

A 3 Dimensional Simulation of the Repeated Load Triaxial Test

Bao Thach Nguyen, Abbas Mohajerani

Abstract—A typical flexible pavement structure consists of the surface, base, sub-base and subgrade soil. The loading traffic is transferred from the top layer with higher stiffness to the layer below with less stiffness. Under normal traffic loading, the behaviour of flexible pavement is very complex and can be predicted by using the repeated load triaxial test equipment in the laboratory. However, the nature of the repeated load triaxial testing procedure is considered time-consuming, complicated and expensive, and it is a challenge to carry out as a routine test in the laboratory. Therefore, the current paper proposes a numerical approach to simulate the repeated load triaxial test by employing the discrete element method. A sample with particle size ranging from 2.36mm to 19.0mm was constructed. Material properties, which included normal stiffness, shear stiffness, coefficient of friction, maximum dry density and particle density, were used as the input for the simulation. The sample was then subjected to a combination of deviator and confining stress and it was found that the discrete element method is able to simulate the repeated load triaxial test in the laboratory.

Keywords—Discrete element method, repeated load triaxial, pavement materials.

I. INTRODUCTION

THE behavior of flexible pavement under the traffic loading conditions is very complex. In recent decades, extensive research work has been undertaken to characterize the behavior of flexible pavement [1]. Under moving wheel loads, overstressing the pavement material can produce unacceptable levels of pavement deflection, which, ultimately, affect the pavement performance during the service life [2]. Therefore, a better understanding of the behavior of flexible pavement under the traffic loading by laboratory tests, through which in-situ stress conditions are adequately considered is strongly required.

A. Repeated Load Triaxial Testing Equipment

The repeated load triaxial test is the most common testing method wherein the behavior of the pavement materials during repeated loading is evaluated [3]. Basically, the repeated load triaxial equipment consists of a loading frame that is powered by either a pneumatic or electro-hydraulic loading system. The apparatus can create a loading waveform in different shapes, such as haversine or rectangular. The schematic of a typical repeated load triaxial test apparatus can be seen in Fig. 1:

Bao Thach Nguyen and Abbas Mohajerani are with RMIT University, Melbourne, VIC 3012 Australia (phone: 613-9925-3847; fax: 613-9925-3921; e-mail: baothach.nguyen@rmit.edu.au, abbas.mohajerani@rmit.edu.au).

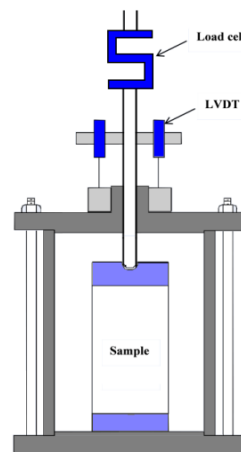


Fig. 1 The schematic of a typical repeated load triaxial test apparatus (modified from [4])

In the test, repeated cyclic axial stress is applied to a cylindrical test specimen. The diameter of the sample is five times larger than the maximum particle size of the tested sample. In addition, the height of the sample is twice that of the diameter for the soil sample having regular platens at both ends. The ratio of the specimen size can be reduced to 1:1 if the frictionless platens are used as reported by Adu-Osei [5]. They also found that the specimens were more stable and practical when the platens were lubricated. The soil specimen can be either undisturbed or compacted fine-grained soil or compacted coarse-grained material. It can be seen from Fig. 1 that the specimen is located inside the triaxial cell and is subjected to a deviator load from the vertical direction, which is measured by the S-shaped load cell. The deviator stress is also referred to as the cyclic stress and is always in a compressive state in the repeated load triaxial test. Besides the deviator stress, the sample is also subjected to the confining stress, which is provided by the confining medium, such as air or water. The sum of the deviator stress and the confining stress is defined as the principle stress, which is applied on the top of the sample in a vertical direction. The main objective of the combination of the deviator stress and confining stress is to simulate traffic loading conditions. The loading values of the deviator stresses and confining stresses are dependent on the relevant testing standards. Moreover, the loading cycle, which consists of the loading and unloading stage, is also pre-determined. For example, in the AASHTO T309 testing standard [6], a loading cycle of 0.1 second of loading and 0.9 second of unloading is suggested in order to simulate a standard vehicle travelling at 60 mph. During the test, the

deformation of the sample is also measured by two linear variable differential transformer transducers, which are externally mounted on top of the triaxial cell. In practice, it is a challenge to carry out the repeated load triaxial test as a daily routine test in the laboratory because performing the repeated load triaxial test is a time-consuming and complicated procedure. Furthermore, a skillful operator is also required to run the test with a high quality control procedure in place. Moreover, the repeated load triaxial test is not a common testing apparatus in the laboratory because the testing equipment is considerably expensive, thus making it less affordable.

B. Discrete Element Method

Pioneered by Cundall and Strack in 1979 [7], DEM was originally developed to investigate the problems in rock mechanics. Since then, it has gained popularity for simulating the dynamics of granular materials. In this method, materials are represented as assemblies of spherical particles (3D) or circular discs (2D). Each of these particles may interact with neighbouring particles or with the boundaries. Newton's equation of motion is employed to characterise these interactions in the translational and rotational directions:

$$m_i x_i = F_i + m_i g \quad (\text{Translational degrees of freedom}) \quad (1)$$

$$I_i \omega_i = M_i \quad (\text{Rotational degrees of freedom}) \quad (2)$$

where:

- m_i = mass of the i^{th} particle
- x_i = translational acceleration of the i^{th} particle
- F_i = $\sum_k f_i^k$ = the total force applied on the i^{th} particle due to the k interaction
- g = the acceleration of gravity
- I_i = the moment of inertia of the i^{th} particle
- ω_i = the angular of the i^{th} particle
- M_i = $\sum_k (l_i^k \times F_i^k + q_i^k)$ = total moment of the i^{th} particle due to the k interaction
- q_i^k = the moment of i^{th} particle at the k interaction

Technically, two mathematical techniques are used to characterize the interactions between the particles, as categorized by Walton [8] – hard sphere and soft sphere. In the former technique, the particles are considered as a rigid element. Therefore, no deformation occurs during the collision of the particles. The interaction is mainly controlled by the momentum exchange and is the function of the change in momentum, coefficient of friction, and coefficient of normal and tangential restitution. In simulation, this technique is particularly well conducted for applications in granular material flow [9]. In contrast, the particles are treated as soft during the collision in the soft sphere approach. When two particles collide, the deformation of the contact is represented as the small overlap, which is a function of the particle velocity, normal stiffness and shear stiffness.

Granular materials, such as crushed rock or recycled concrete, are the main materials used in the base and sub-base layers of the flexible pavement. Generally, the granular materials can be viewed as a very large assembly of

independent particles. Due to the discontinuous and inhomogeneous nature of these materials, the Discrete Element Method (DEM) is commonly employed to examine the behavior of the granular materials ([10]-[13]). The current paper uses DEM to simulate the repeated load triaxial test in the laboratory.

II. REPEATED LOAD TRIAXIAL TEST SIMULATION

In recent years, with the rapid development in the computing area, there is a significant number of discrete element method software available on the Internet. In general, they can be classified as either commercial or open source software. In the current investigation, the open source ESyS-Particle, which was developed by Stefen Abe et al. [14], is used as the main platform to simulate the repeated load triaxial test. Compared with the other available software on the Internet, the main advantage of the ESyS-Particle is that it is categorised as a high performance computing software. This means that ESyS-Particle is comparable with the other commercial grade software. However, ESyS-Particle has one drawback, which is the lack of the graphic user interface. In order to utilise the ESyS-Particle for their application, a certain level of knowledge and experience in the Python programming language is required. The open source ESyS-Particle can be downloaded from the website <https://launchpad.net/esys-particle> [15].

In the current simulation, the DEM model was developed in three dimensions with spherical particles. The sample has a diameter of 100mm and a height of 200mm. Practically, the granular particle is quite rigid during the repeated load triaxial test. Therefore, in the simulation, the particle is assumed to be rigid. It means that each spherical particle has six degrees of freedom. Moreover, only the translation and rotation of the particle centroids are considered in the equilibrium equations. The input of the DEM model includes the normal stiffness, shear stiffness, coefficient of friction, maximum dry density, particle density and particle size. By taking into consideration the current power of the computer as well as the nature of the granular particle size, the current investigation is limited to the application of only the gravel material. According to the soil classification guide, gravel has a minimum particle size of 2.36 mm and a maximum particle size of 19.0mm. The values of these input parameters, which are used in the simulation, are illustrated as follows:

TABLE I
 THE VALUES OF THE INPUT PARAMETERS FOR THE MODEL

Input parameter	Value
Normal stiffness (kN/m)	1000
Shear stiffness (kN/m)	1000
Coefficient of friction (rad)	0.5
Maximum dry density (kg/m ³)	2,200
Particle density (kg/m ³)	2,700
Minimum particle size (mm)	2.36
Maximum particle size (mm)	19.0

After the trial and error process, a total of 24,317 particles were found to be required to construct the sample. The density of the sample after the fabrication process is $1,925 \text{ kg/m}^3$, which is approximately 88% of the input value of the maximum dry density of $2,200 \text{ kg/m}^3$. The 3 dimensional visualisation of the sample after the fabrication, which is produced by the ParaView open source software [16], is illustrated as follows:

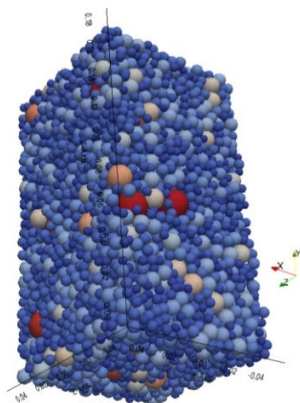


Fig. 2 The three dimensional visualisation of the sample after the fabrication

In the next stage, when the sample fabrication is completed, the contact list for all the particles is created by taking a single sphere and searching in the immediate neighborhood for other objects that are overlapping with it. Practically, the searching procedure starts from the 1st sphere to the last sphere.

The next step is force calculation. The contact between particles in granular materials consists of normal and tangential components of forces. Fig. 3 illustrates the schematic diagram of the contact model used in the current investigation. One spring with normal stiffness k_n models the normal component of the contact and the other, with the shear stiffness k_s , models the tangential component. The friction between two contact components has an inter-particle coefficient of friction μ .

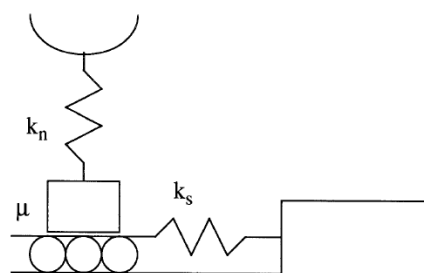


Fig. 3 The schematic diagram of the contact model (modified from [17])

Subsequently, the explicit first order finite difference time integration scheme is used to yield the velocities and position of the sphere at the next time step. In the current simulation, only the compressive state of contact is considered. The neighboring particle search algorithm implemented in the

model is the Verlet list neighbor [18].

As mentioned earlier, the repeated load triaxial test involves applying a different deviator and confining stress stage to the sample. The stress level during the current simulation is controlled by the boundary loading conditions. Technically, there are four types of boundary: rigid, period, membrane and asymmetrical. The current simulation employed the rigid boundary, which is the most widely used. This type of boundary is described as a planar surface and is well suited to simulate the triaxial or direct shear test [19]. Generally, a total of six servo-controlled rigid walls are used. One wall is located at the top of the sample and works as an actuator to provide the cyclic axial load on the sample. One fixed wall is located at the bottom of the sample and works as a pedestal. The other four planar walls are around the sample to provide the confining pressure.

The testing standard, which is used to determine the resilient modulus for the current investigation, is the Protocol P46: "Resilient modulus of unbound granular, base/sub-base materials and subgrade soils" [20]. The Protocol P46 was developed by the U.S. Department of Transportation in 1996 and is partially based on the AASHTO T292-91 test standard: "Resilient modulus of subgrade soils and untreated base/sub-base materials" (1991). Compared with other testing standards, such as the Australian testing procedure AG:PT/T053 [4], a new loading parameter contact stress is introduced. The main purpose of the contact stress is to keep the sample in position during the unloading cycle. The value of the contact stress is normally selected as 10 percent of the maximum axial stress, which is the sum of the cyclic stress and the contact stress. In the protocol P46, the loading cycle comprises 0.1 second of loading and 0.9 second of resting in order to simulate the loading conditions of a vehicle travelling at 60 mph, for the road base and sub-base granular layers. More details about the loading waveform can be seen as follows:

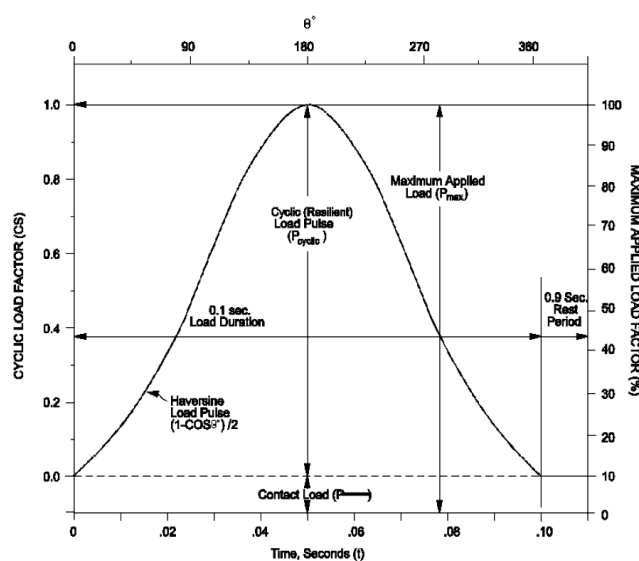


Fig. 4 The loading cycle waveform ([20])

Obviously, from the above figure, the loading waveform, as recommended by the standard, is in haversine shape. The individual loading cycle increases from zero percent to one hundred percent of the maximum applied. The loading value at any time of the loading cycle can be determined from the following equation:

$$S_{pulse} = \left[\frac{1 - \cos(\theta)}{2} \right] \times S_{max} \quad (3)$$

where:

S_{pulse} = Loading value at any time of the loading cycle (kPa)

θ = loading degree (rad)

S_{max} = Maximum axial stress (kPa)

In the current study, for the stress level, the deviator is 90 kPa, the confining stress is 50 kPa and the contact stress is 10 kPa. In order to examine the performance of the repeated load triaxial test simulation, the stress-strain curve is first investigated. The recoverable and permanent strain of the sample reposed to the cyclic loading of the first sequence is illustrated in Fig. 5

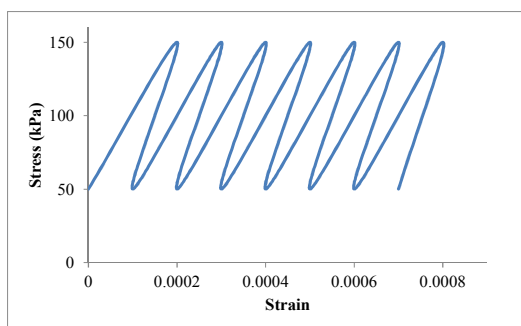


Fig. 5 The illustration of stress-strain curve in the first sequence of cyclic loading

It can be seen from the Fig. 5 that the strain increases when the deviator stress is applied on the top of the sample. In addition, when the deviator stress is released, the sample almost returns back to its previous state. This behavior is literally defined as the resilient behavior of the pavement materials under the repeated load triaxial test. In addition, the increase in the strain reading between two loading cycles presents the permanent deformation of the sample. The typical stress-strain curve for the repeated load triaxial test in the laboratory is illustrated as follows:

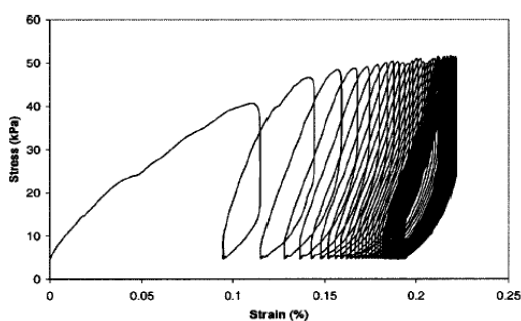


Fig. 6 The typical laboratory stress-strain curve [21]

Clearly, the stress-strain response from the discrete element method simulation conforms to the data for a typical experiment in the laboratory. This means that the numerical method is capable of replicating the resilient behavior of unbound granular materials under cyclic loading.

III. CONCLUSION

In the current study, the discrete element method was employed to simulate the repeated load triaxial test for the granular materials. The sample comprised 24,317 particles with the particle size ranging from 2.36 mm to 19.0 mm. The simulation was then carried out by applying a combination of 16 different stress levels to the sample according to the testing protocol P46: "Resilient modulus of unbound granular, base/sub-base materials and subgrade soils". Based on the observations obtained from the simulation, it is shown that the discrete element method is able to replicate the repeated load triaxial test in the laboratory. The stress-strain response from the discrete element method simulation conforms to the data for a typical experiment in the laboratory. However, due to the complexity of the input parameters, such as normal and shear stiffness, the currently developed DEM model is not capable of simulating the repeated load triaxial test independently. Further research works are required in order to improve the application of the proposed DEM model.

REFERENCES

- [1] F. Lekarp, U. Isacsson and A. R. Dawson, "State of the art. I: Resilient response of unbound aggregates" *Journal of Transportation Engineering*, vol.126 issue 1, pp. 66-75, 2000.
- [2] R. D. Barksdale, "The aggregate handbook" *National Stone Association*, Sheridan Books, Inc., Elliot Place, Washington, D.C, 2001.
- [3] B. Magnusdottir and S. Erlingsson, "Repeated load triaxial testing for quality assessment of unbound granular base course material" *Proceedings from the 9th Nordic Aggregate Research Conference*, Reykjavik, Iceland, 2002.
- [4] Australian testing procedure AG:PT/T053, "Determination of permanent deformation and resilient modulus characteristic of unbound granular materials under drained conditions" *Austrroads Working Group*, Australia, 2007.
- [5] A. Adu-Osei, "Characterization of unbound granular materials" *PhD Dissertation*, Department of Civil Engineering, Texas A&M University, College Station, Texas, 2000.
- [6] AASHTO T 307-99, "Determining the resilient modulus of soils and aggregate materials" AASHTO, Washington, D.C., USA, 1999.
- [7] P. A. Cundall and O. D. L. Strack, "A discrete numerical model for granular assemblies" *Geotechnique*, vol.29, pp.47-65, 1979.
- [8] O. R. Walton, "Explicit particle dynamics model for granular materials" *Numerical methods in Geomechanics*, Z. Eisenstein, ed. A. A. Balkema Rotterdam, pp.1261-1268, 1983.
- [9] C. Campbell and C. Brenan, "Computer simulations of granular shear flow" *Journal of Fluid Mechanics*, vol.151, pp.167-188, 1985.
- [10] R. K. Rajamani, B. K. Mishra, R. Venugopal and A. Datta, "Discrete element analysis of tumbling mills" *Powder Technology*, vol. 09, pp. 105-112, 2000.
- [11] M. Moakher, T. Shinbrot and F. J. Muzzio, "Experimentally validated computations of flow, mixing and segregation of non-cohesive grains in 3D tumbling blenders" *Powder Technology*, vol. 109, pp. 58-71, 2000.
- [12] L. Cui and C. O'Sullivan, "Exploring the macro and micro scale response of an idealised granular material in the direct shear apparatus" *Geotechnique* vol. 56, pp. 455-468, 2006.
- [13] C. Bierwisch, T. Kraft, H. Riedel, and M. Moseler, "Three-dimensional discrete element models for the granular statics and dynamics of powders in cavity filling" *Journal of Mechanics and Physics of Solids*, vol.57 issue 1, pp. 10-31, 2009.

- [14] Steffen Abe, David Place, and Peter Mora, "A parallel implementation of the lattice solid model for the simulation of rock mechanics and earthquake dynamics", *Pure and Applied Geophysics*, vol. 161, pp. 2265-2277, 2004.
- [15] ESyS-Particle: <https://launchpad.net/esys-particle>
- [16] ParaView: <http://www.paraview.org>
- [17] N. V. Rege, "Computational modelling of granular materials" *Doctoral Thesis*, Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, 1996.
- [18] Pöschel and Schwager (2005) "Computational granular dynamics" *Springler-Verlag*, 2005.
- [19] Y. P. Cheng, Y. Nakata and M. D. Bolton, "Discrete element simulation of crushable soil" *Geotechnique*, vol. 53, pp. 633-641, 2003.
- [20] Protocol P46, "Resilient modulus of unbound granular, base/sub-base materials and subgrade soils" *Department of Transportation*, U.S, 1996.
- [21] M. Zeghal, "Discrete-Element method investigation of the resilient behaviour of granular materials" *Journal of Transportation Engineering, ASCE*, vol. 130 issue 4, pp. 503-509, 2004.