

Almost periodicity in a harvesting Lotka-Volterra recurrent neural networks with time-varying delays

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Abstract—By using the theory of exponential dichotomy and Banach fixed point theorem, this paper is concerned with the problem of the existence and uniqueness of positive almost periodic solution in a delayed Lotka-Volterra recurrent neural networks with harvesting terms. To a certain extent, our work in this paper corrects some result in recent years. Finally, an example is given to illustrate the feasibility and effectiveness of the main result.

Keywords—positive almost periodic solution, Lotka-Volterra, neural networks, Banach fixed point theorem, harvesting

I. INTRODUCTION

THE Lotka-Volterra type neural networks, derived from conventional membrane dynamics of competing neurons, provide a mathematical basis for understanding neural selection mechanisms [1]. It was shown that the continuous-time recurrent neural networks can be embedded into Lotka-Volterra models by changing coordinates, which suggests that the existing techniques in the analysis of Lotka-Volterra systems can also be applied to recurrent neural networks [2]. In recent years, there are some papers concerning with the dynamic behaviours of Lotka-Volterra recurrent neural networks [1-4]. In [3], the convergence involving global exponential, or asymptotic, stability of the following Lotka-Volterra recurrent neural networks is discussed:

$$\begin{cases} \dot{x}_i(t) = x_i(t) \left[r_i - \sum_{j=1}^n a_{ij} x_j(t) - \sum_{j=1}^n b_{ij} x_j(t - \tau_{ij}(t)) \right], & t > 0, \\ x_i(t) = \phi_i(t) > 0, & \forall t \in [-\tau, 0], \end{cases} \quad (1)$$

where $x_i(t)$ denotes the state of neuron i th at time t . Real numbers a_{ij} and b_{ij} represent the synaptic connection weights from neuron j to neuron i at time t and $t - \tau_{ij}(t)$, respectively, and r_i denotes the external input. The variable delays $\tau_{ij}(t)$ for $i, j = 1, 2, \dots, n$ are non-negative functions satisfying $\tau_{ij}(t) \in [0, \tau]$ for $t \geq 0$, where τ is a constant.

By using the theory of exponential dichotomy and contraction mapping principle, many scholars increasingly have their eye on the existence and uniqueness of almost periodic solutions of all kinds of neural networks (e.g., Shunting inhibitory cellular neural networks [5], Hopfield neural networks [6], Cohen-Grossberg neural networks [7-8], etc) in the recent ten years. Also, Liu et al. [4] focused on studying the existence and uniqueness of positive almost periodic solution of the

networks extended from network (1):

$$\begin{cases} \dot{x}_i(t) = x_i(t) \left[r_i(t) - \sum_{j=1}^n a_{ij}(t) x_j(t) - \sum_{j=1}^n b_{ij}(t) x_j(t - \tau_{ij}(t)) \right], & t > 0, \\ x_i(t) = \phi_i(t) > 0, & \forall t \in [-\tau, 0]. \end{cases} \quad (2)$$

By using the theory of exponential dichotomy and contraction mapping principle, the authors obtained some sufficient conditions for the existence and uniqueness of almost periodic solution of system (2). Unfortunately, the work in [4] is not perfect (see Remark 3.1 in Section 3).

In many earlier studies, it has been shown that harvesting has a strong impact on dynamic evolution of a population, e.g., see [9-12]. So the study of the population dynamics with harvesting is becoming a very important subject in mathematical bio-economics. This paper is concerning with the almost periodic solution of the following delayed Lotka-Volterra recurrent neural networks with harvesting terms:

$$\begin{aligned} \dot{x}_i(t) = x_i(t) & \left[r_i(t) - \sum_{j=1}^n a_{ij}(t) x_j(t) - \sum_{j=1}^n b_{ij}(t) x_j(t - \tau_{ij}(t)) \right] - h_i(t), \end{aligned} \quad (3)$$

where $r_i(t) > 0$, $a_{ij}(t) > 0$, $b_{ij}(t) > 0$, $h_i(t) > 0$ and $\tau_{ij}(t) > 0$ are all almost periodic functions for each $i, j = 1, 2, \dots, n$, $h_i(t) > 0$ represent harvesting terms. The meanings of the parameters are the same as the corresponding ones mentioned in system (1). From the point of view of biology, we focus our discussion on the existence and uniqueness of positive almost periodic solution of system (3) by using the theory of exponential dichotomy and Banach fixed point theorem. When h_i ($i = 1, 2, \dots, n$) is small enough and close to zero, then system (3) is approximately equivalent to system (2). Therefore, our work in this paper corrects the defect in article [4] to a certain extent.

For any bounded function $f \in C(\mathbf{R})$, $f^+ = \sup_{s \in \mathbf{R}} f(s)$, $f^- = \inf_{s \in \mathbf{R}} f(s)$. We list some assumptions which will be used in this paper.

- (H₁) r_i , a_{ij} , b_{ij} and h_i are nonnegative almost periodic functions with $0 < h_i^- < r_i^+$, $i, j = 1, 2, \dots, n$.
- (H₂) There exist positive constants $\eta_i \in \left[\frac{r_i^+ h_i^+}{r_i^-}, \frac{(r_i^+)^2 h_i^+}{r_i^- h_i^-} \right)$ ($i = 1, 2, \dots, n$) such that

$$\sup_{s \in \mathbf{R}} \left\{ -r_i(s) + \sum_{j=1}^n 2a_{ij}(s) + \sum_{j=1}^n 2b_{ij}(s) \right\} < -\eta_i < 0,$$

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where $i = 1, 2, \dots, n$.

The organization of this paper is as follows. In Section 2, we give some basic definitions and necessary lemmas which will be used in later sections. In Section 3, by using Banach fixed point theorem, we obtain some sufficient conditions ensuring existence and uniqueness of positive almost periodic solution of system (3). Finally, an example is given to illustrate that the result of this paper is feasible.

II. PRELIMINARIES

Now, let us state the following definitions and lemmas, which will be useful in proving our main result.

Definition 2.1. ([13, 14]) $x \in C(\mathbf{R}, \mathbf{R}^n)$ is called almost periodic, if for any $\epsilon > 0$, it is possible to find a real number $l = l(\epsilon) > 0$, for any interval with length $l(\epsilon)$, there exists a number $\tau = \tau(\epsilon)$ in this interval such that $\|x(t+\tau) - x(t)\| < \epsilon, \forall t \in \mathbf{R}$. The collection of those functions is denoted by $AP(\mathbf{R}, \mathbf{R}^n)$.

Definition 2.2. ([13, 14]) Let $y \in C(\mathbf{R}, \mathbf{R}^n)$ and $P(t)$ be a $n \times n$ continuous matrix defined on \mathbf{R} . The linear system

$$\dot{y}(t) = P(t)y(t)$$

is said to be an exponential dichotomy on \mathbf{R} if there exist constants $k, \lambda > 0$, projection S and the fundamental matrix $Y(t)$ satisfying

$$\|Y(t)SY^{-1}(s)\| \leq ke^{-\lambda(t-s)}, \quad \forall t \geq s,$$

$$\|Y(t)(I - S)Y^{-1}(s)\| \leq ke^{-\lambda(s-t)}, \quad \forall t \leq s.$$

Lemma 2.1. ([13, 14]) If the linear system $\dot{y}(t) = P(t)y(t)$ has an exponential dichotomy, then almost periodic system

$$\dot{y}(t) = P(t)y(t) + g(t)$$

has a unique almost periodic solution $y(t)$ which can be expressed as follows:

$$y(t) = \int_{-\infty}^t Y(t)SY^{-1}(s)g(s) ds - \int_t^{+\infty} Y(t)(I - S)Y^{-1}(s)g(s) ds.$$

Lemma 2.2. ([14, 15]) Let $a, b \in AP(\mathbf{R}, \mathbf{R})$. If

$$M(a) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T a(s) ds \neq 0,$$

then $\dot{y}(t) = a(t)y(t) + b(t)$ exists a unique almost periodic solution $y(t)$ can be written as follows

$$y(t) = \begin{cases} \int_{-\infty}^t e^{\int_s^t a(u) du} b(s) ds, & m(a) < 0, \\ -\int_t^{+\infty} e^{\int_s^t a(u) du} b(s) ds, & m(a) > 0. \end{cases}$$

Lemma 2.3. (Banach fixed point theorem [16]) Assume that (\mathbf{B}, ρ) is a complete metric space, $T : (\mathbf{B}, \rho) \rightarrow (\mathbf{B}, \rho)$ is a contraction mapping, i.e., there exists $\lambda \in (0, 1)$, such that

$$\rho(Tx, Ty) \leq \lambda \rho(x, y), \quad \forall x, y \in \mathbf{B}.$$

Then T has a unique fixed point in \mathbf{B} .

III. EXISTENCE AND UNIQUENESS OF POSITIVE ALMOST PERIODIC SOLUTION

In this section, we study the existence and uniqueness of almost periodic solution of system (3) by using Banach fixed point theorem.

Let

$$k_i := \frac{h_i^-}{r_i^+}, \quad l_i := \frac{r_i^+ h_i^+}{r_i^- \eta_i}, \quad i = 1, 2, \dots, n.$$

By (H_2) , it is easy to see that $k_i < l_i \leq 1, i = 1, 2, \dots, n$. Set

$$\mathbf{B} = \left\{ x = (x_1, x_2, \dots, x_n)^T \in AP(\mathbf{R}, \mathbf{R}^n) : k_i \leq x_i(t) \leq l_i, \forall t \in \mathbf{R}, i = 1, 2, \dots, n \right\}$$

with the distance $\rho(x, y)$ from x to y is defined by

$$\rho(x, y) = \max_{1 \leq i \leq n} \sup_{t \in \mathbf{R}} |x_i(t) - y_i(t)|,$$

where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T, y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T \in \mathbf{B}$. Obviously, (\mathbf{B}, ρ) is a complete metric space.

Theorem 3.1. Assume that (H_1) - (H_2) hold, then system (3) has a unique almost periodic solution in \mathbf{B} .

Proof: For $\forall \varphi = (\varphi_1, \varphi_2, \dots, \varphi_n)^T \in \mathbf{B}$, we consider the almost periodic solution of nonlinear almost periodic differential

$$\dot{x}_i(t) = r_i(t)x_i(t) - \varphi_i(t) \left[\sum_{j=1}^n a_{1j}(t)\varphi_j(t) + \sum_{j=1}^n b_{ij}(t)\varphi_j(t - \tau_{ij}(t)) \right] - h_i(t), \quad (4)$$

where $i = 1, 2, \dots, n$. Notice that $M(r_i) > 0, i = 1, 2, \dots, n$. Thus, by Lemma 2.2, we obtain that the system (4) has exactly one almost periodic solution:

$$x^\varphi(t) = (x_1^\varphi(t), x_2^\varphi(t), \dots, x_n^\varphi(t))^T,$$

where

$$x_i^\varphi(t) = \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \left\{ \varphi_i(s) \left[\sum_{j=1}^n a_{ij}(s)\varphi_j(s) + \sum_{j=1}^n b_{ij}(s)\varphi_j(s - \tau_{ij}(s)) \right] + h_i(s) \right\} ds,$$

in which $i = 1, 2, \dots, n$.

Now, we give a mapping T defined on \mathbf{B} by setting

$$T(\varphi) = (T_1(\varphi), T_2(\varphi), \dots, T_n(\varphi))^T = (x_1^\varphi, x_2^\varphi, \dots, x_n^\varphi)^T, \quad \forall \varphi \in \mathbf{B}.$$

First, we prove that the mapping T is a self-mapping from \mathbf{B} to \mathbf{B} . In fact, $\forall \varphi \in \mathbf{B}$, in view of definition of T , we have

$$T_i(\varphi)(t) \geq \int_t^{+\infty} e^{-\int_t^s r_i(u) du} h_i(s) ds \geq \frac{h_i^-}{r_i^+} = k_i, \quad \forall t \in \mathbf{R}, i = 1, 2, \dots, n. \quad (5)$$

On the other hand, it follows that

$$\begin{aligned}
 & T_i(\varphi)(t) \\
 = & \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \left\{ \varphi_i(s) \left[\sum_{j=1}^n a_{ij}(s) \varphi_j(s) \right. \right. \\
 & \left. \left. + \sum_{j=1}^n b_{ij}(s) \varphi_j(s - \tau_{ij}(s)) \right] + h_i(s) \right\} ds \\
 \leq & \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \left\{ l_i \left[\sum_{j=1}^n a_{ij}(s) l_j \right. \right. \\
 & \left. \left. + \sum_{j=1}^n b_{ij}(s) l_j \right] + h_i(s) \right\} ds \\
 \leq & \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \left\{ l_i \left[\sum_{j=1}^n a_{ij}(s) \right. \right. \\
 & \left. \left. + \sum_{j=1}^n b_{ij}(s) \right] + h_i(s) \right\} ds \\
 \leq & \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \left\{ l_i \left[\sum_{j=1}^n 2a_{ij}(s) \right. \right. \\
 & \left. \left. + \sum_{j=1}^n 2b_{ij}(s) \right] + h_i(s) \right\} ds \\
 \leq & \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \left\{ l_i [r_i(s) - \eta_i] + h_i(s) \right\} ds \\
 \leq & \int_t^{+\infty} \left[r_i(s) e^{-\int_t^s r_i(u) du} - \eta_i e^{r_i^+(s-t)} \right] l_i ds + \frac{h_i^+}{r_i^-} \\
 = & l_i \int_t^{+\infty} \left[r_i(s) e^{-\int_t^s r_i(u) du} \right] ds \\
 & - l_i \int_t^{+\infty} \eta_i e^{r_i^+(s-t)} ds + \frac{h_i^+}{r_i^-} \\
 = & -l_i \int_t^{+\infty} \frac{d(e^{-\int_t^s r_i(u) du})}{ds} ds \\
 & - l_i \int_t^{+\infty} \eta_i e^{r_i^+(s-t)} ds + \frac{h_i^+}{r_i^-} \\
 = & l_i - l_i \int_t^{+\infty} \eta_i e^{r_i^+(s-t)} ds + \frac{h_i^+}{r_i^-} \\
 \leq & (1 - \frac{\eta_i}{r_i^+}) l_i + \frac{h_i^+}{r_i^-} = l_i, \quad \forall t \in \mathbf{R}, i = 1, 2, \dots, n. \quad (6)
 \end{aligned}$$

By (5) and (6), we get that T is a self-mapping from \mathbf{B} to \mathbf{B} .

Next, we show that $T : \mathbf{B} \rightarrow \mathbf{B}$ is a contraction mapping. In fact, for $\forall \varphi, \psi \in \mathbf{B}$, we have

$$\begin{aligned}
 & T_i(\varphi) - T_i(\psi) \\
 = & \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \left\{ \varphi_i(s) \left[\sum_{j=1}^n a_{ij}(s) \varphi_j(s) \right. \right. \\
 & \left. \left. + \sum_{j=1}^n b_{ij}(s) \varphi_j(s - \tau_{ij}(s)) \right] + h_i(s) \right\} ds
 \end{aligned}$$

$$\begin{aligned}
 & - \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \left\{ \psi_i(s) \left[\sum_{j=1}^n a_{ij}(s) \psi_j(s) \right. \right. \\
 & \left. \left. + \sum_{j=1}^n b_{ij}(s) \psi_j(s - \tau_{ij}(s)) \right] + h_i(s) \right\} ds,
 \end{aligned}$$

where $i = 1, 2, \dots, n$. Under definition of $\rho(x, y)$, we have

$$\begin{aligned}
 & |T_i(\varphi) - T_i(\psi)|_0 \\
 := & \sup_{t \in \mathbf{R}} |T_i(\varphi)(t) - T_i(\psi)(t)| \\
 = & \sup_{t \in \mathbf{R}} \left| \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \left\{ \varphi_i(s) \left[\sum_{j=1}^n a_{ij}(s) \varphi_j(s) \right. \right. \right. \\
 & \left. \left. + \sum_{j=1}^n b_{ij}(s) \varphi_j(s - \tau_{ij}(s)) \right] + h_i(s) \right\} ds \\
 & - \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \left\{ \psi_i(s) \left[\sum_{j=1}^n a_{ij}(s) \psi_j(s) \right. \right. \\
 & \left. \left. + \sum_{j=1}^n b_{ij}(s) \psi_j(s - \tau_{ij}(s)) \right] + h_i(s) \right\} ds \Big| \\
 \leq & \sup_{t \in \mathbf{R}} \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \\
 & \times \left\{ \sum_{j=1}^n a_{ij}(s) \left| \varphi_i(s) \varphi_j(s) - \psi_i(s) \psi_j(s) \right| \right. \\
 & \left. + \sum_{j=1}^n b_{ij}(s) \left| \varphi_i(s) \varphi_j(s - \tau_{ij}(s)) \right. \right. \\
 & \left. \left. - \psi_i(s) \psi_j(s - \tau_{ij}(s)) \right| \right\} ds \\
 = & \sup_{t \in \mathbf{R}} \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \\
 & \times \left\{ \sum_{j=1}^n a_{ij}(s) \left| \left(\varphi_i(s) - \psi_i(s) \right) \varphi_j(s) \right. \right. \\
 & \left. \left. + \psi_i(s) \left(\varphi_j(s) - \psi_j(s) \right) \right| \right. \\
 & \left. + \sum_{j=1}^n b_{ij}(s) \left| \left(\varphi_i(s) - \psi_i(s) \right) \varphi_j(s - \tau_{ij}(s)) \right. \right. \\
 & \left. \left. + \psi_i(s) \left(\psi_j(s - \tau_{ij}(s)) - \varphi_j(s - \tau_{ij}(s)) \right) \right| \right\} ds \\
 \leq & \sup_{t \in \mathbf{R}} \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \times \left\{ \sum_{j=1}^n a_{ij}(s) \left| \varphi_j(s) + \psi_i(s) \right| \right. \\
 & \left. + \sum_{j=1}^n b_{ij}(s) \left| \varphi_j(s - \tau_{ij}(s)) + \psi_i(s) \right| \right\} ds \cdot \rho(\varphi, \psi) \\
 \leq & \sup_{t \in \mathbf{R}} \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \times \left\{ \sum_{j=1}^n a_{ij}(s) \right. \\
 & \left. + \sum_{j=1}^n b_{ij}(s) \right\} (l_j + l_i) ds \cdot \rho(\varphi, \psi) \\
 \leq & \sup_{t \in \mathbf{R}} \int_t^{+\infty} e^{-\int_t^s r_i(u) du} \left\{ \sum_{j=1}^n 2a_{ij}(s) \right.
 \end{aligned}$$

$$\begin{aligned}
 & + \sum_{j=1}^n 2b_{ij}(s) \Big\} ds \cdot \rho(\varphi, \psi) \\
 \leq & \sup_{t \in \mathbf{R}} \int_t^{+\infty} e^{-\int_t^s r_i(u) du} [r_i(s) - \eta_i] ds \cdot \rho(\varphi, \psi) \\
 \leq & \sup_{t \in \mathbf{R}} \left(\int_t^{+\infty} r_i(s) e^{-\int_t^s r_i(u) du} ds \right. \\
 & \left. - \int_t^{+\infty} e^{r_i^+(s-t)} \eta_i ds \right) \cdot \rho(\varphi, \psi) \\
 \leq & \left(1 - \frac{\eta_i}{r_i^+} \right) \rho(\varphi, \psi), \quad i = 1, 2, \dots, n. \tag{7}
 \end{aligned}$$

It follows from (7) that

$$\rho(T(\varphi), T(\psi)) \leq \max_{1 \leq i \leq n} \left\{ 1 - \frac{\eta_i}{r_i^+} \right\} \rho(\varphi, \psi) = \lambda \rho(\varphi, \psi),$$

where $\lambda = \max_{1 \leq i \leq n} \left\{ 1 - \frac{\eta_i}{r_i^+} \right\} \in [0, 1)$, which implies that the mapping $T : \mathbf{B} \rightarrow \mathbf{B}$ is a contraction mapping. Therefore, the mapping T possesses a unique fixed point

$$x^* = (x_1^*, x_2^*, \dots, x_n^*)^T \in \mathbf{B}, \quad Tx^* = x^*.$$

So system (3) has a unique almost periodic solution. This completes the proof. ■

Remark 3.1. In article [4], by using the contraction mapping principle, Liu et al. obtained that system (2) has a unique positive almost periodic solution in Ω , where

$$\Omega = \left\{ x = (x_1, x_2, \dots, x_n)^T \in AP(\mathbf{R}, \mathbf{R}^n) : \right. \\
 \left. x_i(t) \geq 0^+, \sum_{i=1}^n \rho_i x_i(t) \leq 1, \forall t \in \mathbf{R}, i = 1, 2, \dots, n \right\},$$

in which $\rho_i (i = 1, 2, \dots, n)$ are positive constants and 0^+ is defined as follows:

Definition 3.1. (See Definition 1 in [4]) Define 0^+ as a positive number, which is infinitely close to, yet not equal to, 0, and satisfying

$$0^+ = \alpha 0^+, \quad \forall |\alpha| \in (0, +\infty).$$

However, think carefully and we find that the number 0^+ defined by Definition 3.1 (i.e., Definition 1 of [4]) does not exist. Therefore, Ω defined in [4] is invalid and their work is not perfect.

Remark 3.2. When $h_i (i = 1, 2, \dots, n)$ is small enough and close to zero, then system (3) is approximately equivalent to system (2). Therefore, our work in this paper corrects the defect in article [4] to a certain extent.

IV. AN EXAMPLE

Example 4.1. Consider the following Lotka-Volterra recurrent neural networks with harvesting terms:

$$\begin{aligned}
 \dot{x}_i(t) = & x_i(t) \left[1 - \sum_{j=1}^2 a_{ij}(t) x_j(t) \right. \\
 & \left. - \sum_{j=1}^2 b_{ij}(t) x_j(t-1) \right] - 0.1. \tag{8}
 \end{aligned}$$

where $b_{ij}(s) = 0.1 \sin^2(\sqrt{3}s)$ ($i, j = 1, 2$) and

$$\begin{pmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{pmatrix} = 0.1 \begin{pmatrix} \sin^2(\sqrt{2}s) & \cos^2(\sqrt{3}s) \\ \cos^2(\sqrt{5}s) & \cos^2(\sqrt{7}s) \end{pmatrix}.$$

Then system (8) has a unique positive almost periodic solution.

Proof: Corresponding to system (3), $a_{ij}^+ = 0.1$, $b_i^+ = 0.1$, $r_i^- = 1$ and $h_i^+ = h_i^- = 0.1$, $i, j = 1, 2$. Taking $\eta_1 = \eta_2 = 0.2$. By a easy calculation, we obtain

$$\sup_{s \in \mathbf{R}} \left\{ -r_i(s) + \sum_{j=1}^2 2a_{ij}(s) + \sum_{j=1}^n 2b_{ij}(s) \right\} < -0.2 < 0,$$

where $i = 1, 2$, which implies that (H_2) in Theorem 3.1 holds. It is easy to verify that (H_1) in Theorem 3.1 is satisfied and the result follows from Theorem 3.1. This completes the proof. ■

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