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Fixture Layout Optimization Using Element Strain Energy and Genetic Algorithm

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***Abstract*—**The stiffness of the workpiece is very important to reduce the errors in manufacturing process. The high stiffness of the workpiece can be achieved by optimal positioning of fixture elements in the fixture. The minimization of the sum of the nodal deflection normal to the surface is used as objective function in previous research. The deflection in other direction has been neglected. The 3-2-1 fixturing principle is not valid for metal sheets due to its flexible nature. We propose a new fixture layout optimization method N-3-2-1 for metal sheets that uses the strain energy of the finite elements. This method combines the genetic algorithm and finite element analysis. The objective function in this method is to minimize the sum of all the element strain energy. By using the concept of element strain energy, the deformations in all the directions have been considered. Strain energy and stiffness are inversely proportional to each other. So, lower the value of strain energy, higher will be the stiffness. Two different kinds of case studies are presented. The case studies are solved for both objective functions; element strain energy and nodal deflection. The result are compared to verify the propose method.

***Keywords*—**Fixture layout, optimization, fixturing element, genetic algorithm.

I. INTRODUCTION

MANUFACTURING system consists of many important components and fixture is one of them. A fixture is used to hold and locate the workpiece in the desired orientation during the manufacturing process. The components that hold and locate the workpiece are called fixture elements. The arrangement of these fixture elements is very important to reduce the errors in manufacturing process. According to Prabhaharan et al. the position of the fixturing elements in the fixture is called fixture layout, and the layout, which minimizes the workpiece deformation is called optimal fixture layout [1].

The most usual optimization methods are mathematical programming approaches, penalty function methods, simulated annealing, genetic algorithm, and ant colony algorithm. Menassa et al. suggested a method to determine the position of the fixture supports in a fixture. The minimization of the workpiece deflection at specific points is the objective function. The FEA is used to calculate the deflection. Three numerical examples are presented to verify the method [2]. Meyet et al. used the dynamic conditions to synthesize a fixture. The layout optimization method is solved by linear programming. The minimum clamping force is used to achieve the minimum deflection of the workpiece that is the objective function of the problem [3]. Roy et al. presented a technique based on the qualitative and quantitative reasoning to find the optimal supporting, locating, and clamping positions [4]. Tao et al. presented a computational geometry approach for arbitrarily shaped workpieces. The feasible clamping region with all the possible clamping points is found automatically, and then optimal clamping points are chosen from a feasible clamping region. Case studies are presented to verify the method [5]. Li et al. presented a method to increase the workpiece location accuracy. The fixture-workpiece elastic contact model is used. The problem is solved by nonlinear programming method. The objective is to minimize the rigid body motion of the workpiece [6]. Liao et al. presented a technique for fixture layout optimization and analyzed the parameters affecting the fixturing stability. These parameters are the clamping force magnitude, the application sequence, and the placement of the fixturing clamps. The flexible workpiece deformation with clamping and machining loads is estimated under dynamic conditions [7]. Li et al. presented an approach for fixture layout and clamping force optimization. The workpiece dynamics is considered during machining. The minimization of maximum positional error at the machining point during machining is the objective. The results obtained by an iterative fixture layout and clamping force optimization are verified by simulations [8]. Tan et al. described an approach for the modeling, analysis and verification of optimal fixture design. The methods of force closure, optimization and finite element modeling (FEM) are used in this approach [9]. Amaral et al. developed a method to find the optimum support locations, using finite element analysis (FEA). He analyzed the deformation of the contact area between modular fixture and tool. The 3-2-1 principle is used to place the locators. The objective function is to minimize the maximum resultant deflection and assessing workpiece stability [10].

Most of the above studies are applied to the rigid bodies and use linear or nonlinear programming methods. The few studies applied to sheet metal parts are given. Cai et al. proposed N-2-1 locating principle to find the optimal location of the locators for deformable sheet metal parts. It uses the finite element analysis and nonlinear programming methods. The objective function is the sum of the square of the nodal deflections normal to the surface. The total deformation of the sheet metal is minimized in this way [11]. Li et al. proposed the first method to determine the optimal fixture configuration design

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for sheet metal assembly with laser welding. The number and the location of the locators have been considered first time in this method. A genetic algorithm is used to solve the problem. The effectiveness of the proposed method is verified by case study [12]. Li et al. developed a fixture configuration methodology based on a new proposed locating scheme for sheet metal laser welding. The case study of automotive assembly is investigated by applying the fixture configuration design method [13]. Cai developed a method for fixture optimization for sheet panel assembly. A fixture optimization model is formulated to minimize the assembly dimensional variations under welding gun variations. The method is verified by numerical examples [14]. Ma et al. proposed a new method for compliant fixture layout design using a topology optimization method. The objective function is to minimize the overall deformation of the workpiece. Both 2-D and 3-D numerical examples are presented to verify the effectiveness of the proposed approach [15]. Cheng et al. developed a fixture layout method to minimize the assembly variation of Aeronautical Thin-Walled Structures (ATWS). This approach uses a genetic algorithm and ant's algorithm (GAAA) to optimize the fixture layout [16]. Xiong et al. proposed a new fixture layout optimization method N-2-1-1 for flexible aerospace workpiece. The objective function of the optimization algorithm is to minimize the maximum elastic deformation at the machined point [17].

We propose a new fixture layout optimization method N-3-2-1 using the element strain energy and genetic algorithm. The propose method is verified by two different case studies. The method yields the best fixture optimal layout.

II. N-3-2-1 Fixturing Principle

A rigid body is fully constrained with minimum fixture elements by the 3-2-1 locating principle. This principle is the traditional principle for locating the prismatic shaped workpieces. According to this principle, 3, 2, 1 locators are enough to constrain the workpiece. The locating principle 3-2-1 constrains the rigid body motion (six degree of freedom).

Let us consider a central tunnel. We analyze the deformation under the self weight in 3-2-1 fixturing principle. The deformation results show that deformation of the plate is very high 8.57 mm under self weight as shown in Fig. 1. This high value of deformation produces the geometric errors in manufacturing process, which is not acceptable.

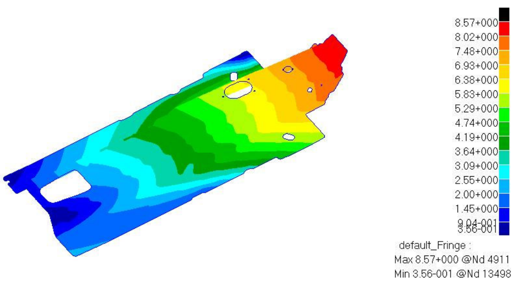


Fig. 1 Deformation in 3-2-1 Fixturing principle

Let us add one more locator in order to analyze the deformation of the plate under self weight. The deformation value obtained is 1.61mm as shown in Fig. 2. The deformation value in this fixturing principle 1-3-2-1 is 5.3 times less than the 3-2-1 fixturing principle, which seems to be acceptable to the manufacturing process. We can reduce this deformation more by adding more locators.

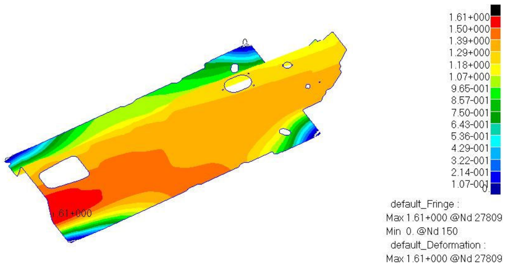


Fig. 2 Deformation in 1-3-2-1 Fixturing principle

The above discussion shows that 3-2-1 fixturing principle is not valid for metal sheet due to their flexible nature. So, more than 3 locators are required to reduce the deflection of the workpiece normal to the surface. When a force is applied to the metal sheet, like a drilling force or a resistance spot welding, the sheet deflects in direction normal to its surface.

We propose an N-3-2-1 locating principle and show that this principle is valid for large sheet metal parts due to their flexible nature. According to this principle, 2-1 locators are enough to constrain the sheet metal in the secondary and tertiary plane, but N+3 locators are required to constrain the metal sheet in the primary plane due to its flexible nature. The value of the N locators must be equal to or greater than 1. This number of locators depends on the geometry and dimensional specification of the workpiece.

N-3-2-1 fixturing principle satisfies the two conditions required for fixturing the workpiece.

1. It constrains the workpiece fully in six degree of freedom.
2. It also satisfies that, deformation of the workpiece is in the elastic range.

The arrangement of locators is very important because the success of this principle depends on it. This arrangement can be achieved by fixture layout optimization method.

III. Fixture Layout Optimization

A. Strain Energy

When a force is applied on the body, this force is directly proportional to the deflection within the elastic limit. This is called Hook's law. The area under the force and deflection curve is the work done. This work is stored in the body as strain energy.

The total strain energy of the body can be written as

$$U = \frac{1}{2}\int_{V} \{\sigma\}^{T} \{\varepsilon\} \, dV \tag{1}$$

where U is the strain energy, σ and ε is the stress and strain vectors and V is the total volume of the body.

Consider the 2D element on which different normal stresses σ_{xx} and σ_{yy} , and shear stresses τ_{xy} and τ_{yx} are acting. The strain energy of this 2D element is given by

$$U = \frac{1}{2}\left\{\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \tau_{xy}\gamma_{xy}\right\} dV \qquad (2)$$

Stiffness is the resistance of an elastic body to deformation by an applied force. This stiffness and strain energy are inversely proportional to each other. If the body has less strain energy values, then it will be stiffer than a body having high strain energy value.

The element strain energy values will be used as objective function in the fixture layout optimization proposed in this paper.

B. Problem Formulation

The fixture layout optimization problem may be defined as: finding the position of the locators, so that the stiffness of the workpiece is maximized. This stiffness is achieved in terms of strain energy of the workpiece.

The formulation of the fixture layout optimization problem is

$$\text{Minimize} \quad F = \sum_{i=1}^{n} u_i \qquad i = 1, 2, 3 \ldots n \qquad (3)$$

$$\text{Subject to} \quad \begin{array}{l} a_j \le x_j \le b_j \\ c_j \le y_j \le d_j \\ g_j \le z_j \le k_j \end{array} \qquad j = 1, 2, 3 \ldots m \qquad (4)$$

where,

F: objective function

u: strain energy of finite elements

n: number of finite elements

m: number of locators

a_i, b_i, c_i, d_i, g_j, k_j : limitation of locator in the x, y, z direction

The objective function of the optimization is defined as the sum of the strain energy of the finite elements. The fixture layout optimization problem is solved by genetic algorithm, which is one of the most efficient optimization algorithms. The design variables are the positions of the locators.

C. Fixture Layout Optimization Process Using Genetic Algorithm

Genetic algorithms are the evolutionary algorithms (EA) which use the techniques inspired by natural evolution. Genetic algorithm is different from traditional gradient based optimization techniques: (1) No gradient information is required, it requires only the fitness value (2) GA does not move sequentially from one point to the next one, but many new points are evaluated during the iteration. The GA convergence is controlled by few parameters: the population size (Ps), the probability of crossover (Pc) and the probability of mutation (Pm).

Genetic algorithm consists of the following steps:

Step 1. Random population of the design variables is generated.

Step 2. The finite element analysis is performed to calculate the fitness or objective function value. This value is passed for fitness evaluation.

Step 3. By using the FEA results, fitness evaluation is performed. Convergence is checked, if the problem is converged, process is terminated. If no, then it will go to the next step.

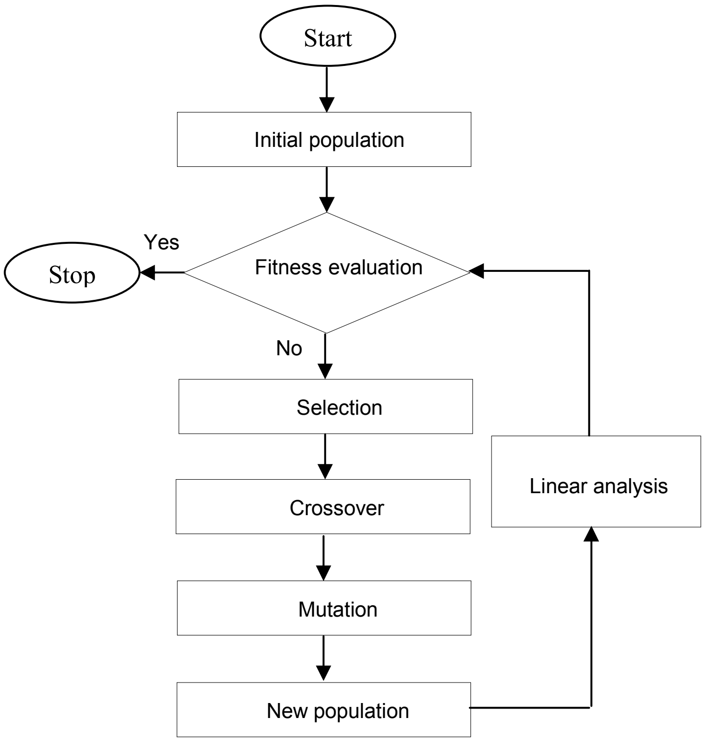


Fig. 3 Flow chart for fixture layout optimization

Step 4. Selection: The parents are selected by tournament selection to generate a new population.

Step 5. Crossover: It is the process of combining two chromosomes with their genetic material to produce a new offspring which have both their characteristics. Single point, two point, multipoint, and uniform crossover are possible. We will use the single point crossover. Single point crossover will create the cut line in the two parents, and combines the first part of the first parent to the second part of the second parent and vice versa to produce two offspring

Step 6. Mutation: Mutation changes one or more string values in a chromosome. The solution may change completely from the previous solution in mutation. Therefore better solution can be achieved by using mutation. The mutation probability defines how often the parts of the chromosome will be mutated. The mutation probability should be set low. If mutation probability value is set too high, the search will turn into a random search.

Step 7. New population has been generated by above process.

Step 8. Go to the step 2 with new population, and repeat the process until the convergence criteria is satisfied.

The population of randomly individual is generated. The fitness or objective function value of every individual in the population is evaluated. The fitness is usually the value of the objective function in the optimization problem. The fitness value is evaluated, if the fitness satisfies the convergence condition, the process is terminated, otherwise it will go for the next iteration. More individual is selected from the current population and mutated to form the new population. This new population is sent to calculate the fitness by finite element analysis and this fitness value is send for evaluation, and process is repeated until the problem is converged. The algorithm converges when number of generations reaches the maximum number of iterations.

IV. Fixture Layout Optimization Case Studies

Two different kinds of case studies are presented to verify the propose method. The preprocessing is done in Hypermesh, analysis in MSC NASTRAN, and optimization in Matlab. The genetic algorithm is used as optimization method.

The fixture layout optimization problem may be defined as: finding the position of the locators, so that the stiffness of the workpiece is maximized. This stiffness is achieved in terms of strain energy, because minimization of strain energy is equal to maximization of the stiffness.

The objective function of the propose method is the sum of the strain energy of the finite elements. The previous studies have the objective function as sum of the nodal deflections normal to the surface of the workpiece. The optimal layout of both the methods will be determined, and strain energy values for both optimal layouts will be calculated. The method with the less strain energy value will be preferred because the workpiece in this layout is stiffer.

It has been verified by FEA that four clamps are enough to constrain it in primary plane for each case study. So, the locating principle will be 1-3-2-1. Three different cases are solved for each case study. In case 1, two locators will be used as design locator, and each locator will be moved independently. In case 2, four locators will be used as design variable, and these four locators L4, L6, L5, and L7 will be moved in two pairs. In case 3, four locators are design variable, and these four locators will be moved together.

The material used for optimization is steel having young's modulus of elasticity 207 GPa and Poisson ratio 0.3. These material properties are used for all case studies. We named the previous method and propose method as A and B to simply describe the method

A. Sum of the nodal deflection normal to the surface as objective function

B. Sum of the element strain energy values as objective function

C. Case Study 1 – Plate Example

The workpiece used here is a sheet metal plate. The dimensions of the sheet metal are 800mm x 600mm x 1mm. The finite element model of the plate is shown in Fig. 4. The finite elements are QUAD4 element with size 10mm x 10mm. O (0, 0, 0) is the origin point. A force of magnitude 50 N is applied on the sheet at (300, 200, 0) in the Z-direction.

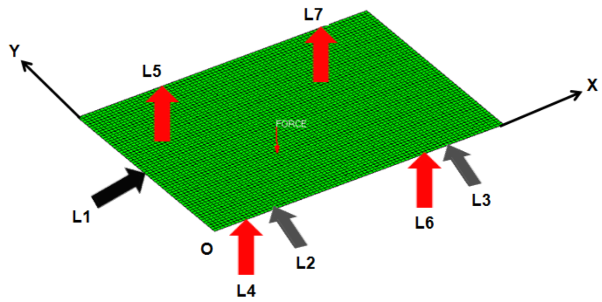


Fig. 4 Finite element model of plate example

TABLE I
Initial and Optimal Clamping Position of the Locators

Cases	Locator type	Locator	Initial position (mm)	Optimal position (mm)	
				A	B
All Cases	Fixed Locator	L1	(0,300,0)		
		L2	(100,0,0)		
		L3	(700,0,0)		
Case-1	Fixed Locator	L5	(200,600,0)		
		L6	(600,0,0)		
	Design Locator	L4	(0,0,0)	(150,0,0)	(150,0,0)
		L7	(800,600,0)	(530,600,0)	(500,600,0)
Case-2	Design Locator	L4	(0,0,0)	(120,0,0)	(150,0,0)
		L5	(0,600,0)	(120,600,0)	(150,600,0)
		L6	(800,0,0)	(510,0,0)	(500,0,0)
		L7	(800,600,0)	(510,600,0)	(500,600,0)
Case-3	Design Locator	L4	(0,0,0)	(200,0,0)	(230,0,0)
		L5	(0,600,0)	(200,600,0)	(230,600,0)
		L6	(800,0,0)	(600,0,0)	(570,0,0)
		L7	(800,600,0)	(600,600,0)	(570,600,0)

The fixed locators L1 constraint the workpiece in X and the locators L2 and L3 constrain the workpiece in the Y direction, while the other locators L4, L5, L6 and L7 constrain the workpiece in the Z direction. The clamping length of each design locator is 300mm along the long edge of the workpiece from the corner point of the workpiece. The two locators L4 and L6 move on the edge along the X axis and the other two locators L5 and L7 move on the other edge parallel to the X axis. The initial position of the locators with their optimal position obtained by fixture layout optimization is given in Table I for all cases.

D. Case Study 2 – Central Tunnel

The Central tunnel is the second case study to verify the propose method. The finite element model of the central tunnel is shown in Fig. 5. The finite elements are QUAD4 and TRIA3 elements. O (0,0,0) is the origin point. The force of magnitude 1000N is applied to the central tunnel in the Z-direction.

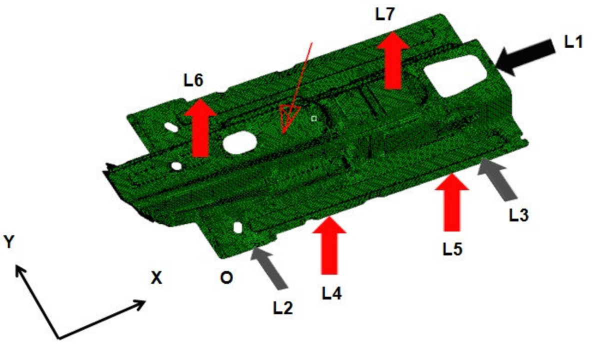


Fig. 5 Finite element model of central tunnel

The fixed locator L1 constraint the workpiece in X and the locators L2 and L3 constrain the workpiece in the Y direction, while the other locators L4, L5, L6 and L7 constrain the workpiece in the Z direction.

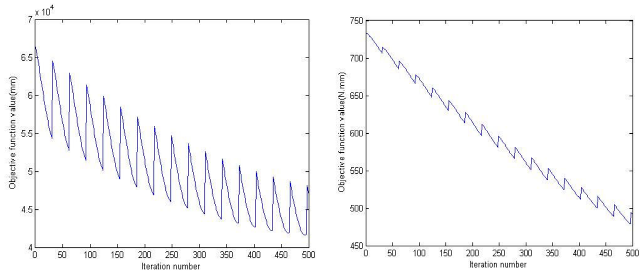
TABLE II
INITIAL AND OPTIMAL CLAMPING POSITION OF THE LOCATORS

Cases	Locator type	Locator	Initial position (mm)	Optimal position (mm)	
				A	B
All Cases	Fixed Locator	L1	(1145,289,0)		
		L2	(71,0,0)		
		L3	(1046,24,0)		
Case-1	Fixed Locator	L5	(897,24,0)		
		L6	(222,579,0)		
	Design Locator	L4	(0,0,0)	(132,0,0)	(122,0,0)
		L7	(1115,579,0)	(857,579,0)	(907,579,0)
Case-2	Design Locator	L4	(0,0,0)	(132,0,0)	(122,0,0)
		L5	(1115,24,0)	(984,24,0)	(984,24,0)
		L6	(0,604,0)	(132,579,0)	(122,604,0)
		L7	(1115,579,0)	(984,579,0)	(984,579,0)
Case-3	Design Locator	L4	(0,0,0)	(272,24,0)	(272,24,0)
		L5	(1115,24,0)	(843,24,0)	(843,24,0)
		L6	(0,604,0)	(272,579,0)	(272,579,0)
		L7	(1115,579,0)	(843,579,0)	(843,579,0)

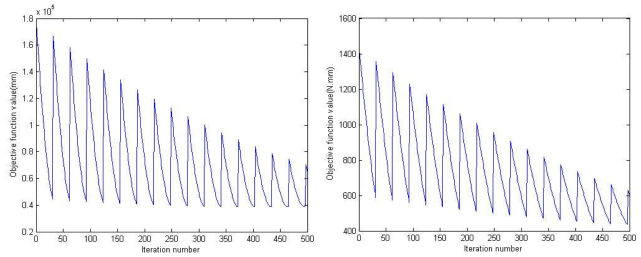
The clamping length of each design locator is 490mm along the long edge of the workpiece from the corner point of the workpiece. The two locators L4 and L5 move along the edge on X axis, while the other two locators L6 and L7 move along the edge parallel to the X axis. The initial position of the locators with their optimal position is given in Table II for all cases.

V. OPTIMIZATION RESULTS

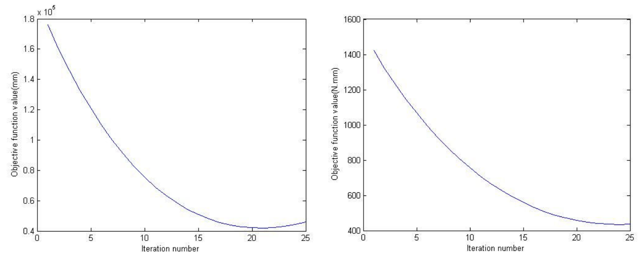
The fixture layout optimization is performed as shown in Fig. 3. The optimization process is started from the initial layout, and an optimum position is achieved in many iterations. The convergence of the objective function value from initial position to optimum position is shown in Figs. 6 and 7 for three cases of the both case studies.



(a) Case 1

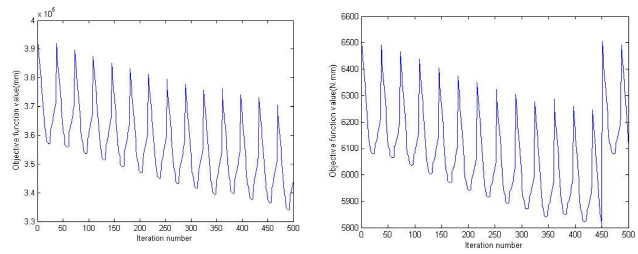


(b) Case 2

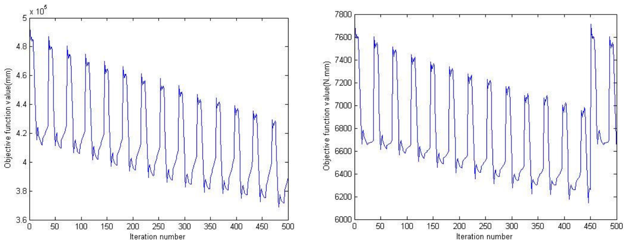


(c) Case 3

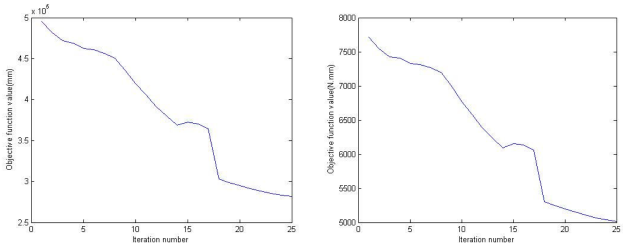
Fig. 6 Plate example convergence results



(a) Case 1



(b) Case 2



(c) Case 3

Fig. 7 Central tunnel convergence results

The fixture layout optimization is performed and different kinds of parameters are calculated like initial and optimal layout, objective function values, and the strain energy of the workpiece in the optimal layout are given in Table III in for both A and B method.

The optimum value of the objective function F is less than the objective function value of the initial position. The strain energy value of the propose method B is less than the strain energy values of the deflection method A. The strain energy and stiffness are the inversely proportional to each other. The lower value of the strain energy with the method B shows that workpiece is stiffer for the method B as compared to the method A. This reduces the errors in the manufacturing process. The convergence criterion is the maximum number of iterations.

VI. Conclusion

A new fixture layout optimization method N-3-2-1 for metal sheets is introduced, that uses the strain energy of the finite elements. This method combines the genetic algorithm and finite element analysis. The objective function is to minimize the sum of all the element strain energy. The deformations in all the directions have been considered by using the concept of element strain energy. Two different kinds of case studies are solved by using element strain energy and nodal deflection as objective functions. The less value of strain energy in the optimal layout of the propose method shows that workpiece is more stiff as compared to the method using nodal deflection as objective function.

TABLE III
Optimization Results of Two Case Studies

Sr. No.	Parameter	Case-1		Case-2		Case-3	
		A	B	A	B	A	B
Case Study 1	F (Initial)	6.63E +04	7.33 E+02	1.76 E+05	1.42 E+03	1.76 E+05	1.42 E+03
	F Optimum	4.16E +04	4.79 E+02	3.87 E+04	4.35 E+02	4.20 E+04	4.34 E+02
	No. of Iterations	500	500	500	500	25	25
	Strain energy	4.80E+ 02	4.79 E+02	4.61 E+02	4.35 E+02	4.46 E+02	4.34 E+02
Case Study 2	F (Initial)	3.92E +05	6.50 E+03	4.95 E+05	7.71 E+03	4.95 E+05	7.71 E+03
	F Optimum	3.33E +05	5.81 E+03	3.68 E+05	6.14 E+03	2.81 E+05	5.00 E+03
	No. of Iterations	500	500	500	500	25	25
	Strain energy	6.37E+ 03	5.81 E+03	6.67 E+03	6.14 E+03	5.00 E+03	5.00 E+03

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