World Academy of Science, Engineering and Technology
International Journal of Physical and Mathematical Sciences
Vol:7, No:3, 2013

Uniform Solution on the Effect of Internal Heat Generation on Rayleigh-Benard Convection in Micropolar Fluid

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***Abstract*—**The effect of internal heat generation is applied to the Rayleigh-Benard convection in a horizontal micropolar fluid layer. The bounding surfaces of the liquids are considered to be rigid-free, rigid-rigid and free-free with the combination of isothermal on the spin-vanishing boundaries. A linear stability analysis is used and the Galerkin method is employed to find the critical stability parameters numerically. It is shown that the critical Rayleigh number decreases as the value of internal heat generation increase and hence destabilize the system.

***Keywords*—**Internal heat generation, micropolar fluid, rayleigh-benard convection.

I. Introduction

THE microfluids, a subclass of generalized fluids are introduced and developed for the first time by Eringen [1]. These are fluids body moments and are influenced by the spin inertia. The stress tensor for such fluids is non-symmetric. Eringen's theory has provided a good model to study a number of complicated fluids, including the flow of low concentration suspensions, liquid crystal, blood and turbulent shear flows. Several special forms of this theory [2]-[3] have also been given by Eringen. Later, Eringen [4] has also developed a continuum theory of dense rigid suspensions, when the substructure particles are assumed to be rigid; the special form of the above theory is called the theory of micropolar fluids. Eringen [5] introduced a subclass of micro-fluids named micropolar fluids, which exhibit micro-rotational inertia. This class of fluids posses a certain simplicity and elegance in their mathematical formulation and are more easily amenable to solution, which obviously is more attractive for applied mathematicians. Thermal effects in microfluids have also been discussed by Eringen [6]. These have been again specialized to generate a theory of isotropic thermomicropolar fluids suspensions, polymeric fluids, turbulent and blood flow.

The classical Rayleigh problem on the onset of convective instabilities in a horizontal layer of fluid heated from below has its origin in the experimental observations of Benard [7] and [8]. Convection has been the subject of many investigations due to the related engineering applications such as crystal growth, weld penetration and in coating process. A theory and modeling of material processing in the laboratory may include the mechanism of suppressing free convection driven by both buoyancy and surface tension forces. Rayleigh's paper is the pioneering work for almost all modern theories of convection. Rayleigh [9] showed that Benard convection, which is caused by buoyancy effects, will occur when the Rayleigh number exceeds a critical value. Walzer [10] analyzed the problem of convective instability of a micropolar fluid layer confined between two rigid boundaries and pointed out that the analysis of the instability finds applications in the area of geophysics. One of the examples is the understanding of the phenomena of the rising of volcanic liquid with bubbles and convective processes inside the earth's mantle. The onset of convection for a heat conducting micropolar fluid layer between two rigid boundaries is investigated by Rama Rao [11]. Sastry and Rao [12], Qin and Kaloni [13] studied the instability of a rotating micropolar fluid. The universal stability of magneto micropolar fluids was studied by Ahmadi and Shahinpoor [14]. Effect of through flow on Marangoni convection in micropolar fluid is investigated by Murty and Rao [15]. The magnetoconvection in micropolar fluid is micropolar fluid was studied by Siddeshwar and Pranesh [16].

Vidal and Acrivos [17], Debler and Wolf [18] and Nield [19] studied the effect of a non-uniform temperature gradient on the onset of Marangoni convection. Rudraiah and Siddeshwar [20] analyzed the effects of non-uniform temperature gradients of parabolic and stepwise-types on the onset of Marangoni convection. Rudraiah [21] and Friedrich and Rudraiah [22] have examined the combine effect of rotation and non-uniform basic temperature gradient on Marangoni convection. The effects of non-uniform basic temperature gradient on Benard-Marangoni convection were studied by Lebon and Cloot [23]. The combined effects of non-uniform temperature gradient and Coriolis force (due rotation) on the Benard-Marangoni convection were analyzed by Rudraiah and Ramachandramurty [24]. As for the non-uniform basic temperature in a micropolar fluid, these researchers [25]-[30] have studied the individual effects.

The nonlinear temperature distribution in a horizontal fluid layer arising due to internal heat generation has been studied theoretically by Sparrow et.al [31] and Roberts [32]. The

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effects of quadratic basic state temperature gradient caused by uniform internal heat generation were first addressed by Char and Chiang [33] for Benard-Marangoni convection. Wilson [34] used a combination of analytical and numerical techniques to analyze the effect of internal heat generation on the onset of Marangoni convection. Bachok and Arifin [35] studied the feedback control of the Marangoni-Benard convection in the presence of internal heat generation. The inverted parabolic temperature profile in a simulated microgravity as it increases critical Marangoni number making the system more stable. Also the electrical conducting micropolar fluid layer heated from below is more stable compared to the electrically conducting viscous fluid.

The present study deals with internal heat generation on the onset of Rayleigh-Benard convection in micropolar fluid. This analysis based on the linear stability theory and the resulting eigenvalue problem is solved numerically using the Galerkin technique with lower and upper boundary conditions that are rigid-free, rigid-rigid and free-free.

II. Mathematical Formulation

Consider an infinite horizontal layer of micropolar fluid of depth *d*, where the fluid is heated from below with the internal heat generation exists with the fluid system. The stability of a horizontal layer of micropolar fluid in the presence of internal heat generation is examined. The no-spin boundary condition is assumed for micro-rotation at the boundaries. Let ΔT be the temperature difference between the lower and upper surfaces with the lower boundary at a higher temperature than the upper boundary. These boundaries maintained at constant temperature.

The upper free surface is assumed to be non-deformable and the governing equations for the Rayleigh-Benard situation in Boussinesquian micropolar fluid are continuity equation;

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

conservation of linear momentum

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q} \cdot \nabla \right) \vec{q} \right] = -\nabla p - \rho g \hat{k} + \left(2\zeta + \eta \right) \nabla^2 \vec{q} + \zeta \left(\nabla \times \vec{\omega} \right), \tag{2}$$

conservation of angular momentum

$$\rho_0 I \left[\frac{\partial \vec{\omega}}{\partial t} + \left(\vec{q} \cdot \nabla \right) \vec{\omega} \right] = \left(\lambda' + \eta' \right) \nabla \left(\nabla \cdot \vec{\omega} \right) + \left(\eta' \nabla^2 \vec{\omega} \right) + \zeta \left(\nabla \times \vec{q} - 2\vec{\omega} \right), \tag{3}$$

conservation of energy

$$\frac{\partial T}{\partial t} + \left(\vec{q} \cdot \nabla T \right) = \frac{\beta}{\rho_0 C_v} \left(\nabla \times \vec{\omega} \right) \cdot \nabla T + \kappa \nabla^2 T + h_g, \tag{4}$$

density equation of state

$$\rho = \rho_0 \left[1 - \alpha \left(T - T_a \right) \right], \tag{5}$$

where $\vec{q}$ is the velocity, ρ_0 is the density at T_a, t is the time, p is the pressure term, g is the gravity, $\hat{k}$ is the unit vector in z-direction, ζ is the coupling viscosity coefficient for vortex viscosity, λ and η is the bulk and shear kinematic viscosity coefficients, $\vec{\omega}$ is the micro rotation, I is the moment of inertia, λ' and η' is the bulk and shear spin-viscosity coefficients, T is the temperature, β micropolar heat conduction parameter, C_v is the specific heat, κ is the thermal conductivity, h_g is the overall uniformly distributed volumetric internal heat generation within the micropolar fluid layer.

The basic state of the fluid is quiescent and described by

$$\vec{q}_b = (0,0,0),\ \vec{\omega}_b = (0,0,0),\ p = p_b(z), \text{ and } T = T_b(z), \tag{6}$$

where the subscript *b* denotes the basic state. Substituting (6) into (2) and (4), we get the basic state governing equations as

$$\frac{dp_b}{dz} = -\rho_b g, \tag{7}$$

$$\frac{d^2 T_b}{dz^2} = -\frac{h_g}{\kappa}, \tag{8}$$

with

$$\rho = \rho_0 \left[1 - \alpha (T - T_a) \right]. \tag{9}$$

Subject to the boundary conditions, $T_b = T_0$ and $z = 0$, and $T_b = T_0 - \Delta T$ at $z = d$, then (8) is solved and we obtained

$$T_b(z) = -\frac{h_g}{2\kappa} z^2 + \left(\frac{h_g d}{2\kappa} - \frac{\Delta T}{d} \right) z + T_0. \tag{10}$$

Take note that (10) is a parabolic distribution with the liquid layer height due to the existence of the internal heat generation; $Q = 0$, the basic state temperature distribution in the fluid layer is linear. Let the basic state be distributed by an infinitesimal thermal perturbation and we now have

$$\vec{q} = \vec{q}_b + \vec{q}',\ \vec{\omega} = \vec{\omega}_b + \vec{\omega}',\ \vec{p} = \vec{p}_b + \vec{p}', \text{ and } T = T_b + T', \tag{11}$$

where the primes indicate that the quantities are infinitesimal perturbations. Substituting (11) into (1)-(4), we obtained the linearized equations in the form

$$\nabla \cdot q' = 0, \tag{12}$$

$$-\nabla p' - \rho' g \hat{k} + (2\zeta + \eta)\nabla^2 q' + \zeta(\nabla \times \omega') = 0, \tag{13}$$

$$(\lambda' + \eta')\nabla(\nabla \cdot \omega') + \eta' \nabla^2 \omega' + \zeta(\nabla \times q' - 2\omega') = 0, \tag{14}$$

$$\frac{\beta}{\rho_0 C_v}\left[\nabla \times \omega' \cdot \left(-\frac{\Delta T}{d}\right)\hat{k}\right] \cdot \nabla T + \kappa \nabla^2 T' + \left(\frac{z h_g}{\kappa} - \frac{d h_g}{2\kappa} + \frac{\Delta T}{d}\right) W = -W \frac{\Delta T}{d}. \tag{15}$$

The perturbations of (11)-(15) are non-dimensionalised using the following definitions

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right), \; W^* = \frac{W'}{x/d}, \; \Omega^* = \frac{(\nabla \times \omega')z}{x/d^2},$$

and $T^* = \dfrac{T'}{\Delta T}$. (16)

Substituting (16) into (13)-(15), eliminating the pressure term by operating curl twice on the resulting (13), operating curl once on (14) and non-dimensionalising, we get

$$(1 + N_1)\nabla^4 W + N_1 \nabla^2 \Omega + Ra\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] = 0, \tag{17}$$

$$N_3 \nabla^2 \Omega - 2N_1 \Omega - N_1 \nabla^2 W = 0, \tag{18}$$

$$\nabla^2 \Theta + \left[1 - Q(1 - 2z)\right] W - N_5 \Omega = 0, \tag{19}$$

where the asterisks have been dropped for simplicity. Here

$N_1 = \dfrac{\zeta}{\eta + \zeta}$ is the coupling parameter,

$N_3 = \dfrac{\eta'}{(\eta + \zeta) d^2}$ is the couple stress parameter,

$N_5 = \dfrac{\beta}{\rho_0 C_v d^2}$ is the micropolar heat conduction parameter,

$Ra = \dfrac{\alpha g \Delta T \rho_0 d^3}{(\eta + \zeta)\chi}$ is the Rayeligh number,

and $Q = \dfrac{h_g d^2}{2\kappa \nabla T}$ is the heat source strength.

The perturbation quantities in a normal mode form are

$$(W, T, \Omega) = \left[W(z), \Theta(z), G(z)\right] \exp\left[i(a_x x + a_y y)\right], \tag{20}$$

where $W(z)$, $Q(z)$ and $G(z)$ are amplitudes of the perturbations of vertical velocity, temperature and spin, and $a = \sqrt{a_x^2 + a_y^2}$ is the wave number of the disturbances at the liquid layer. Substituting (20) into (17)-(19), we get

$$(1 + N_1)(D^2 - a^2)^2 W + N_1 (D^2 - a^2) G = Ra^2 \Theta, \tag{21}$$

$$N_1\left[(D^2 - a^2) W + 2G\right] - N_3 (D^2 - a^2) G = 0, \tag{22}$$

$$(D^2 - a^2)\Theta - \left[Q(1 - 2z) - 1\right] W - N_5 G = 0, \tag{23}$$

where $D = \dfrac{d}{dz}$.

Equations (21)-(23) are solved subject to appropriate boundary conditions that are

$$W = DW = G = 0 \text{ at } z = 0, \tag{24}$$

<u>For upper free boundary</u>

$$W = D\Theta = G = D^2 W = 0 \text{ at } z = 1, \tag{25}$$

<u>For upper rigid boundary</u>

$$W = D\Theta = G = DW = 0 \text{ at } z = 1. \tag{26}$$

III. Method of Solution

Equations (21)-(23) are solved subject to the appropriate boundary conditions. The single-term Galerkin technique is used to find the critical eigenvalue. Multiply (21) by $W_i(z)$, (22) by $\Theta_i(z)$ and (23) by $G_i(z)$ respectively. Perform the integration by parts with respect to z between $z = 0$ and $z = 1$ for the resulting equations. By using the appropriate boundary conditions, the expression for the Rayleigh number is given by

$$Ra = \frac{\left[C_4 (C_3 C_2 - C_1 C_6)\right]}{\left[a^2 C_8 (C_1 C_5 - C_2 C_7)\right]}, \tag{27}$$

where

$C_1 = -N_1\left[\langle DW \rangle \langle DG \rangle + a^2 \langle WG \rangle\right]$,

$C_2 = \langle G^2 \rangle (2N_1 + N_3 a^2) + N_3 \langle (DG)^2 \rangle$,

$C_3 = -(1 + N_1)\left[\langle (D^2 W)^2 \rangle + 2a^2 \langle (DW)^2 \rangle + a^4 \langle W^2 \rangle\right]$,

$C_4 = \langle (D\Theta)^2 \rangle + a^2 \langle \Theta^2 \rangle$,

$C_5 = -N_5 \langle G\Theta \rangle$,

$C_6 = N_1\left[\langle DG \rangle \langle DW \rangle + a^2 \langle GW \rangle\right]$,

$C_7 = \left[1 - Q(1 - 2z)\right]\langle W\Theta \rangle$,

$C_8 = \langle \Theta W \rangle$,

where the angle bracket $\langle ... \rangle$ denotes the integration by parts with respect to z from 0 to 1.

IV. Result and Discussion

The criterion for the onset of Rayleigh-Benard convection in micropolar fluid in the presence of internal heat generation is investigated theoretically. The sensitiveness of critical Rayleigh number; Ra_c to the changes in the micropolar fluid parameters; N_1, N_3 and N_5 are also studied. Three cases are involved in the system, which are rigid-free, rigid-rigid and free-free surfaces respectively.

Table I shows the comparison of the critical Rayleigh number; Ra_c for different values of coupling parameter; N_1 and internal heat generation effect; Q, when $N_3 = 2$ and $N_5 = 1$. Our findings are compared with Siddeshwar and Pranesh [16] for $Q = 0$ and the results are in a good agreement. We proceed by substituting $Q = 1$, $Q = 3$, $Q = 5$ and we found that the critical Rayleigh number decreasing when we increase the values of Q, for all N_1 values considered. Treating Ra as the critical parameter, we find that the effect of increasing Q in micropolar fluid is to destabilize the system.

TABLE I
COMPARISON OF CRITICAL RAYLEIGH NUMBER IN MICROPOLAR FLUID FOR $N_3 = 2$, $N_5 = 1$

N_1	Ra_c				
	Siddheshwar and Pranesh [16]	Present Study			
	$Q = 0$	$Q = 0$	$Q = 1$	$Q = 3$	$Q = 5$
0.5	2700.06	2700.06	1914.64	1210.39	884.90
1.0	4743.52	4743.52	3076.68	1806.84	1278.97
1.5	8466.87	8466.87	4769.71	2545.56	1735.93
2.0	16976.05	16976.05	7403.13	3471.99	2267.26

Table II shows the comparison of the critical Rayleigh number; Ra_c with internal heat generation; Q for three different types of surfaces. It can be clearly seen that the critical Rayleigh number; Ra_c values decreases as the values of internal heat generation; Q increase for all cases considered. This shows that the effect of internal heat; Q in micropolar fluid is to enhance onset of convection either in rigid-free, rigid-rigid and free-free surfaces. This investigation revealed that the free-free surface is the most unstable system when we increase the value of Q.

TABLE II
COMPARISON OF CRITICAL RAYLEIGH NUMBER FOR DIFFERENT TYPES OF LOWER-UPPER SURFACES

Q	Ra_c		
	Rigid-Free	Rigid-Rigid	Free-Free
0	2700	2863	793
0.1	2593	2826	779
0.2	2495	2789	765
0.3	2404	2754	752
0.4	2319	2719	740
0.5	2240	2686	726
0.6	2166	2653	716
0.7	2097	2621	705
0.8	2032	2589	694
0.9	1972	2559	683
1.0	1914	2529	673

The variation values of Rayleigh number; Ra with wave number; a and $Q = 0, 2, 4$ in three different cases is shown in Fig. 1. The parameters chosen are $N_1 = 0.5$, $N_3 = 2$ and $N_5 = 1$. It is found that the internal heat generation has a rapid impact on the stability of the system especially in the rigid-free and rigid-rigid surfaces. This can be proved by looking at the difference in the Rayleigh number; Ra where Ra number recorded between $Q = 0$ and $Q = 2$ are much larger compared with the value of $Q = 4$. It is proven that the internal heat generation is a destabilizing factor.

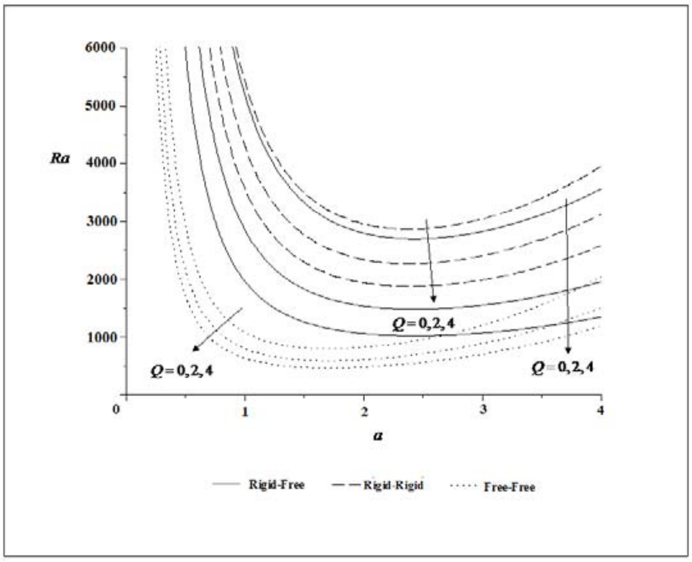

Fig. 1 Variation of Ra with a for different values of Q

Fig. 2 shows the plots of the critical Rayleigh number; Ra_c versus the coupling parameter; N_1 for various values of internal heat generation; Q when $N_3 = 2$ and $N_5 = 1$. In every case considered, we found that the effect of increasing the internal heat generation is to decreases the critical Rayleigh number and thus destabilize the system. However, increasing the value of N_1 helps to delay the onset of convection in the system. In each of these plots, the critical number increases with increasing of N_1 for all values of Q in three different cases considered. N_1 indicates the concentration of microelements, and increasing of N_1 is to elevate the concentration of microelements number. When this happened, a greater part of the energy of the system is consumed by these elements in developing gyrational velocities of the fluid and thus delayed the onset of convection.

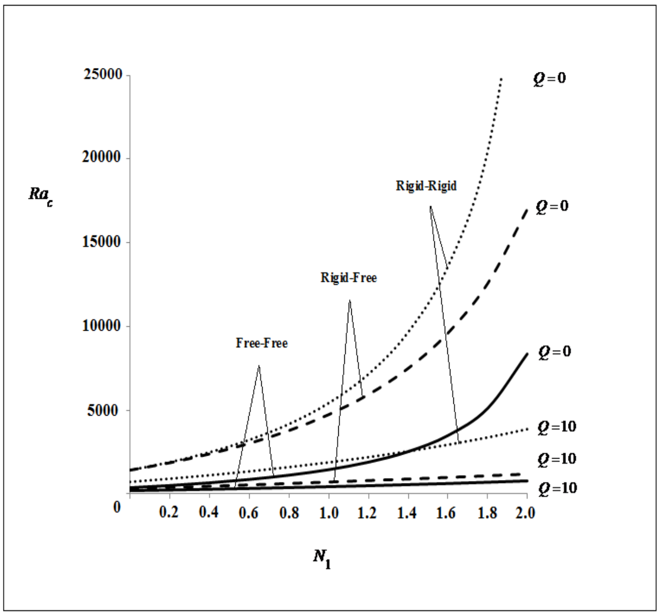

Fig. 2 The critical Rayleigh number; Ra_c as a function of coupling parameter; N_1 for different values of Q

Fig. 3 is the illustration of the couple stress parameter; N_3 when N_1 = 0.1 and N_5 = 10 for various values of internal heat generation; Q. It is found that an increased of Q and N_3 decrease the values of Ra_c for all cases considered and therefore the system become more unstable.

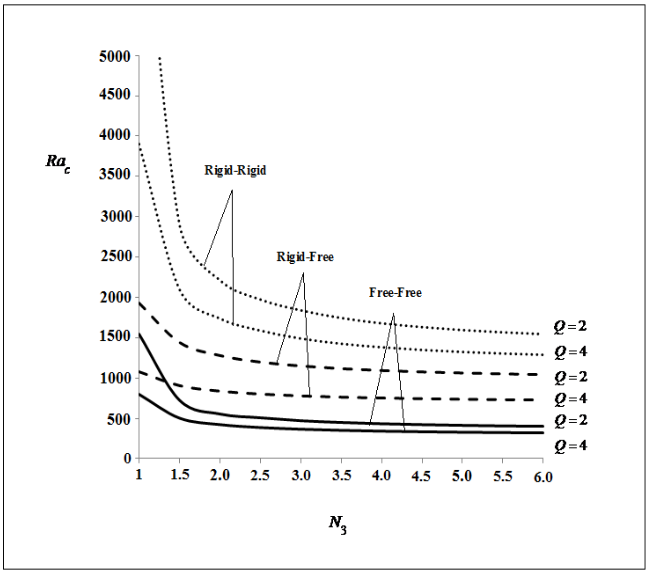

Fig. 3 The critical Rayleigh number; Ra_c as a function of couple stress parameter; N_3 for different values of Q

Fig. 4 shows the plot of Ra_c versus micropolar heat conduction parameter; N_5 when N_1 = 0.1 and N_5 = 2 with various values of Q. Although the effect of internal heat generation is still to destabilize the system, an increasing of the micropolar heat conduction; N_5, promotes stability in the system. The reason behind this is, when N_5 increases, the heat induced into the fluid due to the microelements is also increased and thus reducing the heat transfer from the bottom to the top of the system. The decrease in heat transfer is responsible for delaying the onset of convection. Thus increasing N_5 promotes stability in the micropolar fluid.

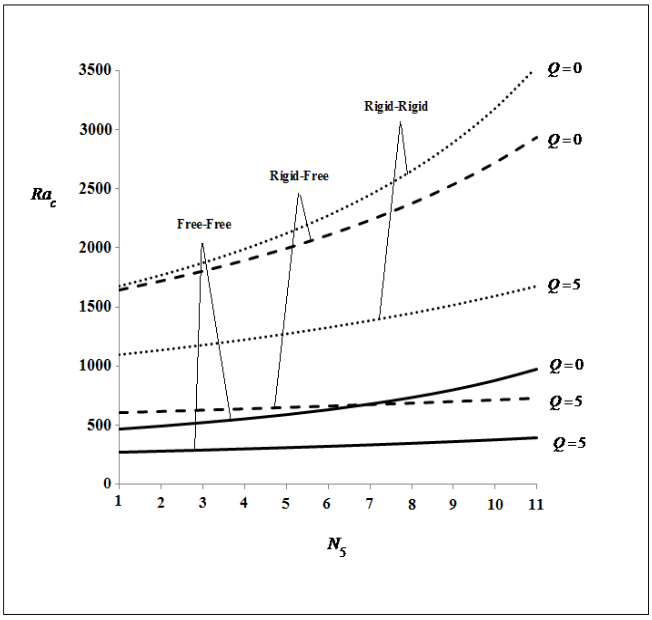

Fig. 4 The critical Rayleigh number; Ra_c as a function of heat conduction parameter; N_5 for different values of Q

V. Conclusion

The stability analysis of the Rayleigh-Benard convection in micropolar fluid with internal heat generation is investigated theoretically. It is found that the effect of internal heat generation; Q in the micropolar fluid has a significant influence on the Rayleigh-Benard convection and is clearly a destabilizing factor to make the system more unstable. For three cases considered, rigid-free, rigid- rigid and free-free surfaces, it is found that the critical values of the Rayleigh number in rigid-rigid surfaces are the highest. This show that the used of rigid-rigid surfaces can delay the onset of convection. The coupling parameter; N_1, couple stress parameter; N_3 and micropolar heat conduction; N_5, has a significant effect on the onset of the Rayleigh-Benard convection. Although the effect of internal heat generation; Q is to destabilize the system, the increase of the microelement concentration; N_1 and N_5 helps to slow down the process of destabilizing.

Acknowledgment

Authors gratefully acknowledge the financial support received from Universiti Putra Malaysia under RUG Scheme.

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