta B = \Delta D = 0,$$

For $\tau_l = 0$, the stable problem of this system was researched in [17], [21]. When $\dot{h}(t) = 0.2$, the maximum upper bound delay of τ_u obtained in [21] is 1.8344 , and the maximum upper bound delay of τ_u obtained in [17] is 1.8412. However, by using Corollary 3.1, we can get the maximum upper bound delay of τ_u is 1.9423. Additional, we do not require $\dot{h}(t)$ is bounded, it even can be non-differentiable. This implying that our criteria established in this paper are less conservative and more effective than the existing ones.

V. Conclusions

Combined with Lyapunov stable theory and double integral inequality, we researched a class of neutral Lur'e systems with interval time-varying delays. Different from previous work on this topic, the property of convex function is introduced to research the stability of Lur'e system, and a new Lyapunov functional including triple integral has been proposed to derive some less conservative delay-dependent stability criteria. Numerical examples show that the new criteria derived in this paper are less conservative than some previous results obtained in the references cited therein.

Acknowledgment

This research was supported by China Postdoctoral Science Foundation Grant (2012 M521718), and Soft Science Research Project in Guizhou Province ([2011]LKC2004).

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