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Bi-linear Complementarity Problem

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Abstract—In this paper, we propose a new linear complementarity problem named as bi-linear complementarity problem (BLCP) and the method for solving BLCP. In addition, the algorithm for error estimation of BLCP is also given. Numerical experiments show that the algorithm is efficient.

Keywords—Bi-linear complementarity problem, Linear complementarity problem, Extended linear complementarity problem, Error estimation, P-matrix, M-matrix.

I. Introduction

The linear complementarity problem is of interest in a wide range of applications such as, free boundary problems [1], a Nash-equilibrium in bimatrix games [2], the interval hull of linear systems of interval equations [3], contact problems with friction [4], optimal stopping in Markov chains [5], circuit simulation [6], linear and quadratic programming [7] and economies with institutional restrictions upon prices [8].

Different algorithms have been proposed for complementarity problem. The generalized linear complementarity problem has been presented and an algorithm has been found for all its solutions [9]. The extended linear complementarity problem has been solved by a smoothing Levenberg-Marquardt method in [10]. A smoothing damped Gauss-Newton method was exhibited for nonlinear complementarity problem [11]. Besides, there are some other papers contribute to the linear complementarity problem. Conjugate gradient method for solving the linear complementarity problem with S-matrix was also considered in [12]. An iterative method for linear complementarity problems with interval data has been given [13]. For effectiveness, an algorithm for linear complementarity problems with interval data has been presented to improve the precision of the solutions [14].

However, with the development of science technologies and their applications, the problem need to be more generalized and their results need to be more precise. In this paper, we present a new linear complementarity problem named as bi-linear complementarity problem (BLCP). In addition, the method for error estimation of BLCP is also given.

The rest of the paper is organized as follows. In Section 2, we review some notations [13], several definitions and an algorithm [14]. In Section 3, bi-linear complementarity problem is proposed and some properties of the bi-linear complementarity problem are also analyzed. In addition, the method for BLCP is also exhibited in the corollary. Section 4 gives the relations of LCP and BLCP, generalized linear complementarity problem and extended linear complementarity problem. The algorithm for the error estimation of BLCP is presented in Section 5. In Section 6, numerical experiments on three examples are reported to demonstrate the effectiveness of our algorithm. Finally, concluding remarks are given in Section 7.

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II. Preliminaries and notations

Denote that $\mathbb{IR}, \mathbb{IR}^n, \mathbb{IR}^{n\times n}$ are the sets of intervals, the set of interval vectors with n components, the set of $n \times n$ matrices with interval data, respectively. Interval vectors and interval matrices are vectors and matrices with interval entries, respectively. We write point intervals with brackets which we identify with the element being contained. We denote $[a] = [\underline{a}, \overline{a}]$ for $[a] \in \mathbb{IR}$, $[x] = [\underline{x}, \overline{x}] = ([x_i]) = ([\underline{x}_i, \overline{x}_i]) \in \mathbb{IR}^n$ for $\forall\ i \in \{1, \cdots, n\}$ and $[A] = [\underline{A}, \overline{A}] = ([a_{ij}]) = ([\underline{a}_{ij}, \overline{a}_{ij}]) \in \mathbb{IR}^{n\times n}$ for $\forall\ i, j \in \{1, \cdots, n\}$. (The notations can also be found in [13].)

Definition 1: [15] Let $A \in \mathbb{R}^{n\times n}$, if $\max\limits_{1\le i\le n} x_i(Mx)_i > 0$ for $\forall\ x \neq 0$, A is a P-matrix.

Definition 2: [15] Let $A \in \mathbb{R}^{n\times n}$, $a_{ij} \le 0$, for $\forall\ i \neq j$, then A is a Z-matrix.

Definition 3: [15] Let $A \in \mathbb{Z}^{n\times n}$, $A^{-1} \ge 0$, then A is an M-matrix.

Definition 4: [13] Internal matrix $[A] \in \mathbb{IR}^{n\times n}$ is called
(1) regular, if $\forall\ A \in [A]$ is nonsingular;
(2) an M-matrix, if $\forall\ A \in [A]$ is an M-matrix.

Definition 5: [16] Let $A = (a_{i,j}) \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$, the linear complementarity problem denoted as $LCP(A, b)$ is to find a vector $x \in \mathbb{R}^n$ such that

$$Ax - b \ge 0,\ x \ge 0,\ x^{\mathrm{T}}(Ax - b) = 0, \qquad (1)$$

or conclude that there is no such x exist.

In the following part, we introduce an algorithm with monotonicity for solving LCP.

Let $N = 1, 2, \cdots, n$. We define the index set of x as

$$J_k = \{i \in N | (x_n)_i > 0\}, \quad for \quad \forall\ n \in \{0, \cdots, t-1\}.$$

Suppose that x^* is the solution of $LCP(A, b)$, and a_1^{T}, a_2^{T}, $\cdots$, a_n^{T} are the rows of A.

In [12], Li et al. gave a way constructed a finite sequence of index set $\{J_k\}_{k=0}^t$ satisfies

$$J_0 \subset J_1 \subset \cdots \subset J_{t-1}.$$

Evidently, we can convert the $LCP(A, b)$ into the system of linear equation. The set J_{k+1} depends on the solution of lower-dimension system of linear equations

$$\begin{cases} a_i^{\mathrm{T}} x - b_i = 0, & i \in J_k, \\ x_i = 0, & i \in I_k. \end{cases}$$

Then they got Algorithm 1 with monotonicity to solve $LCP(A, b)$. The monotonicity of Algorithm 1 has been proved in [14].

Algorithm 1: **Step 1** Let $J_0 = \{i|b_i > 0\}$, $I_0 = N/J_0$, $k = 0$. If J_0 is empty, $I_0 = N = \{1, 2, \cdots, n\}$.

Step 2 Solving the linear equation $A_{J_kJ_k}x_{J_k} - b_{J_k} = 0$, we obtain $x^*_{J_k}$.

Step 3 Let $x^* = \begin{pmatrix} x^*_{J_k} \\ 0_{I_k} \end{pmatrix}$, $I_k = N \backslash J_k$, if $A_{J_kJ_k}x^*_{J_k} - b_{J_k} \geq 0$, then we get the solution of $LCP(A, b)$ is x^*. Otherwise, go to Step 4.

Step 4 Let $J_{k_0} = \{i| \sum_{i \in I_k} a_{ij}x^*_j - b_i < 0\}$, and $J_{k+1} = J'_k \cup J_{k_0}$, $k = k + 1$, go to the Step 2.

III. Bi-linear complementarity problem

Definition 6: Let A, $C \in \mathbb{R}^{n\times n}$, b, $d \in \mathbb{R}^n$, the bi-linear complementarity problem, abbreviated as BLCP is to find a pair of vectors x, $y \in \mathbb{R}^n$, satisfies

$$\left\{ \begin{array}{l} x \geq 0, \quad y \geq 0, \\ Ax - b \geq 0, \quad Cy - d \geq 0, \\ x^{\mathrm{T}}y = 0, \end{array} \right. \tag{2}$$

or

$$\left\{ \begin{array}{l} x \geq 0, \quad y \geq 0, \\ Ax - b \geq 0, \quad Cy - d \geq 0, \\ (Ax - b)^{\mathrm{T}}(Cy - d) = 0, \end{array} \right. \tag{3}$$

or

$$\left\{ \begin{array}{l} x \geq 0, \quad y \geq 0, \\ Ax - b \geq 0, \quad Cy - d \geq 0, \\ x^{\mathrm{T}}(Cy - d) = 0, \end{array} \right. \tag{4}$$

or

$$\left\{ \begin{array}{l} x \geq 0, \quad y \geq 0, \\ Ax - b \geq 0, \quad Cy - d \geq 0, \\ (Ax - b)^{\mathrm{T}}y = 0, \end{array} \right. \tag{5}$$

we called them as bi-linear complementarity problem 1, bi-linear complementarity problem 2, bi-linear complementarity problem 3, bi-linear complementarity problem 4 and abbreviated as BLCP1, BLCP2, BLCP3, BLCP4, respectively.

Definition 7: Let A, $C \in \mathbb{R}^{n\times n}$, b, $d \in \mathbb{R}^n$, the simple bi-linear complementarity problem, abbreviated as $SBLCP$ is to find a pair of vectors x, $y \in \mathbb{R}^n$, satisfies

$$\left\{ \begin{array}{l} x \geq 0, \quad y \geq 0, \\ Ax - b \geq 0, \quad Cy - d \geq 0, \\ x^{\mathrm{T}}(Ax - b) = 0, \quad y^{\mathrm{T}}(Cy - d) = 0, \\ x^{\mathrm{T}}y = 0. \end{array} \right. \tag{6}$$

Definition 8: Let A, $C \in \mathbb{R}^{n\times n}$, b, $d \in \mathbb{R}^n$, the general simple bi-linear complementarity problem, abbreviated as GBLCP is to find a pair of vectors x, $y \in \mathbb{R}^n$, satisfies

$$\left\{ \begin{array}{l} x \geq 0, \quad y \geq 0, \\ Ax - b \geq (\leq)0, \quad Cy - d \geq (\leq)0, \\ x^{\mathrm{T}}y = 0. \end{array} \right. \tag{7}$$

Definition 9: Let A, $C \in \mathbb{R}^{n\times n}$, b, $d \in \mathbb{R}^n$, the extended linear complementarity problem, abbreviated as ELCP is to find a pair of vectors x, $y \in \mathbb{R}^n$, satisfies

$$\left\{ \begin{array}{l} x \geq 0, \quad y \geq 0, \\ Ax - Cy \geq b - d = e, \\ x^{\mathrm{T}}y = 0. \end{array} \right. \tag{8}$$

Theorem 1: Let A, $C \in \mathbb{R}^{n\times n}$ be P-matrices, two vectors $b, d \in \mathbb{R}^n$, $I^* = \{i \in N|x^* > 0\}$, $J^* = \{i \in N|y^* > 0\}$, x^*, y^* be the solutions of

$$\left\{ \begin{array}{l} x \geq 0, \\ Ax - b \geq 0, \\ x^{\mathrm{T}}(Ax - b) = 0, \end{array} \right. \tag{9}$$

and

$$\left\{ \begin{array}{l} y \geq 0, \\ Cy - d \geq 0, \\ y^{\mathrm{T}}(Cy - d) = 0, \end{array} \right. \tag{10}$$

we can obtain the following conclusions:

a)If

$$\left\{ \begin{array}{l} I^* \cap J^* = \emptyset, \\ I^* \cup J^* = N, \end{array} \right.$$

then SBLCP has only one solution.

b)If

$$\left\{ \begin{array}{l} I^* \cap J^* = \emptyset, \\ I^* \cup J^* \subset N, \end{array} \right.$$

then the solution of SBLCP is not unique.

c)If

$$I^* \cap J^* \neq \phi,$$

then SBLCP has no solution.

Proof: a) If SBLCP satisfies

$$\left\{ \begin{array}{l} I^* \cap J^* = \emptyset, \\ I^* \cup J^* = N, \end{array} \right.$$

then $x^{\mathrm{T}}y = 0$. x, y are the unique solutions of two linear complementarity problems. Thus, the SBLCP has only one solution.

b) Consider SBLCP satisfies

$$\left\{ \begin{array}{l} I^* \cap J^* = \emptyset, \\ I^* \cup J^* = N_1 \subset N. \end{array} \right.$$

If we chose the solution $x_i = a \neq 0$, $y_i = 0$ for $\forall\ i \in N_1,\ a \in \mathbb{R}$, x, y are still the solution of SBLCP. Then the solution of SBLCP is not unique.

c) Suppose that SBLCP satisfies

$$I^* \cap J^* \neq \emptyset,$$

it has $x^{\mathrm{T}}y \neq 0$. Then the solution of SBLCP is not exist. ■

Theorem 2: Let x^* be the solution of SBLCP, then x^* is the solution of BLCP1.

Proof: SBLCP is a special case of BLCP1. ■

Theorem 3: Let A, C be M-matrices, b, $d \in \mathbb{R}^n$, then BLCP1, BLCP2, BLCP3, BLCP4 are all equivalent.

Proof: First, we prove BLCP1 and BLCP4 are equivalent. From (2), BLCP1 has the following form

$$\left\{ \begin{array}{l} x \geq 0, \quad y \geq 0, \\ Ax - b \geq 0, \quad Cy - d \geq 0, \\ x^{\mathrm{T}}y = 0. \end{array} \right.$$

From the condition of this theorem, it has $A^{-1} \geq 0$, $C^{-1} \geq 0$. We take $\begin{cases} x_1 = Ax - b, \\ y_1 = Cy - d. \end{cases}$ into (2), it has

$$\begin{cases} x_1 \geq 0, \quad y_1 \geq 0, \\ A^{-1}x_1 + A^{-1}b \geq 0, \\ C^{-1}y_1 + C^{-1}d \geq 0, \\ (A^{-1}x_1 + A^{-1}b)^{\mathrm{T}}(C^{-1}y_1 + C^{-1}d) = 0. \end{cases}$$

If $A_1 = A^{-1}$, $b_1 = A^{-1}b$, $C_1 = C^{-1}$, $d_1 = C^{-1}d$, we get

$$\begin{cases} x_1 \geq 0, \quad y_1 \geq 0, \\ A_1x_1 + b_1 \geq 0, \quad C_1y_1 + d_1 \geq 0, \\ (A_1x_1 + b_1)^{\mathrm{T}}(C_1y_1 + d_1) = 0. \end{cases}$$

Then BLCP1 and BLCP4 are equivalent.

With the same method, BLCP1, BLCP2, BLCP3, BLCP4 are all equivalent. ■

Theorem 4: Let A, $C \in \mathbb{R}^{n\times n}$ be M-matrices,

$$I' = \{i | b_i < 0\}, \quad J' = \{i | d_i < 0\}, \quad \text{for} \quad \forall\, i \in N = \{1, \cdots, n\}.$$

If $I' = N$, $J' = N$, then SBLCP has only one solution

$$x, y = 0.$$

Theorem 5: Let $A \in \mathbb{R}^{n\times n}$ be an M-matrix, b, $d \in \mathbb{R}^n$, $q_1 \in \mathbb{R}$, $q_1 > 0$, vector $\varepsilon = q_1 \cdot \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$. If x^* is the solution of the linear complementarity problem

$$\begin{cases} x \geq 0, \\ Ax - b \geq 0, \\ x(Ax - b) = 0, \end{cases} \tag{11}$$

then $x_1 = x_* + \varepsilon$ is the solution of following linear system

$$\begin{cases} x \geq 0, \\ Ax - b \geq 0, \\ x(Ax - b) \geq 0. \end{cases} \tag{12}$$

Proof: From the above conditions, we get $\varepsilon > 0$. $x^* \geq 0$. Then $x^* + \varepsilon \geq 0$. For A is an M-matrix, A is main diagonal dominance, we obtain

$$A\varepsilon = A \cdot q_1 \cdot \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} > 0.$$

For x^* is the solution of (11), we get

$$A(x^* + \varepsilon) - b = Ax^* - b + A\varepsilon \geq 0.$$

Therefore, it has

$$x(Ax - b) \geq 0.$$

The conclusion of Theorem 5 is proved. ■

Theorem 6: Suppose that A is an M-matrix, a vector $b \in \mathbb{R}^n$, x'' is the solution of

$$\begin{cases} x \geq 0, \\ Ax - b \geq 0, \\ x(Ax - b) \geq 0, \end{cases} \tag{13}$$

x' is the solution of

$$\begin{cases} x \geq 0, \\ Ax - b \geq 0, \\ x(Ax - b) = 0, \end{cases} \tag{14}$$

then we have

$$\min x'' = x'.$$

Proof: Let $[X]$ be the set of the solutions in (13). From the above two complementarity problem, we obtain $x' \in [X]$. Suppose that $\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{pmatrix} \in \mathbb{R}^n$, where $\forall\, (\varepsilon)_i > 0$, $i \in N = \{1, \cdots n\}$, and

$$\hat{x}' = x' - \varepsilon,$$

is the solution satisfies (13).

Let $I = \{i | x'_i > 0\}$, $J = N \backslash I$. For $\forall\, i \in I$, we obtain

$$\sum_{i\in I} A_{ij}x'_j - b_i = 0.$$

According to $A\hat{x}' < Ax'$ which means that $\sum_{i\in I} A_{ij}\hat{x}'_j < \sum_{i\in I} A_{ij}x'_j$, then

$$\sum_{i\in I} A_{ij}\hat{x}'_j - b_i < 0. \tag{15}$$

For (15) is contracted with the condition of (13), we have x' is the minimum solution of (13). ■

Corollary 1: Let A, $C \in \mathbb{R}^{n\times n}$ be M-matrices, vector b, $d \in \mathbb{R}^n$. If x', y', x'', y'' are the solutions of BLCP1 and SBLCP, then $\min x' = x''$, $\min y' = y''$.

Corollary 2: If SBLCP has no solution, then LCP1 has no solution.

Proof: Suppose that x'', y'' are the solutions of BLCP1, SBLCP has no solutions, x', y' are the solutions of Algorithm 1 from SBLCP. Then $x'y' \neq 0$. From Theorem 5, it has $x'' \geq x'$, $y'' \geq y'$. Thus $x''y'' \neq 0$ which is conflict with BLCP1 has solutions. Then the conclusion of this corollary is obtained. ■

Corollary 3: Let x', y' are the solutions of SBLCP, then

$$x'' = \{x | x_i \geq x'_i \text{ for } \forall\, x'_i \neq 0, \quad x_i = 0 \text{ for } \forall\, x'_i = 0\},$$
$$y'' = \{y | y_i \geq y'_i \text{ for } \forall\, y'_i \neq 0, \quad y_i = 0 \text{ for } \forall\, y'_i = 0\},$$

are the solutions of BLCP1.

IV. Relations between LCP, BLCP1, GBLCP and ELCP

A. Relation between BLCP1 and LCP

$BLCP1$ is to find a pair of vectors x, $y \in \mathbb{R}^n$, when A, $B \in \mathbb{R}^{n\times n}$, $b, d \in \mathbb{R}^n$, satisfies

$$\begin{cases} x \geq 0, \quad y \geq 0, \\ Ax - b \geq 0, \quad Cy - d \geq 0, \\ x^{\mathrm{T}}y = 0. \end{cases}$$

BLCP1 is LCP when $d = 0$, $c = I$, $y = Ax - b$. Because LCP is to find a pair of vectors $x, y \in \mathbb{R}^n$, when A, $B \in \mathbb{R}^{n\times n}$, $b, d \in \mathbb{R}^n$, satisfies

$$\begin{cases} x \geq 0, \\ Ax - b \geq 0, \\ x^{\mathrm{T}}(Ax - b) = 0. \end{cases}$$

B. Relation between GBLCP and ELCP

GBLCP is to find a pair of vectors x, $y \in \mathbb{R}^n$, when A, $B \in \mathbb{R}^{n\times n}$, $b, d \in \mathbb{R}^n$, satisfies

$$\begin{cases} x \geq 0, \quad y \geq 0, & \\ Ax - b \geq 0, & (4.1) \\ Cy - d \leq 0, & (4.2) \\ x^{\mathrm{T}} y = 0. & \end{cases}$$

Consider (4.1)-(4.2), GCLCP is transformed into ELCP.

ELCP is to find a pair of vectors x, $y \in \mathbb{R}^n$, when A, $B \in \mathbb{R}^{n\times n}$, $b, d \in \mathbb{R}^n$, satisfies

$$\begin{cases} x \geq 0, \quad y \geq 0, & \\ Ax - Cy \geq b - d = e, & (4.1) - (4.2) \\ x^{\mathrm{T}} y = 0. & \end{cases}$$

V. Algorithm for error estimation of BLCP

Normally, it is difficult for us to find the precise data of A, C, b, and d, but they can be included in element-wise. So it is very useful for us to find the solutions of SBLCP and BLCP with interval data. Consider $[A]$, $[C] \in \mathbb{IR}^{n\times n}$ are M-matrices, $[b]$, $[d] \in \mathbb{R}^n$ and $I'_k = \{i \in N | \underline{x}^* > 0\}$, $J'_k = \{i \in N | \underline{y}^* > 0\}$, we can get the solutions of SBLCP. At the same time, the solutions of BLCP can also be obtained.

Algorithm 2: **Step 1** Let $J'_0 = \{i | \overline{b}_i > 0\}$, $k = 0$. If $J'_0 = \Phi$, go to Step 2.

Step 2 Solving the linear equations, $\underline{A}_{J'_k J'_k} x_{J'_k} - \overline{b}_{J'_k} = 0$, we obtain $x^*_{J'_k}$.

Step 3 Let $x^* = \begin{pmatrix} x^*_{J'_k} \\ 0_{I'_k} \end{pmatrix}$, $I_k = N \backslash J'_k$, if $\underline{A}_{J'_k J'_k} x^*_{J'_k} - \overline{b}_{J'_k} \geq 0$, then x^* is the solution of $LCP(A, b)$. Go to Step 5. Otherwise, go to Step 4.

Step 4 Let $J'_{k_0} = \{i | \sum\limits_{i \in J'_k} \underline{A}_{ij} x^*_j - \overline{b}_i < 0\}$, $J'_{k+1} = J'_k \cup J'_{k_0}$, $k = k + 1$, go to Step 2.

Step 5 $x^* = \overline{x}$. Let $\underline{A} = \overline{A}$, $\overline{b} = \underline{b}$, go to Step 1. We get x^* is $\underline{x}$. Then $[\underline{x}, \overline{x}]$ is the solution of the $LCP([A], [b])$.

Step 6 With the same method, we get $[\underline{y}, \overline{y}]$. From Theorem 1, we can obtain that the solution of $SBLCP$ $[\underline{x}, \overline{x}]$ and $[\underline{y}, \overline{y}]$ exist or not. Moreover, if $X = \{x | x > \underline{x}\}$, $Y = \{y | y > \underline{y}\}$ satisfy

$$\begin{cases} I^* \cap J^* = \emptyset, \\ I^* \cup J^* \subset N, \end{cases}$$

where

$$I_* = \{i \in N | x \geq 0\}, \quad J_* = \{i \in N | y \geq 0\},$$

then X, Y are the solutions of BLCP.

VI. Numerical experiments

In this section, the numerical results obtained with a Matlab 7.0.1 implementation on window XP with 2.39 GHz 64-bit processor. The matrices $[A]$, $[C]$ of the experiments are M-matrices throughout this section.

Example 1 In BLCP1, A, $C \in \mathbb{R}^{9\times 9}$,

$$A = \begin{pmatrix} 4 & -1 & & & \\ -1.1 & 4 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1.1 & 4 & -1 \\ & & & -1.1 & 4 \end{pmatrix}_{9\times 9},$$

$$b = \begin{pmatrix} 1 \\ 1 \\ -0.1 \\ -0.01 \\ -0.001 \\ -0.1 \\ \vdots \end{pmatrix}_{9\times 1}, \quad C = A, \quad d = b.$$

With Algorithm 1, we get the solution of two linear complementarity problem (9), (10)

$$x = y = (0.3410, 1.3638, 0.0803, 0.0210, 0.0055, 0, 0, 0, 0)^{\mathrm{T}}$$

after three iterations. Obviously, it has

$$I \cap J = \{1, 2, 3, 4, 5\} \neq \phi,$$

SBLCP has no solution. From Corollary 2, then BLCP1 has no solution.

Example 2. In BLCP1, A, $C \in \mathbb{R}^{9\times 9}$,

$$A = C = \begin{pmatrix} 4 & -1 & & & \\ -1.1 & 4 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1.1 & 4 & -1 \\ & & & -1.1 & 4 \end{pmatrix}_{9\times 9},$$

$$b = (1, 1, -0.1, -0.01, -0.001, -0.1, -0.1, -0.1, -0.1)^{\mathrm{T}}_{1\times 9},$$
$$d = (-0.1, -0.1, -0.1, -0.1, -0.1, -0.1, 1, -0.1, -0.01)^{\mathrm{T}}_{1\times 9}.$$

With Algorithm 1, the solutions of two linear complementarity problem (9), (10) are obtained where

$$x = (0.3410, 1.3638, 0.0803, 0.0210, 0.0055, 0, 0, 0, 0)^{\mathrm{T}},$$
$$y = (0, 0, 0, 0, 0, 0.0439, 0.2755, 0.0538, 0.0123)^{\mathrm{T}}.$$

It is obviously that

$$I \cap J = \phi, \quad I \cup J = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} = N.$$

According to Theorem 1, SBLCP in Example 2 has only one solutions. From Corollary 3, the solution of BLCP1 is

$$x = (x_1, x_2, x_3, x_4, x_5, 0, 0, 0, 0)^{\mathrm{T}},$$
$$y = (0, 0, 0, 0, 0, y_6, y_7, y_8, y_9)^{\mathrm{T}},$$

where

$$x_1 \geq 0.3410, x_2 \geq 1.3638, x_3 \geq 0.0803, x_4 \geq 0.0210,$$
$$x_5 \geq 0.0055, y_6 \geq 0.0439, y_7 \geq 0.2755, y_8 \geq 0.0538,$$
$$y_9 \geq 0.0123.$$

Example 3 In BLCP1, $[A]$, $[C] \in \mathbb{IR}^{9\times 9}$,

$$[A] = [C] =$$

$$\begin{pmatrix} [3.9,4] & [-1.1,-1] & & \\ [-1.2,-1.1] & [3.9,4] & [-1.1,-1] & \\ & \ddots & \ddots & \ddots \\ & & [-1.2,-1.1] & [3.9,4] \end{pmatrix}_{9\times 9},$$

$$[b] = \begin{pmatrix} [1,1.1] \\ [1,1.1] \\ [-1,-0.09] \\ [-0.009,-0.01] \\ [-0.001,-0.0009] \\ [-0.1,-0.09] \\ [-0.1,-0.09] \\ [-0.1,-0.09] \\ [-0.1,-0.09] \end{pmatrix}, [d] = \begin{pmatrix} [-0.1,-0.09] \\ [-0.1,-0.09] \\ [-0.1,-0.09] \\ [-0.1,-0.09] \\ [-0.1,-0.09] \\ [-0.1,-0.09] \\ [1,1.1] \\ [-0.1,-0.09] \\ [-0.01,0.009] \end{pmatrix}.$$

With Algorithm 2, we obtain

$$[x] = \begin{pmatrix} [0.3410,0.4068] \\ [0.3638,0.4422] \\ [0.0803,0.1240] \\ [0.0210,0.0392] \\ [0.0055,0.0118] \\ [0.0000,0.0000] \\ [0.0000,0.0000] \\ [0.0000,0.0000] \\ [0.0000,0.0000] \end{pmatrix}, [y] = \begin{pmatrix} [0.0000,0.0000] \\ [0.0000,0.0000] \\ [0.0000,0.0000] \\ [0.0000,0.0000] \\ [0.0000,0.0000] \\ [0.0439,0.0692] \\ [0.2755,0.3271] \\ [0.0538,0.0842] \\ [0.0123,0.0236] \end{pmatrix}.$$

Then $[x]$, $[y]$ are the error estimation of SBLCP. From Corollary 2, the solution of BLCP1 is

$$x = (x_1, x_2, x_3, x_4, x_5, 0, 0, 0, 0)^{\mathrm{T}},$$
$$y = (0, 0, 0, 0, 0, y_6, y_7, y_8, y_9)^{\mathrm{T}},$$

where

$$x_1 \geq [0.3410, 0.4068],\ x_2 \geq [0.3638, 0.4422],$$
$$x_3 \geq [0.0803, 0.1240],\ x_4 \geq [0.0210, 0.0392],$$
$$x_5 \geq [0.0055, 0.0118],\ y_6 \geq [0.0439, 0.0692],$$
$$y_7 \geq [0.2755, 0.3271],\ y_8 \geq [0.0538, 0.0842],$$
$$y_9 \geq [0.0123, 0.0236].$$

VII. Conclusion

In this paper, for generalizing the linear complementarity problem, we propose a new linear complementarity problem named as bi-linear complementarity problem (BLCP). Moreover, some properties of BLCP1 are analyzed and the method for solving BLCP1 is also presented. In addition, the algorithm for error estimation of BLCP1 is also given. Numerical experiments show that the method for BLCP1 and the algorithm for error estimation of BLCP1 are efficient.

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