Average Switching Thresholds and Average Throughput for Adaptive Modulation using Markov Model

Essam S. Altubaishi

Abstract—The motivation for adaptive modulation and coding is to adjust the method of transmission to ensure that the maximum efficiency is achieved over the link at all times. The receiver estimates the channel quality and reports it back to the transmitter. The transmitter then maps the reported quality into a link mode. This mapping however, is not a one-to-one mapping. In this paper we investigate a method for selecting the proper modulation scheme. This method can dynamically adapt the mapping of the Signal-to-Noise Ratio (SNR) into a link mode. It enables the use of the right modulation scheme irrespective of changes in the channel conditions by incorporating errors in the received data. We propose a Markov model for this method, and use it to derive the average switching thresholds and the average throughput. We show that the average throughput of this method outperforms the conventional threshold method.

Keywords—Adaptive modulation and coding, CDMA, Markov model.

I. INTRODUCTION

NEW cellular communication systems (e.g., third generation and beyond), are characterized by supporting different services. These services require high quality, high speed, and high flexibility as well as temporary and spatial control of traffic under severe fading environments, the so-called conventional system design concept will be insufficient. Providing a wide range of multimedia services, over wireless networks would need a different network design, centered on a packet-switched architecture, and a new set of power and bandwidth allocation algorithms. This new system must allocate network resources according to the benefit that each user would receive, considering the type of service required.

The most important factors that influence capacity in Code Division Multiple Access (CDMA) system are the target SIR, transmission rate, channel activity, and number of users. Note that in [1], the authors report that for outage probability of 5 percent, users can be doubled when the activity is decreased from 1 to 0.5. Moreover, the lower the transmission rate, the higher the number of users admitted in the system. Accurate and fast power control can compensate for bad radio channel conditions and keep the received SNR above the target level. However, in a bad radio link, the source can transmit with high power levels, causing extensive interference to other users. Therefore, another option is needed to compensate during the bad radio channel. One possibility is to limit the maximum transmit power, and reduce the transmission rate, this is investigated in [2]. Another concept in transmission rate control is to relate the rate to the measured SNR. It can be assumed that there exists a one to one relation between rate and SNR for a given BER target. In CDMA systems, the transmission rate can be controlled by using adaptive modulation and coding (AMC), which is a promising method for high data rate access.

In [3], modulation and power control are combined to achieve a specified range of packet error rate for real time applications. Another paper investigates the joint optimization of modulation and power in cellular systems [4]. It is shown that a large gain can be achieved using adaptive modulation. In [5], a variable-power variable-rate modulation scheme using M-ary Quadrature Amplitude Modulation (MQAM) is proposed. It is shown that the proposed technique outperforms the variable-rate fixed-power modulation. However, the effect of peak power constraint is not addressed in this work. A similar concept based on adaptation technique [6], where the peak power is limited, has been proposed. The results show that there is a small loss in spectral efficiency as compared to unconstrained case.

The channel capacity of various adaptive transmission techniques is evaluated in [7]. The performance of these techniques is also investigated in conjunction with space diversity. The presented results show that using adaptive transmission techniques and space diversity can improve the spectral efficiency for a fading channel. It is also found that when the transmission rate is varied continuously according to the channel condition, adapting the transmit power at the same time has minimal effect.

As a mean of supporting multiple rates, a trellis-coded modulation has drawn some interest [8-11]. A variable rate adaptive trellis-coded QAM is discussed in [8]. An adaptive modulation and coding scheme is proposed in [9]. This technique puts a trellis code on top of the uncoded modulation. Simulation results show that, an effective coding gain can be obtained. In [11], adaptive coded modulation is applied to a model of urban microcellular network. Results show that the adaptive modulation and coding scheme that utilizes a set of trellis codes provide significant advantages.

Essam S. Altubaishi is with the Department of Electrical Engineering, King Saud University, Saudi Arabia (e-mail: etubashi@ksu.edu.sa).
over a traditional non-adaptive coded modulation scheme in terms of the average spectral efficiency and decoding delay.

In such systems as High Data Rate (HDR), also known as 1xEV-DO [12,13] or High Speed Downlink Packet Access (HSDPA) [14,15], AMC is used to support high transmission rate. The work in [16] derives the distribution of the received Signal-to-Interference Ratio (SIR) by mobile station located at arbitrary cell position, and evaluates the average BER performance of the common channel of HDR system employing the adaptive modulation. In [17], a performance evaluation of modulation and coding schemes proposed for HSDPA is presented.

The performance of AMC depends mainly on the method used at the receiver to estimate the channel condition. Any prediction error of the channel condition can have an effect on the decision making of the appropriate modulation level to be used; as a result the performance of the system would be degraded. In the literature, many articles present AMC methods together with considering the effect of prediction errors in decision making, such as [5], [8], and [18]. The work in [8] uses pilot symbols to estimate channel state at the receiver to improve the prediction accuracy.

Recent studies have considered finite-state first-order Markov modeling for describing a wireless communication channel [19-21]. In [19], a two-states Markov model is presented with parameters, directly related to correlation parameters of a fading channel. The work in [21] proposes a new method for selecting the appropriate Modulation and Coding Scheme (MCS) according to the estimated channel condition using a first-order finite-state Markov model. Results show that this new method outperforms the conventional threshold-based decision-making approach. However, this method requires a training time to set the transition probabilities of the Markov model.

The success of adaptive modulation and coding process depends primarily on the accuracy of mapping the SNR into a link mode. In the conventional threshold method, the resultant regions are created based on the SNR/BLER curves. Unfortunately, there is not a one-to-one mapping between the measured SNR and the MCS to achieve a desired Block Error Rate (BLER). This is because the achieved BLER depends primarily on the accuracy of mapping the SNR into a link mode. In the conventional threshold method, the resultant regions are created based on the SNR/BLER curves. Therefore the rule of selection can be given as

\[ \text{mode}(n) \text{ is chosen, when } \gamma_n \leq \text{SNR} < \gamma_{n+1} \]  

No transmission takes place, when \( \gamma_0 \leq \text{SNR} < \gamma_1 \), which corresponds to the mode \( n = 0 \). Based on (1), mode \( n \) will be chosen with probability

\[ P_{\text{mode}(n)} = \int_{\gamma_n}^{\gamma_{n+1}} p_f(\gamma) d\gamma \]  

where \( p_f(\gamma) \) is the channel distribution function. The channel model considered in this paper is a Rayleigh fading channel with additive white Gaussian noise.

The conventional threshold Method has previously been considered in the literature, such as [12-16]. In this method, the adaptive modulation scheme has a set of \( N \) transmission modes. The relation between probabilities of error versus SNR is generated for each mode. These probability values can be graphically represented, where the target error rate is intersecting with each curve. Therefore, The values of the SNR corresponding to the intersection points are chosen as the switching threshold denoted by \( \gamma_n \), where \( n \in \{0, 1, \ldots, N-1\} \). These threshold points partition the range of SNR into \( N \) regions. Therefore the rule of selection can be given as

\[ \text{mode}(n) \text{ is chosen, when } \gamma_n \leq \text{SNR} < \gamma_{n+1} \]  

No transmission takes place, when \( \gamma_0 \leq \text{SNR} < \gamma_1 \), which corresponds to the mode \( n = 0 \). Based on (1), mode \( n \) will be chosen with probability

\[ P_{\text{mode}(n)} = \int_{\gamma_n}^{\gamma_{n+1}} p_f(\gamma) d\gamma \]  

The paper is organized as follows. In Section II, we describe our system model and problem formulation. In Section III, the proposed Markov model is presented. Results and discussions are presented in section IV. Finally, conclusions are drawn in section V.
where $P_f(\gamma)$ is the probability density function (pdf) of the received SNR. Then, the average throughput $T$ expressed in terms of bit per symbol (BPS) can be described as

$$T = \sum_{n=0}^{N-1} b_n \int_{\gamma_n}^{\gamma_{n+1}} P_f(\gamma) d\gamma$$

(3)

where $b_n$ is the BPS throughput of the individual modes.

III. MARKOV MODEL

The DRM can be modeled as a discrete-time Markov chain. The states in this model represent the received SIR in dB. These states form a set $\{S_M, \ldots, S_1\}$ of $2M+1$ states. We assume that the connectivities between states are shown in Fig. 2, where:

- N-mode with N regions is defined in each state.
- State 0 is the initial state where region $n$ can be defined as $\gamma_n \leq \text{SNR} < \gamma_{n+1}$. Note that state 0 represent the threshold method.
- The region limits from state to state differ by a multiple of step size, $\Delta$ in dB. Therefore, the region $n$ in state $(0+j)$ is $(\gamma_n + j\Delta) \leq \text{SIR} < (\gamma_{n+1} + j\Delta)$.
- $P_j$ is the probability of an upward transition to state $(j+K)$ due to an error in the received block of data. It is therefore the probability of a block error in state $j$.
- The transition probability from state $j$ to $(j-1)$ is $(1-P_j)$.

$K \geq 1$ is an integer that indicates the jump in the region limits when a block is in error, as a multiple of $\Delta$. This method is aim to keep the BLER always less than or equal to $1/(K+1)$. Hence, $K$ should be chosen to be $1/(\text{req. BLER})-1$ [22].

The general idea of this method is to adjust each region in a particular state according to the BLER. If there is an error, the regions limits will be increased by $K\Delta$, and if not, the regions limits will be decreased by $\Delta$. In each case the received SNR will be mapped to a specific mode based on the new regions limits. It is clear that; the adjustment of regions depends mainly on the value of the step size, $\Delta$. If the value is not determined properly, the selection of the mode will be affected, and as a result, the throughput of the system will be degraded. For the analysis purposes, the mean of the region can be computed from the stationary distribution. The average of the threshold values can be computed. And as a result the average throughput can be determined.

In order to compute the stationary probability in state $j$, we first need to derive the average bit error rate for each mode in state $j$, which can be defined as

$$P_{b/mode(n), j} = \int_{\gamma_n}^{\gamma_{n+1}} \text{BER}_{mode(n)}(\gamma) P_f(\gamma) d\gamma$$

(5)

where $\text{BER}_{mode(n)}$ is the BER of the $n$th uncoded modulation mode in the presence of additive white Gaussian noise (AWGN), and $j\Delta$ as well as $\gamma_n$ are given as non-dB value. $\text{BER}_{mode(n)}$ can be written as [25,26]

$$\text{BER}_{mode(n)} = \sum_i A_i Q(\sqrt{B_i})$$

(6)

The system of equations from (4) can be solved to yield the stationary probabilities. From the stationary distribution, the average of the threshold values can be computed. And as a result the average throughput can be determined.
where \( A_i, B_i \) is a set of modulation mode dependent constants, and \( Q(x) \) is the Gaussian Q-function defined as

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} \, dt
\]

(7)

then, \( P_{b_{\text{mode}(n)}j} \) can be written as

\[
P_{b_{\text{mode}(n)}j} = \sum_i A_j \int_{\beta_{x_{n}}}^{\infty} Q(\sqrt{B_i} \gamma) p_{i}(\gamma) \, d\gamma
\]

(8)

And with the assumption of a Rayleigh fading channel, \( P_{b_{\text{mode}(n)}j} \) can be given as

\[
P_{b_{\text{mode}(n)}j} = \sum_i A_j \int_{\beta_{x_{n}}}^{\infty} Q(\sqrt{B_i} \gamma) e^{-\gamma/\gamma} \, d\gamma
\]

(9)

where \( \gamma \) is the average SNR. Then, the average bit error rate of mode \( n \) can be expressed using the result of Appendix A as

\[
P_{b_{\text{mode}(n)}j} = b_j \sum_i \left[ -e^{\beta_{x_{n}}/\gamma} Q(\sqrt{B_{i}} \gamma) + e^{\beta_{x_{n}}/\gamma} Q(\sqrt{B_{i}} \gamma) + \frac{B_{i} \gamma}{B_{i} + 2} \left( Q(\sqrt{B_{i}} \gamma) + Q \left( \frac{2 \sqrt{B_{i}} \gamma}{\gamma} \right) \right) \right]
\]

(10)

From the average bit error rate of mode \( n \), the average block error rate of mode \( n \) in state \( j \) can be computed as

\[
P_{b_{\text{mode}(n)}j} = 1 - (1 - b_j P_{b_{\text{mode}(n)}j})^{N_s}
\]

(11)

where \( N_s \) is the number of symbol in a block.

Now the error in a block depends first on the selection of the mode, then on a block error rate of that mode. And by the fact that the selection of the mode are mutually exclusive, then block error rate \( P_j \) (also the probability of an upward transition in state \( j \)) can be defined using theorem on total probability as

\[
P_j = \sum_{n=0}^{N-1} P_{b_{\text{mode}(n)}j} \cdot P_{\text{mode}(n)j}
\]

(12)

Similarly as explained earlier, \( P_{\text{mode}(n)j} \) is the probability of selection mode \( n \) in state \( j \), which can be defined in a case of Rayleigh fading channel as

\[
P_{\text{mode}(n)j} = \int_{\beta_{x_{n}}}^{\infty} e^{-\gamma/\gamma} \, d\gamma = e^{-\beta_{x_{n}}/\gamma} - e^{-\beta_{x_{n}}/\gamma}
\]

(13)

With the transition probabilities known, the stationary distribution is readily computed from equation (4). Finally, the average of the threshold values can be given as

\[
\bar{\tau}_n = \sum_{j=1}^{M} \pi_{n,j} \tau_{n,j}
\]

(14)

where \( \tau_{n,j} = j \Delta \gamma_n \) is the threshold values in state \( j \), then the average throughput of our model is

\[
T = \sum_{n=0}^{N-1} \int_{\beta_{x_{n}}}^{\infty} p_{j}(\gamma) d\gamma
\]

(15)

IV. RESULTS AND DISCUSSIONS

In this investigation, a five-mode adaptive modulation system is used. These modes are No Transmission, BPSK, QPSK, 16QAM and 64QAM. To generate the threshold values for each mode in state 0 at specific block error rate, we first need to obtain the BER for each modulation mode, which can be written as [26]

\[
\text{BER}_{\text{BPSK}} = \frac{3}{4} Q(\sqrt{2} \gamma)
\]

\[
\text{BER}_{\text{QPSK}} = \frac{1}{2} Q(\sqrt{2} \gamma)
\]

\[
\text{BER}_{\text{16QAM}} = \frac{7}{12} Q(\sqrt{2} \gamma) + \frac{1}{6} Q(\sqrt{2} \gamma) - \frac{1}{12} Q(\sqrt{25/21} \gamma)
\]

\[
\text{BER}_{\text{64QAM}} = \frac{1}{12} Q(\sqrt{8/2} \gamma) - \frac{1}{12} Q(\sqrt{169/21} \gamma)
\]

(16)

then, by using \( \text{BLER} = 1 - (1 - b_j \text{BER})^{N_s} \), and assuming the target BLER = 0.1 and \( N_s = 160 \) Symbols, we have the following results as shown in Table I.

For the Markov model, as mentioned earlier to have a BLER = 0.1, \( K \) should be chosen to be 9. The number of states is chosen to be large compared to the value of \( K \), therefore \( M \) is taken to be 10K. Fig. 3 shows the resultant average throughput in terms of BPS for both threshold method and Markov model.

| TABLE I  |
|-----------------|-----------------|-----------------|
| \( n \) | Mode | Region (dB) | BN (BPS) |
| 0 | No Transmission | SNR < 7.12 | 0 |
| 1 | BPSK | 7.12 ≤ SNR < 10.64 | 1 |
| 2 | QPSK | 10.64 ≤ SNR < 17.91 | 2 |
| 3 | 16QAM | 17.91 ≤ SNR < 24.24 | 4 |
| 4 | 64QAM | 24.24 ≤ SNR | 6 |
From Fig. 3, it is clear that DRM has better average throughput than threshold method. Average SNR is an indication of the quality of the channel; therefore for low average SNR, the powerful of using the DRM appears. Specifically, the average throughput gain is more than double in the case of average SNR is below 5dB.

To evaluate the effect of the step size, $\Delta$, Fig. 4 shows the average throughput for both methods as a function of $\Delta$.

Based on Fig. 4 the average throughput of the DRM is decreasing with increasing the step size. In fact, the reduction of the throughput due to increasing in the step size is expected. Because, if there is an error, the next jump will shift the regions aggressively by $K\Delta$ to the lower mode. Therefore, the value of the step size should be small to make sure that the following transmissions will use a mode that can be supported.

V. CONCLUSION

In this work, we investigated the performance of the adaptive modulation method employing various phase shift keying (PSK) and square-QAM constellations. We proposed a discrete-time Markov model for the adaptive modulation method that incorporates errors in the received data for selecting the best modulation scheme; this method can be called Dynamic Range Method (DRM). From this model we derived the average switching thresholds as well as the average throughput. Results show that, this method proves to have better performance than the conventional threshold method in terms of throughput. Also it is noticed that, the efficiency of DRM comes from selecting higher modulation scheme if there is no impact on the target quality.

APPENDIX A

Solving the integration of Eq. (9)

This equation can be written as

$$P = \int_{\gamma}^{B\gamma} Q(\sqrt{B\gamma}) \frac{e^{-\gamma}}{\gamma} d\gamma$$  \hspace{0.5cm} (17)

The above integration can be solved using integration by part, which can be defined as

$$\int u dv = uv - \int v du$$  \hspace{0.5cm} (18)

where

$$u = Q(\sqrt{B\gamma}) \hspace{1cm} dv = \frac{e^{-\gamma}}{\gamma} d\gamma$$

$$\frac{du}{d\gamma} = -\frac{1}{2\sqrt{2\pi\gamma}} e^{-\frac{B\gamma}{2}}$$

$$\frac{dv}{d\gamma} = \frac{1}{\gamma}$$

then

$$P = \int_{\gamma}^{B\gamma} Q(\sqrt{B\gamma}) \frac{e^{-\gamma}}{\gamma} d\gamma$$  \hspace{0.5cm} (19)

the second part of (19) can be written as

$$I = \int_{\gamma}^{B\gamma} \frac{1}{2\sqrt{2\pi\gamma}} e^{-\frac{B\gamma}{2\gamma}} d\gamma$$  \hspace{0.5cm} (20)

Let $x^2 = \frac{B\gamma}{B\gamma + 2}$, and $dy = 2\sqrt{\frac{B\gamma}{B\gamma + 2}} dx$. Then

(20) can be expressed as
\[
I = \frac{B_\gamma}{B_\gamma^2 + 2} \frac{1}{2\pi} \int e^{-x^2/2} \, dx
\]  
(21)
\[
\text{this yields}
\]
\[
I = \frac{B_\gamma}{B_\gamma^2 + 2} \left( Q\left( B_\gamma + \frac{2s_1}{\gamma} \right) - Q\left( B_\gamma + \frac{2s_2}{\gamma} \right) \right)
\]  
(22)
\[
\text{and finally (19) becomes}
\]
\[
P = -e^{-x^2/2} Q\left( B_\gamma \right) + e^{-x^2/2} Q\left( B_\gamma + \frac{2s_1}{\gamma} \right) + e^{-x^2/2} Q\left( B_\gamma + \frac{2s_2}{\gamma} \right)
\]  
(23)

REFERENCES


Essam S. Altuhaishi received the B.Sc. with First Class Honor and M.Sc. from King Saud University, Riyadh, saudi Arabia, all in electrical engineering, in 1996 and 2004, respectively. From 1996 to 1999, he was with Advanced Electronics Company (AEC) in Riyadh, as a Test Engineer where he has been involved in development and provision of In-circuit and functional tests for several systems, such as military transceiver from Racal, Signal Data recorder Reproducer (SDDR) of F-15 aircraft from Smiths Industries, and F-15 aircraft communications system from Northrop Grouman.

In 1999, he joined the Department of Electrical Engineering, King Saud University, where he is currently a lecturer. His research interests include design and analysis of the future wideband communications system.