World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:7, No:10, 2013

Design of Two-Channel Quadrature Mirror Filter Banks Using Digital All-Pass Filters

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***Abstract*—**The paper deals with the minimax design of two-channel linear-phase (LP) quadrature mirror filter (QMF) banks using infinite impulse response (IIR) digital all-pass filters (DAFs). Based on the theory of two-channel QMF banks using two IIR DAFs, the design problem is appropriately formulated to result in an appropriate Chebyshev approximation for the desired group delay responses of the IIR DAFs and the magnitude response of the low-pass analysis filter. Through a frequency sampling and iterative approximation method, the design problem can be solved by utilizing a weighted least squares approach. The resulting two-channel QMF banks can possess approximately LP response without magnitude distortion. Simulation results are presented for illustration and comparison.

***Keywords*—**Chebyshev approximation, Digital All-Pass Filter, Quadrature Mirror Filter, Weighted Least Squares.

I. Introduction

FOR many communication and signal processing systems, quadrature mirror filter banks have been widely used to achieve the goals of subband coding and short-time spectral analysis [1]-[4]. Generally, we use a QMF bank to decompose a signal into subbands and decimate the subband signals in the analysis system by an integer equal to the number of subbands. Moreover, two-channel QMF banks are usually used for constructing M-channel QMF banks based on a tree structure.

In the literature, several techniques have been presented for designing two-channel QMF banks with IIR analysis filters and approximately linear phase (LP) based on the least-squares (L_2) error criteria [5]-[9]. These IIR QMF banks are designed with the LP property imposed on the analysis filters. In contrast, a technique has been proposed in [10] for designing an IIR QMF bank with arbitrary group delay optimal in the minimax (L_∞) sense. Recently, the design results for IIR LP QMF banks based on real all-pass sections have been reported in [8], [9], [11]. The main advantage of using all-pass sections is that the designed IIR QMF banks can possess approximately LP response without magnitude distortion.

In this paper, we present a method based on the weighted least squares (WLS) algorithm [12] for the minimax design of two-channel LP QMF banks using real IIR digital all-pass filters (DAFs). The design problem is formulated by using the minimax error criteria on the phase approximation and the magnitude response of the low-pass analysis filter to obtain an appropriate objective function. The optimization of the objective function can be solved by utilizing the well-known WLS algorithm of [12] and a linear approximation scheme. The WLS solution provides the required increment for updating the filter coefficients during the iteration process. Simulation results showing the effectiveness of the proposed method are also provided.

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This work was supported by the National Science Council under Grant NSC100-2221-E002-200-MY3.

II. Problem Formulation

A. QMF Bank with Linear-Phase Response

Consider the two-channel filter bank with a system architecture shown in Fig. 1. $H_0(z)$ and $H_1(z)$ designate the low-pass and high-pass analysis filters, respectively, and $F_0(z)$ and $F_1(z)$ designate the low-pass and high-pass synthesis filters, respectively. Setting the synthesis filters $F_0(z) = H_1(-z)$ and $F_1(z) = -H_0(z)$ eliminates the aliasing term. As the mirror-image symmetry about the frequency $\omega = \pi/2$ exists between $H_0(z)$ and $H_1(z)$, we have $H_0(z) = H_1(-z)$. It has been shown in [11] that the input-output relationship in the Z-transform is given by

$$\hat{X}(z) = \frac{1}{2}[H_0^2(z) - H_0^2(-z)]X(z) \tag{1}$$

Let $T(e^{j\omega})$ denote the frequency response of the QMF bank. Equation (1) reveals that producing a reconstructed signal $\hat{x}(n)$ that is a delayed replica of $x(n)$ requires

$$T(e^{j\omega}) = H_0^2(e^{j\omega}) - H_0^2(e^{j(\omega+\pi)}) = e^{-jg_d\omega} \text{ for all } \omega \tag{2}$$

where g_d is the system delay of the QMF bank. This imposes constraints not only that $H_0(z)$ should be an ideal low-pass analysis filter, but also that its behavior for all ω should satisfy the condition given in (2). The designs of QMF banks using conventional FIR or IIR structures for $H_0(z)$ usually induce both magnitude and phase distortions.

B. Digital All-Pass Based QMF Bank

Here, we consider the two-channel QMF bank with analysis and synthesis structures shown by Figs. 2 and 3, respectively, where $A_1(z^2)$ and $A_2(z^2)$ are two real IIR DAFs. Hence, we have

$$H_0(z)=\frac{A_1(z^2)+z^{-1}A_2(z^2)}{2}$$

and

$$H_1(z)=\frac{A_1(z^2)-z^{-1}A_2(z^2)}{2} \tag{3}$$

Substituting (3) into (2) yields the frequency response of the QMF bank as follows:

$$T(e^{j\omega})=\frac{1}{2}e^{-j\omega}A_1(e^{j2\omega})A_2(e^{j2\omega}) \tag{4}$$

Equation (4) reveals that the resulting QMF bank possesses perfect magnitude response, i.e., there is no magnitude distortion. Moreover, $H_0(z)$ and $H_1(z)$ satisfy the all-pass complementary and power complementary properties. They are termed the doubly-complementary (DC) filter pair [13]. Therefore, the design problem is to find the real coefficients for the IIR DAFs $A_1(z^2)$ and $A_2(z^2)$ such that the resulting phase response $Arg\{T(e^{j\omega})\}$ of the DC-based QMF bank can approximate a desired phase characteristic in the minimax sense. The real IIR DAFs $A_1(z^2)$ and $A_2(z^2)$ with frequency responses are given by

$$A_1(e^{j2\omega})=e^{-j2N_1\omega}\frac{\sum_{n=0}^{N_1}a_1(n)e^{j2n\omega}}{\sum_{n=0}^{N_1}a_1(n)e^{-j2n\omega}}=e^{j\theta_1(\omega)}$$

and

$$A_2(e^{j2\omega})=e^{-j2N_2\omega}\frac{\sum_{n=0}^{N_2}a_2(n)e^{j2n\omega}}{\sum_{n=0}^{N_2}a_2(n)e^{-j2n\omega}}=e^{j\theta_2(\omega)} \tag{5}$$

respectively. Moreover, without loss of generality, both of the coefficients $a_1(0)$ and $a_2(0)$ can be set to one. Then, the phase responses $\theta_i(\omega)$, i = 1,2, are given by

$$\theta_i(\omega)=-2N_i\omega-2\phi_i(\omega)=-2N_i\omega-2\tan^{-1}\left\{\frac{-\sum_{n=1}^{N_i}a_i(n)\sin(2n\omega)}{1+\sum_{n=1}^{N_i}a_i(n)\cos(2n\omega)}\right\} \tag{6}$$

Substituting (6) into (3) yields

$$H_0(e^{j\omega})=\frac{1}{2}\left[e^{j\theta_1(\omega)}+e^{-j\omega}e^{j\theta_2(\omega)}\right]$$
$$=\exp\left(j\frac{\theta_1(\omega)+\theta_2(\omega)-\omega}{2}\right)\cos\left(\frac{\theta_1(\omega)-\theta_2(\omega)+\omega}{2}\right) \tag{7}$$

and

$$H_1(e^{j\omega})=\frac{1}{2}\left[e^{j\theta_1(\omega)}-e^{-j\omega}e^{j\theta_2(\omega)}\right]$$
$$=j\exp\left(j\frac{\theta_1(\omega)+\theta_2(\omega)-\omega}{2}\right)\sin\left(\frac{\theta_1(\omega)-\theta_2(\omega)+\omega}{2}\right)\cdot \tag{8}$$

In order to guarantee that $H_0(z)$ and $H_1(z)$ are LP low-pass and high-pass filters, respectively, we can impose the following conditions on $\theta_i(\omega)$, i = 1,2, based on (7) and (8):

Case (i): For $N_1 = N_2 = N$:

$$\begin{cases}\theta_1(\omega)=-2N\omega-\omega/2\\ \theta_2(\omega)=-2N\omega+\omega/2\end{cases},\ \text{for } 0\leqq\omega\leqq\omega_p,$$
$$\begin{cases}\theta_1(\omega)=-2N\omega-\omega/2+\pi/2\\ \theta_2(\omega)=-2N\omega+\omega/2-\pi/2\end{cases},\ \text{for } \omega_s\leqq\omega\leqq\pi. \tag{9}$$

Case (ii): For $N_1 = N_2 + 1$:

$$\begin{cases}\theta_1(\omega)=-2N_1\omega+\omega/2\\ \theta_2(\omega)=-2N_2\omega-\omega/2\end{cases},\ \text{for } 0\leqq\omega\leqq\omega_p,$$
$$\begin{cases}\theta_1(\omega)=-2N_1\omega+\omega/2-\pi/2\\ \theta_2(\omega)=-2N_2\omega-\omega/2+\pi/2\end{cases},\ \text{for } \omega_s\leqq\omega\leqq\pi, \tag{10}$$

where ω_p and ω_s are the passband and stopband edge frequencies of $H_0(z)$, respectively. Equations (9) and (10) reveal that the above conditions also satisfy the following stability constraints for the real IIR DAFs $A_i(z^2)$ [13]: $\theta_i(\omega)$ is monotonically decreasing and $\theta_i(\pi) = \theta_i(0) - 2N_i\pi$, for i = 1, 2. The frequency response of the DC-based QMF bank becomes

$$T(e^{j\omega})=\frac{1}{2}\exp[j(-\omega+\theta_1(\omega)+\theta_2(\omega))]$$
$$=\frac{1}{2}\exp[-j(2N_1+2N_2+1)\omega]\cdot \tag{11}$$

Equation (11) shows that the QMF bank possesses a LP with group delay $g_d = 2N_1 + 2N_2 + 1$ and without magnitude distortion. We formulate the design problem as follows:

i. From (4) and (11), we have a constraint on the group delays of the IIR DAFs $A_1(z^2)$ and $A_2(z^2)$ as follows:

$$\mathrm{GD}_1(\omega)+\mathrm{GD}_2(\omega)+1=g_d, \tag{12}$$

where

$$\mathrm{GD}_i(\omega)=-\frac{d}{d\omega}\theta_i(\omega)$$
$$=-2\left(\frac{\sum_{n=0}^{N_i}(n-N_i)a_i(n)e^{j2n\omega}}{\sum_{n=0}^{N_i}a_i(n)e^{j2n\omega}}+\frac{\sum_{n=0}^{N_i}na_i(n)e^{-j2n\omega}}{\sum_{n=0}^{N_i}a_i(n)e^{-j2n\omega}}\right) \tag{13}$$

for $\omega\in[0,\pi/2]$ and i = 1, 2.

ii. The magnitude of the low-pass analysis filter must be zero in $\omega\in[\pi/2,\pi]$, i.e.,

$$|H_0(e^{j\omega})| = \left|\frac{A_1(e^{j2\omega}) + e^{-j\omega}A_2(e^{j2\omega})}{2}\right| = \frac{1}{2}\left|\frac{\sum_{n=0}^{N_1} a_1(n)e^{j(n-N_1)2\omega}}{\sum_{n=0}^{N_1} a_1(n)e^{-jn2\omega}} + \frac{e^{-j\omega}\sum_{n=0}^{N_2} a_2(n)e^{j(n-N_2)2\omega}}{\sum_{n=0}^{N_2} a_2(n)e^{-jn2\omega}}\right| = 0 \quad (14)$$

The objective function based on the Chebyshev criteria can be formulated as follows:

$$\underset{\underline{a}}{Minimize} \left\|\mathrm{Aprx}_1(\boldsymbol{a},\omega)\right\|_{\infty,\omega\in[0,\frac{\pi}{2}]} + \alpha\left\|\mathrm{Aprx}_2(\boldsymbol{a},\omega)\right\|_{\infty,\omega\in[\omega_s,\pi]} \quad (15)$$

where $\|x\|_\infty$ denotes the Chebyshev norm of x and $\boldsymbol{a} = [\boldsymbol{a}_1^{\mathrm{T}}\ \boldsymbol{a}_2^{\mathrm{T}}]^{\mathrm{T}}$ with $\boldsymbol{a}_i = [a_i(1), a_i(2), \ldots, a_i(N_i)]^{\mathrm{T}}$ the filter coefficient vector.

$$\mathrm{Aprx}_1(\boldsymbol{a},\omega) = -2\left(\frac{\sum_{n=0}^{N_1}(n\text{-}N_1)a_1(n)e^{j2n\omega}}{\sum_{n=0}^{N_1}a_1(n)e^{j2n\omega}} + \frac{\sum_{n=0}^{N_1}na_1(n)e^{-j2n\omega}}{\sum_{n=0}^{N_1}a_1(n)e^{-j2n\omega}}\right),$$

$$-2\left(\frac{\sum_{n=0}^{N_2}(n\text{-}N_2)a_2(n)e^{j2n\omega}}{\sum_{n=0}^{N_2}a_2(n)e^{j2n\omega}} + \frac{\sum_{n=0}^{N_2}na_2(n)e^{-j2n\omega}}{\sum_{n=0}^{N_2}a_2(n)e^{-j2n\omega}}\right) + 1 - g_d$$

$$\mathrm{Aprx}_2(\boldsymbol{a},\omega) = \frac{1}{2}\left(\frac{\sum_{n=0}^{N_1}a_1(n)e^{j2(n-N_1)\omega}}{\sum_{n=0}^{N_1}a_1(n)e^{-j2n\omega}} + \frac{e^{-j\omega}\sum_{n=0}^{N_2}a_2(n)e^{-j2(n-N_2)\omega}}{\sum_{n=0}^{N_2}a_2(n)e^{-j2n\omega}}\right) \quad (16)$$

α is a preset relative weight between the two error terms.

III. Proposed Design Method

The design method is based on the WLS algorithm of [12] for solving the resulting minimization problem of (15). This is through a frequency sampling and iterative approximation scheme to find the optimal filter coefficients $a_i(n)$, $n = 1, 2, \ldots, N_i$, $i = 1,2$, for the real IIR DAFs shown by (5).

A. Frequency Sampling and Approximation Scheme

Let $\Omega_1 = [\omega_1 = 0, \omega_2, \ldots, \omega_L = \pi/2]$ and $\Omega_2 = [\omega_{L+1} = \omega_s, \omega_{L+2}, \ldots, \omega_M = \pi]$ represent the two dense grids of frequency bands in $[0, \pi]$. Each of them has grid points uniformly distributed in the individual frequency band. The design process of the proposed technique is then performed on $\Omega_d = \Omega_1 \bigcup \Omega_2$ with S = L+M grid points. If the number S of grid points is sufficiently large, the obtained best approximation solution of the objective function based on Ω_d will be close to the best solution found based on $\Omega = [0, \pi/2] \bigcup [\omega_s, \pi]$. This conclusion can be justified by the theorem due to Cheney [14, Chapter 3]. Next, we utilize a linearization scheme to approximate the related errors (15) due to a perturbation in the filter coefficient vector in the linear subspace spanned by the gradient matrix associated with $\mathrm{Aprx}_i(\boldsymbol{a}_k, \omega)$ at the kth iteration. As a result, the approximation for minimizing (15) can be formulated as finding the increments $\delta\boldsymbol{a}_k = \left[\delta\boldsymbol{a}_{1k}^{\mathrm{T}}\ \delta\boldsymbol{a}_{2k}^{\mathrm{T}}\right]^{\mathrm{T}}$ with $\delta\boldsymbol{a}_{ik} = [\delta a_{ik}(1)\ \delta a_{ik}(2) \ldots \delta a_{ik}(N_i)]^{\mathrm{T}}$ of the filter coefficient vectors $\boldsymbol{a}$ at the kth iteration such that

$$\|\mathrm{Aprx}_1(\boldsymbol{a}_k,\omega_l) + \delta\boldsymbol{a}_k^{\mathrm{T}}\nabla\,\mathrm{Aprx}_1(\boldsymbol{a}_k,\omega_l)\|_\infty + \alpha\|\mathrm{Aprx}_2(\boldsymbol{a}_k,\omega_l) + \delta\boldsymbol{a}_k^{\mathrm{T}}\nabla\,\mathrm{Aprx}_2(\boldsymbol{a}_k,\omega_l)\|_\infty \quad (17)$$

is minimized for $\omega_l \in \Omega_d$, where the subscript k denotes the kth iteration and the $(N_1 + N_2)\times 1$ gradient vector of $\mathrm{Aprx}_i(\boldsymbol{a}_k,\omega_l)$ is given by

$$\nabla\,\mathrm{Aprx}_i(\boldsymbol{a}_k,\omega_l) = [\,\psi_{i1}(\omega_l,1)\ \psi_{i1}(\omega_l,2) \ldots \psi_{i1}(\omega_l,N_1)\ \psi_{i2}(\omega,1) \ldots \psi_{i2}(\omega,N_2)\,]^{\mathrm{T}}, \quad (18)$$

where $\psi_{i1}(\omega_l, j) = \partial\mathrm{Aprx}_i(\boldsymbol{a}_k,\omega_l)/\partial a_{1k}(j)$ denotes the jth gradient component of $\mathrm{Aprx}_i(\boldsymbol{a}_k,\omega_l)$ and $\psi_{i2}(\omega_l, j) = \partial\mathrm{Aprx}_i(\boldsymbol{a}_k,\omega_l)/\partial a_{2k}(j)$ the $(N_1 + j)$th gradient component of $\mathrm{Aprx}_i(\boldsymbol{a}_k,\omega_l)$. For details, we rewrite (17) as follows:

$$\|\mathrm{Aprx}_1(\boldsymbol{a}_k,\omega_l) + \sum_{n=1}^{N_1}\delta a_{1k}(n)\psi_{11}(\omega_l,n) + \sum_{n=1}^{N_2}\delta a_{2k}(n)\psi_{12}(\omega_l,n)\|_\infty + \alpha\|\mathrm{Aprx}_2(\boldsymbol{a}_k,\omega_l) + \sum_{n=1}^{N_1}\delta a_{1k}(n)\psi_{21}(\omega_l,n) + \sum_{n=1}^{N_2}\delta a_{2k}(n)\psi_{22}(\omega_l,n)\|_\infty. \quad (19)$$

B. Minimax Design of QMF Using WLS Algorithm

We reformulate the design problem of minimizing (19) based on the WLS criteria as follows:

$$\underset{\delta\boldsymbol{a}_k}{Minimize}\ \boldsymbol{W}_1\left|\boldsymbol{U}_1\delta\boldsymbol{a}_k - \boldsymbol{d}_1\right|^2_{\omega_l\in\Omega_1} + \alpha\boldsymbol{W}_2\left|\boldsymbol{U}_2\delta\boldsymbol{a}_k - \boldsymbol{d}_2\right|^2_{\omega_l\in\Omega_2} \quad (20)$$

where $\boldsymbol{U}_i$ is a $S\times(N_1+N_2)$ matrix with the (l,n)th entry given by $U_i(l,n) = \psi_{i1}(\omega_l,n)$, $1 \le l \le S$, $1 \le n \le N_1$, and $U_i(l,n) = \psi_{i2}(\omega_l,n)$, $1 \le l \le S$, $1 \le n \le N_2$, $\boldsymbol{d}_i$ is a $S \times 1$ column vector with the lth entry given by $d_i(l) = -\mathrm{Aprx}_i(\boldsymbol{a}_k, \omega_l)$, $1 \le l \le S$, for $i = 1,2$. $\boldsymbol{W}_i = diag\{W_i(\omega_1), W_i(\omega_2), \ldots, W_i(\omega_S)\}$ denotes the $S \times S$ diagonal matrix containing the required least-squares weighting function calculated on the set $\Omega_d = \{\omega_1=0, \omega_2, \ldots, \omega_S=\pi\}$ of the S frequency grid points for $i = 1,2$. Clearly, the optimal solution for minimizing (20) is given by

$$\delta\boldsymbol{a}_k = \{\mathrm{real}\,(\boldsymbol{U}_1^{\mathrm{H}}\boldsymbol{W}_1\boldsymbol{U}_1 + \alpha\boldsymbol{U}_2^{\mathrm{H}}\boldsymbol{W}_2\boldsymbol{U}_2)\}^{-1}\times\{\mathrm{real}(\boldsymbol{U}_1^{\mathrm{H}}\boldsymbol{W}_1\boldsymbol{d}_1 + \alpha\boldsymbol{U}_2^{\mathrm{H}}\boldsymbol{W}_2\boldsymbol{d}_2)\}. \quad (21)$$

The suitable least-squares weighting function $W_i(\omega)$, $i = 1, 2$, required in (21) for a minimax design can be obtained by using the WLS algorithm presented in [12].

C. Iterative Design Procedure

Step 1.

<1.1> Determine the design parameters: the orders N_1 and N_2, the relative weight α, pass-band edge frequency ω_p and the stop-band edge frequency ω_s.

<*1.2*> Compute an initial guess $\boldsymbol{a}_{i0}$ for the filter coefficient vector $\boldsymbol{a}_i = [a_i(1), a_i(2), \ldots, a_i(N_i)]^T$, $i = 1, 2$, as described in [10]. Set the iteration number $k = 0$.

Step 2. Perform a test for stopping the iteration process:

<*2.1*> Compute

$$V_k = \|\mathrm{Aprx}_1(\boldsymbol{a}_k,\omega_l)\|_{\infty\ \omega_l \in \Omega_1} + \alpha\|\mathrm{Aprx}_2(\boldsymbol{a}_k,\omega_l)\|_{\infty\ \omega_l \in \Omega_2} \quad (22)$$

<*2.2*> Terminate the design process and take the coefficient vector $\boldsymbol{a}_k$ as the designed filter coefficients if $|V_k - V_{k-1}|/|V_{k-1}| \leqq \varepsilon$, where ε is a preset small positive real number. Otherwise, go to *Step 3* to perform an inner iterative process to find the best increment $\delta\boldsymbol{a}_{ik}$ of the filter coefficient vector $\boldsymbol{a}_{ik}$.

Step 3. Calculate the increment $\delta\boldsymbol{a}_{ik}$ of the filter coefficient vector $\boldsymbol{a}_{ik} = [a_{ik}(1), a_{ik}(2), \ldots, a_{ik}(N_i)]^T$ at the kth iteration based on (21) according to the WLS algorithm as follows:

<*3.1*> Set the initial weighting matrix $\boldsymbol{W}_i$ to the S × S identity matrix $\boldsymbol{I}$ and an iteration index $p = 0$.

<*3.2*> Compute the WLS solution $\delta\boldsymbol{a}_k$ from (21).

<*3.3*> Compute the error functions $e_{ik}(\omega) = |\boldsymbol{U}_i\delta\boldsymbol{a}_k - \boldsymbol{d}_i|$, $i = 1, 2$.

<*3.4*> If $|\max\{e_{ik}(\omega)\} - \max\{e_{ik-1}(\omega)\}| / \max\{e_{ik-1}(\omega)\} \leqq \eta_i$, where each η_i is a preset small positive real number, the WLS solution $\delta\boldsymbol{a}_k$ is used for obtaining the optimal solutions $\delta\boldsymbol{a}_{ik}$, This ends the inner iterative process. Then, go to *Step 4*. Otherwise, go to <*3.5*>.

<*3.5*> Update the least-squares weighting function $W_i(\omega)$ according to the systematical approach as described in [12] and set the iteration index $p = p + 1$. Then, go to <*3.2*>.

Step 4. Update the filter coefficient vector as follows:

<*4.1*> Use the obtained optimal solution $\delta\boldsymbol{a}_k$ to find the best increment such that

$$\|\mathrm{Aprx}_1(\boldsymbol{a}_k+\beta\delta\boldsymbol{a}_k,\omega_l)\|_{\infty\ \omega_l \in \Omega_1} + \alpha\|\mathrm{Aprx}_2(\boldsymbol{a}_k+\beta\delta\boldsymbol{a}_k,\omega_l)\|_{\infty\ \omega_l \in \Omega_2}, \ \forall \beta \geqq 0$$

is minimized.

<*4.2*> Employ the Nelder and Mead simplex algorithm [15] to perform the line search for finding the best value of β. Let the best value of β be β_k.

<*4.3*> We update the filter coefficient vector according to $\boldsymbol{a}_{(k+1)} = \boldsymbol{a}_k + \beta_k\delta\boldsymbol{a}_k$.

<*4.4*> Set $k = k + 1$ and go to *Step 2*.

IV. Simulation Results

The design results of using the proposed method are compared with the design results of [11] in terms of peak stop-band ripple of $H_0(z)$ (PSR), the maximal variation of pass-band group delay of $H_0(z)$ (MVPGD), the maximal variation of the group delay (MVGD) and maximum variation of the phase response (MVPR) of the designed filter bank $\hat{T}(e^{j\omega})$, and the maximal variation of the filter-bank response (MVFBR). They are defined as follows:

$$\mathrm{PSR} = 20\log_{10}\left(\max_{\omega_l \in [\omega_s,\pi]} |H_0(e^{j\omega_l})|\right) \ \text{(dB)}$$

$$\mathrm{MVPGD} = \max_{\omega_l \in [0,\omega_p]} \left| GD\{H_0(e^{j\omega_l})\} - (N_1 + N_2 + \frac{1}{2}) \right| \ \text{(sample)}$$

$$\mathrm{MVGD} = \max_{\omega_l \in [0,\pi]} \left| GD\{\hat{T}(e^{j\omega_l})\} - (2N_1 + 2N_2 + 1) \right| \ \text{(sample)}$$

$$\mathrm{MVPR} = \max_{\omega_l \in [0,\pi]} \left| Phase\{\hat{T}(e^{j\omega_l})\} + (2N_1 + 2N_2 + 1)\omega_l \right| \ \text{(radian)}$$

$$\mathrm{MVFBR} = \max_{\omega_l \in [0,\pi]} \left| \hat{T}(e^{j\omega_l}) - \frac{1}{2} e^{j(2N_1+2N_2+1)\omega_l} \right| \quad (23)$$

Example: We use the same specifications as those of [11] for this design: the real IIR DAFs $A_1(z)$ and $A_2(z)$ with orders N_1 and N_2 equal to 9 and 8, respectively, the low-pass analysis filter $H_0(z)$ with a passband edge frequency $\omega_p = 0.4\pi$ and a stopband edge frequency $\omega_s = 0.6\pi$. The spacing between two adjacent frequency grid points is set to $\pi/(8N_1+1) = \pi/73$. Moreover, the parameters $\varepsilon = 0.001$, $\eta_1 = \eta_2 = 0.00001$, and $\alpha = 300$. These parameters are selected by experiment. Table I lists the significant design results for comparison and Table II shows the resulting filter coefficients designed by using the proposed method. Figs. 4 ~ 9 plot the frequency responses associated with the design results of using the proposed method and the method of [11]. From the simulation results, we observe that the proposed method can provide much better phase response and more equiripple magnitude response of the QMF bank, though its PSR is about 4 dB higher than that of [11].

V. Conclusion

This paper has presented a method for the minimax design of two-channel linear-phase (LP) quadrature mirror filter (QMF) banks. The QMF bank is constructed by using infinite impulse response (IIR) digital all-pass filters (DAFs). Utilizing the theory of two-channel QMF banks with two IIR DAFs, the design problem is appropriately formulated in an appropriate Chebyshev approximation for the desired group delay responses of the IIR DAFs and the magnitude response of the low-pass analysis filter. As a result, the design problem can be solved by using a well-known weighted least squares algorithm. Simulation results have confirmed the effectiveness of the proposed method.

TABLE I
The Significant Design Results

	α=300	Method of [11]
PSR(dB)	- 50.6398	- 54.7222
MVPGD	0.0535	0.1359
MVPR	0.0093	0.1366
MVFBR(dB)	- 46.6620	- 23.3214
MVFBR	0.0046	0.0682
MVGD	0.1069	2.0120
Number of Iterations	5	4, 4

TABLE II
THE RESULTING FILTER COEFFICIENTS

n	$a_1(n)$	$a_2(n)$
0	1.000000000000000	1.000000000000000
1	0.241863193218369	-0.240227789564765
2	-0.078478266139255	0.136706003937396
3	0.035637365828341	-0.087478032820367
4	-0.016300710495321	0.056618413232281
5	0.006203534762817	-0.035675609568455
6	-0.001458305138989	0.020861719906423
7	-0.000920400535139	-0.011193882146515
8	0.001975443673669	0.006005740894830
9	-0.001261582024461	

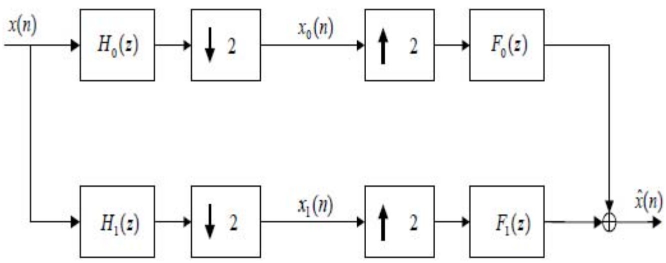

Fig. 1 The two-channel QMF bank

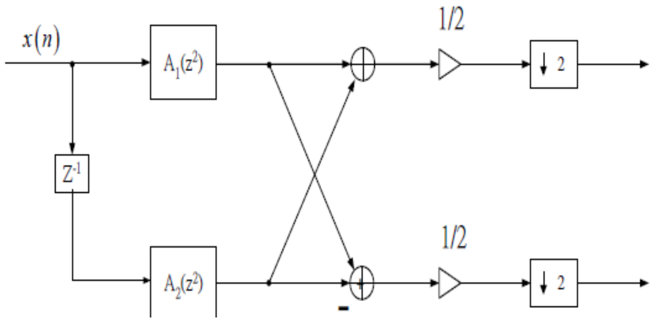

Fig. 2 The analysis system of the DC-based QMF bank

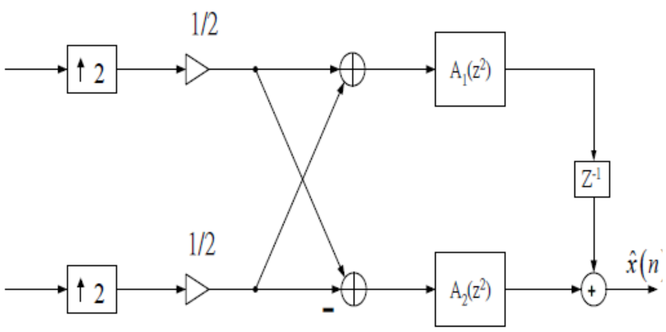

Fig. 3 The synthesis system of the DC-based QMF bank

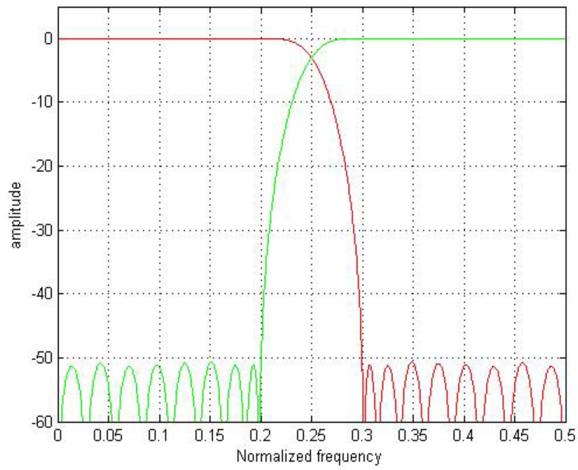

Fig. 4 The magnitude responses of H_0(z) (Low-pass Response Curve) and H_1(z) (High-pass Response Curve) using the proposed method

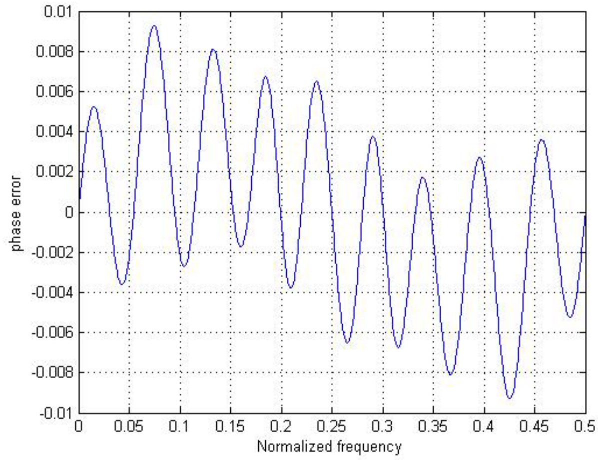

Fig. 5 Phase error of the filter bank using the proposed method

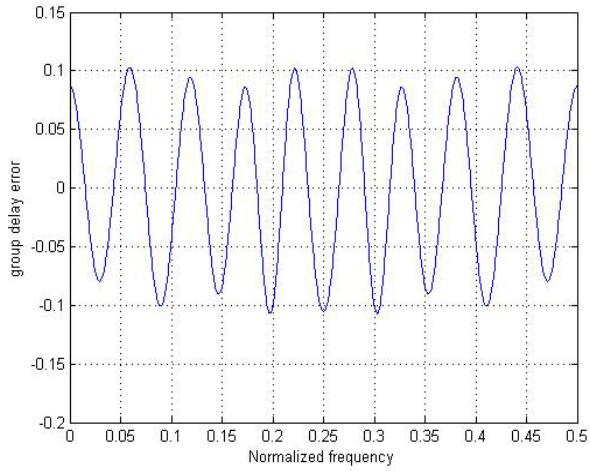

Fig. 6 Group delay error of the filter bank using the proposed method

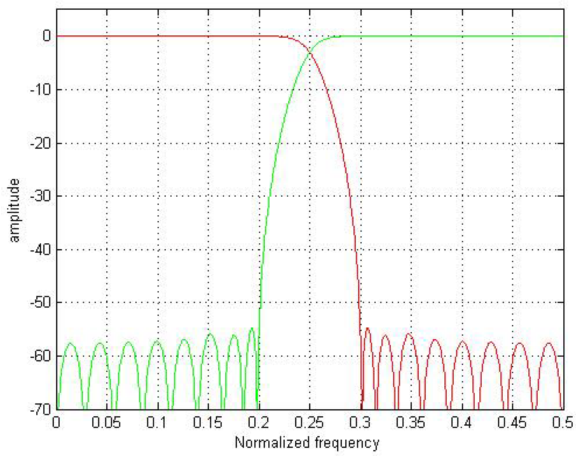

Fig. 7 The magnitude responses of $H_0(z)$ (Low-pass Response Curve) and $H_1(z)$ (High-pass Response Curve) using the method of [11]

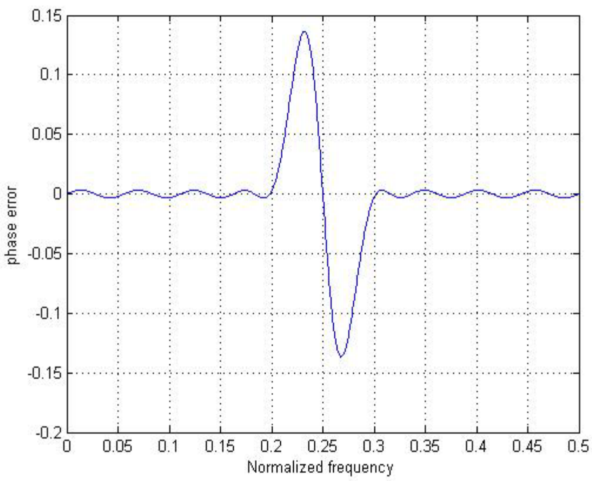

Fig. 8 Phase error of the filter bank using the method of [11]

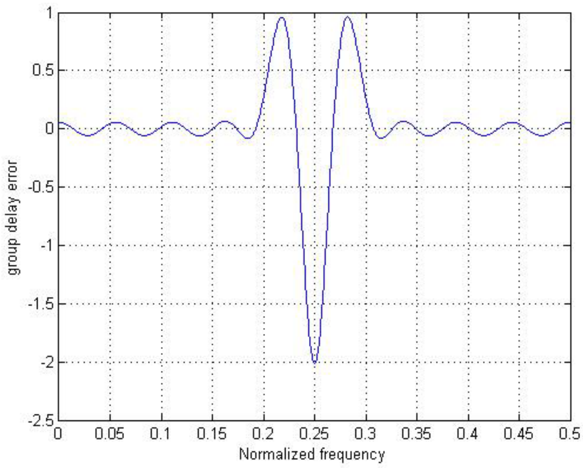

Fig. 9 Group delay error of the filter bank using the method of [11]

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