A Family of Entropies on Interval-valued Intuitionistic Fuzzy Sets and Their Applications in Multiple Attribute Decision Making

Min Sun, Jing Liu

Abstract—The entropy of intuitionistic fuzzy sets is used to indicate the degree of fuzziness of an interval-valued intuitionistic fuzzy set(IvIFS). In this paper, we deal with the entropies of IvIFS. Firstlt, we propose a family of entropies on IvIFS with a parameter $\lambda \in [0,1]$, which generalize two entropy measures defined independently by Zhang and Wei, for IvIFS, and then we prove that the new entropy is an increasing function with respect to the parameter λ. Furthermore, a new multiple attribute decision making(MADM) method using entropy-based attribute weights is proposed to deal with the decision making situations where the alternatives on attributes are expressed by IvIFS and the attribute weights information is unknown. Finally, a numerical example is given to illustrate the applications of the proposed method.

Keywords—Interval-valued intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy entropy, multiple attribute decision making

I. INTRODUCTION

TO deal with vagueness and uncertainty in many real-life areas, Zadeh[1] introduced fuzzy set(FS), which has achieved a great success in various fields, such as group decision, medical diagnosis, pattern recognition. After that, many scholars have investigated FS and a lot of generalized forms have been proposed. Among them, intuitionistic fuzzy set(IFS) proposed Atanassov[2] and interval-valued intuitionistic fuzzy sets(IvIFS) introduced by Atanassov and Gargov[3] are two well-known generalizations of the conventional fuzzy set theory. Both of them alleviate some drawbacks of Zadeh's fuzzy set. Entropy is a very important notion for measuring uncertain information, which was first mentioned by Zadeh[1]. Later, many researchers have investigated entropy from different aspects, such as Deluca and Termin[4], Kaufmann[5], Yager[6], Szmidt and Kacprzyk[7] introduced different entropies on fuzzy set. As for intuitionistic fuzzy set, Bustince and Burillo[8] firstly introduced an entropy on IFS in 1996, and then Hung[9], Zhang[10], Vlachos and Sergiadis[11], Zeng[12] presented different entropies on IFS from different aspects. In respect to IvIFS, Zhang[13], Ye[14], Zhang et al.[15], Wei et al.[16] proposed some entropies on IvIFS. Although IvIFS is an important extension of FS, there are few works involving the entropy on it. Therefore, it is worth studying the entropy on IvIFS.

Mainly motivated by the entropies proposed by Zhang et al.[15] and Wei et al.[16], in this paper, we will present a

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family of entropies on IvIFS by introducing a parameter $\lambda \in [0,1]$ to the entropy presented by Wei et al.[16]. Thus, a set of entropy measures (depending on $\lambda \in [0,1]$) is defined, which include the entropies in [15] and [16] as special cases. That is, if $\lambda = 0$, then our entropy is reduced to the entropy in [15], and if $\lambda = 1$, then our entropy is reduced to the entropy in [16].

A multiple attribute decision making(MADM) problem is to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and qualitative[17]. In order to choose a desirable solution, decision maker often provide his/her preference information which takes the form of numerical values, such as exact values, interval-number values, FS, IFS and IvIFS. MADM is an interesting research topic having received more and more attention from researchers during the last several years. However, many researchers mainly focus on the MADM problems with known or incompletely known attribute weight information under interval-valued intuitionistic fuzzy environment, and there is little research on MADM with completely unknown attribute weight information in the existing literature[18]. Thus, in this paper, we will utilize the proposed IvIFS entropy to assess attribute weights based on the IvIFS decision making matrix.

The rest of the paper is organized as follows. In Section 2, we introduce the concept of IvIFS and some basic relations, and propose a set of IvIFS entropies. In Section 3, a new MADM method using entropy-based attribute weights under IvIFS environment is constructed. A numerical example is given to demonstrate the effectiveness of the proposed method in Section 4. Concluding remarks are drawn in Section 5.

II. A FAMILY OF ENTROPIES ON INTERVAL-VALUED INTUITIONISTIC FUZZY SETS

In this paper, let $[I]$ denote the set of all the closed subintervals of $[0,1]$.

Definition 2.1.[5] Let $[a_1, b_1], [a_2, b_2] \in [I]$, we define

$$[a_1, b_1] \leq [a_2, b_2], \text{ iff } a_1 \leq a_2, b_1 \leq b_2;$$

$$[a_1, b_1] \preceq [a_2, b_2], \text{ iff } a_1 \leq a_2, b_1 \geq b_2; [a_1, b_1] = [a_2, b_2], \text{ iff } a_1 = a_2, b_1 = b_2.$$

Definition 2.2.[1] Let X be a universe of discourse. An intuitionistic fuzzy set(IFS) in X is an object having the form:

$$A = \{\langle x, \mu_A(x), v_A(x) \rangle | x \in X\},$$

where

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$$\mu_A : X \to [0,1], v_A : X \to [0,1]$$

with the condition

$$0 \leq \mu_A(x) + v_A(x) \leq 1, \forall x \in X.$$

The numbers $\mu_A(x)$ and $v_A(x)$ denote the degree of membership and non-membership of x to A, respectively.

For convenience, we denote by IFS(X) the set of all the IFS in X.

Definition 2.3.[2] An interval-valued intuitionistic fuzzy set(IvIFS) A in the finite universe X is expressed by the form

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\},$$

where $\mu_A(x) = [\mu_A^-(x), \mu_A^+(x)] \in [I]$ is called membership interval of element x to IvIFS A, while $v_A(x) = [v_A^-(x), v_A^+(x)] \in [I]$ is the non-membership interval of that element to the set A, and the condition $0 \leq \mu_A^+(x) + v_A^+(x) \leq 1$ must hold for any $x \in X$.

A point x is said to be crossover point of IvIFS A, if $\mu_A(x) = v_A(x)$.

An IvIFS A is viewed as a most vague IvIFS, also denoted by A_F, if $\mu_A(x) = v_A(x)$ for all $x \in X$.

For convenience of notations, we denote by IvIFS(X) the set of all the IvIFS in X.

We call the interval

$$[1 - \mu_A^+(x) - v_A^+(x), 1 - \mu_A^-(x) - v_A^-(x)],$$

abbreviated by $[\pi_A^-(x), \pi_A^+(x)]$ and denoted by $\pi_A(x)$, the interval-valued intuitionistic index of x in A, which is a hesitancy degree of x to A.

Definition 2.4.[2] Let $A, B \in$ IvIFS(X), then some operations can be defined as follows:

$$A \cup B = \{\langle x_i, [\mu_A^-(x_i) \vee \mu_B^-(x_i), \mu_A^+(x_i) \vee \\ \mu_B^+(x_i)], [v_A^-(x_i) \wedge v_B^-(x_i), v_A^+(x_i) \wedge v_B^+(x_i)]\rangle | x_i \in X\};$$

$$A \cap B = \{\langle x_i, [\mu_A^-(x_i) \wedge \mu_B^-(x_i), \mu_A^+(x_i) \wedge \\ \mu_B^+(x_i)], [v_A^-(x_i) \vee v_B^-(x_i), v_A^+(x_i) \vee v_B^+(x_i)]\rangle | x_i \in X\},$$

where $\vee, \wedge$ stand for max and min operators, respectively.

$$A^C = \{\langle x_i, [v_A^-(x_i), v_A^+(x_i)], [\mu_A^-(x_i), \mu_A^+(x_i)]\rangle | x_i \in X\};$$

$$A \subseteq B, \text{ iff } [\mu_A^-(x_i), \mu_A^+(x_i)] \leq [\mu_B^-(x_i), \mu_B^+(x_i)],$$

$$\text{and } [v_A^-(x_i), v_A^+(x_i)] \geq [v_B^-(x_i) v_B^+(x_i)] \quad x_i \in X;$$

$$A \preceq B, \text{ iff } [\mu_A^-(x_i), \mu_A^+(x_i)] \preceq [\mu_B^-(x_i), \mu_B^+(x_i)],$$

$$\text{and } [v_A^-(x_i), v_A^+(x_i)] \succeq [v_B^-(x_i) v_B^+(x_i)] \quad x_i \in X.$$

Now we give the entropy concept of IvIFS which is similar to the work of Zhang et al.[15].

Definition 2.5. A real function $E : \text{IvIF}(X) \to [0,1]$ is named an entropy on IvIFSs, if E satisfies all the following properties:

(E1) $E(A) = 0$ iff A is a crisp set;

(E2) $E(A) = 1$ iff $\mu_A(x_i) = v_A(x_i), \forall x_i \in X$;

(E3) $E(A) = E(A^C)$;

(E4) $E(A) \leq E(B)$ if A is less fuzzy than B, which is defined as

$$\mu_A(x_i) \leq \mu_B(x_i), v_A(x_i) \geq v_B(x_i), \text{ for } \mu_B(x_i) \leq v_B(x_i);$$

$$\mu_A(x_i) \geq \mu_B(x_i), v_A(x_i) \leq v_B(x_i), \text{ for } \mu_B(x_i) \geq v_B(x_i);$$

$$\mu_A(x_i) \preceq \mu_B(x_i), v_B(x_i) \succeq v_A(x_i), \text{ for } \mu_B(x_i) \preceq v_B(x_i);$$

$$\mu_A(x_i) \succeq \mu_B(x_i), v_B(x_i) \preceq v_A(x_i), \text{ for } \mu_B(x_i) \succeq v_B(x_i).$$

Now we recall some entropy formulas for an IvIFS. For an IvIFS $A = \{\langle x_i, \mu_A(x_i), v_A(x_i)\rangle | x_i \in X\}$, Zhang et al.[15] defined he following entropy E_{ZJ} for A:

$$E_{\text{ZJ}}(A) = \frac{1}{n} \sum_{i=1}^{n} \frac{\mu_A^-(x_i) \wedge v_A^-(x_i) + \mu_A^+(x_i) \wedge v_A^+(x_i)}{\mu_A^-(x_i) \vee v_A^-(x_i) + \mu_A^+(x_i) \vee v_A^+(x_i)}. \quad (1)$$

Especially, when $[\mu_A^-(x_i), \mu_A^+(x_i)] = [0,0], [v_A^-(x_i), v_A^+(x_i)] = [0,0], \forall x_i \in X$, they put $E_{\text{ZJ}}(A) = 1$.

For an IvIFS A, Wei et al.[16] gave a different entropy formula by

$$E_{\text{WW}}(A) =$$

$$\frac{1}{n} \sum_{i=1}^{n} \frac{\mu_A^-(x_i) \wedge v_A^-(x_i) + \mu_A^+(x_i) \wedge v_A^+(x_i) + \pi_A^-(x_i) + \pi_A^+(x_i)}{\mu_A^-(x_i) \vee v_A^-(x_i) + \mu_A^+(x_i) \vee v_A^+(x_i) + \pi_A^-(x_i) + \pi_A^+(x_i)}. \quad (2)$$

Example 2.1. Let $X = \{x_1, x_2, \cdots, x_n\}$ be a universe of discourse. Let $A_1 = \{\langle x_i, [0.1, 0.2], [0.2, 0.4]\rangle | x_i \in X\}, A_2 = \{\langle x_i, [0.2, 0.2], [0.3, 0.5]\rangle | x_i \in X\}$. Calculate the entropies of A_1 and A_2.

From the entropy formula $E_{\text{ZJ}}(A)$, we have:

$$E_{\text{ZJ}}(A_1) = \frac{0.1 + 0.2}{0.2 + 0.4} = 0.5, E_{\text{ZJ}}(A_2) = \frac{0.2 + 0.2}{0.3 + 0.5} = 0.5.$$

Therefore $E_{\text{ZJ}}(A_1) = E_{\text{ZJ}}(A_2)$, then $E_{\text{ZJ}}(A)$ can not distinguish the fuzzyness of A_1 and A_2.

Example 2.2. Let $X = \{x_1, x_2, \cdots, x_n\}$ be a universe of discourse. Let $A_3 = \{\langle x_i, [0.2, 0.4], [0,0]\rangle | x_i \in X\}$, $A_4 = \{\langle x_i, [3/10, 2/5], [0, 1/7]\rangle | x_i \in X\}$. Calculate the entropies of A_3 and A_4.

From the entropy formula $E_{\text{WW}}(A)$, we have:

$$E_{\text{WW}}(A_3) = \frac{0 + 0 + 1.4}{0.2 + 0.4 + 1.4} = 0.7.$$

$$E_{\text{WW}}(A_4) = \frac{0 + 1/7 + (2 - 3/10 - 2/5 - 1/7)}{3/10 + 2/5 + (2 - 3/10 - 2/5 - 1/7))} = 0.7.$$

Therefore $E_{\text{WW}}(A_3) = E_{\text{WW}}(A_4)$, then $E_{\text{WW}}(A)$ can not distinguish the fuzzyness of A_3 and A_4.

Motivated by the entropies (1) and (2), now we give an entropy measure for IvIFSs. For each $A \in \text{IvIFS}(X)$, define $E(A, \lambda)$ by

$$E(A, \lambda) =$$

$$\frac{1}{n} \sum_{i=1}^{n} \frac{\mu_A^-(x_i) \wedge v_A^-(x_i) + \mu_A^+(x_i) \wedge v_A^+(x_i) + \lambda(\pi_A^-(x_i) + \pi_A^+(x_i))}{\mu_A^-(x_i) \vee v_A^-(x_i) + \mu_A^+(x_i) \vee v_A^+(x_i) + \lambda(\pi_A^-(x_i) + \pi_A^+(x_i))}, \quad (3)$$

where $\lambda \in [0,1]$ is a parameter.

Obviously, if $\lambda = 0$, then $E(A, \lambda)$ is reduced to the entropy $E_{\text{ZJ}}(A)$; Meanwhile, if $\lambda = 1$, then $E(A, \lambda)$ degenerates to the entropy $E_{\text{WW}}(A)$. In the following, we will restrict the parameter $\lambda \in (0, 1)$.

Theorem 2.1. The mapping $E(A, \lambda)$, defined by (3), is an entropy measure for IvIFS, i.e., it satisfies all the properties in Definition 2.5.

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Proof. We only need to prove that all the properties in Definition 2.5 hold.

(E1) If A is a crisp set, then for each $x_i \in X$, we have

$$[\mu_A^-(x_i), \mu_A^+(x_i)] = [0,0], [v_A^-(x_i), v_A^+(x_i)] = [1,1];$$

or

$$[\mu_A^-(x_i), \mu_A^+(x_i)] = [1,1], [v_A^-(x_i), v_A^+(x_i)] = [0,0],$$

So, $\pi_A(x_i) = [0,0]$ for each $x_i \in X$. From (3) we obtain that $E(A, \lambda) = 0$.

On the other hand, suppose now that $E(A, \lambda) = 0$. From (3) and $\lambda \in (0,1)$, we have

$$\mu_A^-(x_i) \wedge v_A^-(x_i) = 0, \mu_A^+(x_i) \wedge v_A^+(x_i) = 0,$$
$$\pi_A^-(x_i) = 0, \pi_A^+(x_i) = 0.$$

This set of equations implies that A is a crisp set.

(E2) If $\mu_A(x_i) = v_A(x_i)$, $\forall x_i \in X$; i.e.,

$$\mu_A^-(x_i) = v_A^-(x_i), \mu_A^+(x_i) = v_A^+(x_i), \forall x_i \in X,$$

then

$$\mu_A^-(x_i) \wedge v_A^-(x_i) + \mu_A^+(x_i) \wedge v_A^+(x_i) = \\ \mu_A^-(x_i) \vee v_A^-(x_i) + \mu_A^+(x_i) \vee v_A^+(x_i).$$

Thus, $E(A, \lambda) = 1$.

On the other hand, if $E(A, \lambda) = 1$, then for each $x_i \in X$, we immediately obtain that

$$\mu_A^-(x_i) \wedge v_A^-(x_i) = \mu_A^-(x_i) \vee v_A^-(x_i), \mu_A^+(x_i) \wedge v_A^+(x_i) = \\ \mu_A^+(x_i) \vee v_A^+(x_i).$$

Hence, $\mu_A^-(x_i) = v_A^-(x_i), \mu_A^+(x_i) = v_A^+(x_i)$, which means that $\mu_A(x_i) = v_A(x_i)$.

(E3) It is obvious that $E(A^C, \lambda) = E(A, \lambda)$ from $A^C = \{\langle x_i, [v_A^-(x_i), v_A^+(x_i)], [\mu_A^-(x_i), \mu_A^+(x_i)]\rangle | x_i \in X\}$.

(E4) To prove it, we need to separate X into four parts as follows:

$$X_1 = \{x_i \in X | \mu_B(x_i) \leq v_B(x_i)\},$$
$$X_2 = \{x_i \in X | \mu_B(x_i) \geq v_B(x_i)\},$$
$$X_3 = \{x_i \in X | \mu_B(x_i) \preceq v_B(x_i)\},$$
$$X_4 = \{x_i \in X | \mu_B(x_i) \succeq v_B(x_i)\}.$$

Suppose A is less fuzzy than B. If $x_i \in X_1$, then $\mu_A(x_i) \leq \mu_B(x_i), v_A(x_i) \geq v_B(x_i)$, for $\mu_B(x_i) \leq v_B(x_i)$; i.e.,

$$\mu_A^-(x_i) \leq \mu_B^-(x_i) \leq v_B^-(x_i) \leq v_A^-(x_i),$$
$$\mu_A^+(x_i) \leq \mu_B^+(x_i) \leq v_B^+(x_i) \leq v_A^+(x_i).$$

Therefore, we have

$$\mu_A^-(x_i) + \mu_A^+(x_i) + \lambda(\pi_A^-(x_i) + \pi_A^+(x_i))$$
$$= 2\lambda + (1-\lambda)\mu_A^-(x_i) + (1-\lambda)\mu_A^+(x_i) - \lambda v_A^-(x_i) \\ - \lambda v_A^+(x_i)$$
$$\leq 2\lambda + (1-\lambda)\mu_B^-(x_i) + (1-\lambda)\mu_B^+(x_i) - \lambda v_B^-(x_i) \\ - \lambda v_B^+(x_i)$$
$$= \mu_B^-(x_i) + \mu_B^+(x_i) + \lambda(\pi_B^-(x_i) + \pi_B^+(x_i)).$$

$$v_A^-(x_i) + v_A^+(x_i) + \lambda(\pi_A^-(x_i) + \pi_A^+(x_i))$$
$$= 2\lambda + (1-\lambda)v_A^-(x_i) + (1-\lambda)v_A^+(x_i) - \lambda\mu_A^-(x_i) \\ - \lambda\mu_A^+(x_i)$$
$$\geq 2\lambda + (1-\lambda)v_B^-(x_i) + (1-\lambda)v_B^+(x_i) - \lambda\mu_B^-(x_i) \\ - \lambda\mu_B^+(x_i)$$
$$= v_B^-(x_i) + v_B^+(x_i) + \lambda(\pi_B^-(x_i) + \pi_B^+(x_i)).$$

Hence by (3), we have $E(A, \lambda) \leq E(B, \lambda)$. Similarly, we can prove that the above inequality holds for all $x_i \in X_2, x_i \in X_3, x_i \in X_4$, respectively. This completes the proof.

Remark 2.1. If $A \in \text{IFS}(X)$, then $\mu_A = \mu_A^-(x_i) = \mu_A^+(x_i), v_A = v_A^-(x_i) = v_A^+(x_i)$. Hence the entropy measure formula given by (3), reduces to the entropy formulas on $\text{IFS}(X)$, which is new. So we have the following corollary.

Corollary 2.1. The mapping $E(A, \lambda)$, defined by

$$E(A, \lambda) = \frac{1}{n}\sum_{i=1}^{n}\frac{\mu_A(x_i) \wedge v_A(x_i) + \lambda\pi_A(x_i)}{\mu_A(x_i) \vee v_A(x_i) + \lambda\pi_A(x_i)},$$

where $\lambda \in [0,1]$ is a parameter, is an entropy on $\text{IFS}(X)$.

The proposed entropy $E(A, \lambda)$ also has the following important property.

Theorem 2.2. Let $A, B \in \text{IvIFS}(X)$ and if they satisfy that for any $x_i \in X$, either $A(x_i) \subseteq B(x_i)$ or $B(x_i) \subseteq A(x_i)$, then we can get

$$E(A, \lambda) + E(B, \lambda) = E(A \cap B, \lambda) + E(A \cup B, \lambda),$$

for any $\lambda \in [0,1]$.

Proof. Its proof is similar to that of Theorem 2 in [15].

Theorem 2.3. The proposed IvIFS entropy as equation (3) is an increasing function of λ on [0,1].

Proof. The proof is obvious. Thus it is omitted.

To validate the Theorem 2.3, we introduce the following numerical example.

Example 2.3. Assume that there are three IvIFS $A_i(i = 1, 2, 3)$ on $X = \{x\}$. We adopt (3) to calculate the entropies of $A_i(i = 1, 2, 3)$ by choosing different values of λ. The IvIFS $A_i(i = 1, 2, 3)$ and their entropies based on different values of λ are listed in Table 1(A_1=([0.3,0.7],[0.1,0.3]), A_2=([0.4,0.4],[0.5,0.5]), A_3=([0.2,0.5],[0.1,0.2])). The results

TABLE I
IvIFS ENTROPIES OF $A_i(i = 1, 2, 3)$ WITH DIFFERENT λ.

	A_1	A_2	A_3
$E(A_i, 0.1)$	0.43	0.8039	0.50
$E(A_i, 0.2)$	0.46	0.8077	0.56
$E(A_i, 0.3)$	0.49	0.8113	0.60
$E(A_i, 0.4)$	0.52	0.8148	0.64
$E(A_i, 0.5)$	0.54	0.8181	0.67
$E(A_i, 0.6)$	0.56	0.8214	0.69
$E(A_i, 0.7)$	0.58	0.8246	0.71
$E(A_i, 0.8)$	0.59	0.8276	0.73
$E(A_i, 0.9)$	0.61	0.8305	0.75

of Example 2.3 confirm Theorem 2.3. In fact, a set of entropy measures is defined in Equation (3), which depends on the parameter $\lambda \in [0,1]$, and the choice of λ mainly depends on the specific application.

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III. MULTIPLE ATTRIBUTE DECISION MAKING METHOD

In this section, we present a multiple attribute decision making method using entropy-based attribute weights with alternatives on attributes denoted by IvIFS, and the attribute weights information for alternatives is unknown. Let $A = \{A_1, A_2, \cdots, A_m\}$ be a discrete set of alternatives, and $G = \{G_1, G_2, \cdots, G_n\}$ be the set of attributes. The IvIFS decision D of A on G is as below:

$$D = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix},$$

where $a_{ij} = ([\mu_{ij}^-, \mu_{ij}^+], [v_{ij}^-, v_{ij}^+])(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ denotes an IvIFS value. Now we give a method of MADM based on the proposed entropy formula.

Step 1. In order to eliminate the impact of different physical dimension to the decision making result, we need to normalize each attribute value $\tilde{a}_{ij}$ in the matrix D into a corresponding element in the matrix $R = (\tilde{r}_{ij})_{m \times n} = ([\tilde{\mu}_{ij}^-, \tilde{\mu}_{ij}^+], [\tilde{v}_{ij}^-, \tilde{v}_{ij}^+])_{m \times n}$. Consider that there are generally benefit attributes and cost attributes. The normalizing methods are shown as follows[18]:

$$\begin{cases} \tilde{\mu}_{ij}^- = \mu_{ij}^- / \sqrt{\sum_{i=1}^n (2 - v_{ij}^- - v_{ij}^+)^2} \\ \tilde{\mu}_{ij}^+ = \mu_{ij}^+ / \sqrt{\sum_{i=1}^n (2 - v_{ij}^- - v_{ij}^+)^2} \\ \tilde{v}_{ij}^- = 1 - (1 - v_{ij}^-) / \sqrt{\sum_{i=1}^n (\mu_{ij}^- + \mu_{ij}^+)^2} \\ \tilde{v}_{ij}^+ = 1 - (1 - v_{ij}^+) / \sqrt{\sum_{i=1}^n (\mu_{ij}^- + \mu_{ij}^+)^2} \end{cases}$$

for benefit attributes $G_j, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$, and

$$\begin{cases} \tilde{\mu}_{ij}^- = (1 - v_{ij}^-)^{-1} / \sqrt{\sum_{i=1}^n ((1/\mu_{ij}^-) + (1/\mu_{ij}^+))^2} \\ \tilde{\mu}_{ij}^+ = (1 - v_{ij}^+)^{-1} / \sqrt{\sum_{i=1}^n ((1/\mu_{ij}^-) + (1/\mu_{ij}^+))^2} \\ \tilde{v}_{ij}^- = 1 - (1/\mu_{ij}^-) / \sqrt{\sum_{i=1}^n ((1 - v_{ij}^-)^{-1} + (1 - v_{ij}^+)^{-1})^2} \\ \tilde{v}_{ij}^+ = 1 - (1/\mu_{ij}^+) / \sqrt{\sum_{i=1}^n ((1 - v_{ij}^-)^{-1} + (1 - v_{ij}^+)^{-1})^2} \end{cases}$$

for cost attributes $G_j, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.

Step 2. Set $\lambda \in [0, 1]$. Then based on the proposed entropy formula (3), we obtain the entropy matrix $E = (e_{ij})_{m \times n}$ of the normalized decision matrix R, where $e_{ij} = E(\tilde{r}_{ij}, \lambda)$ for $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$. Then the information entropy of attribute G_j is defined as follows[18]:

$$E_j = \frac{1}{m} \sum_{i=1}^m e_{ij},$$

Thus the attribute weight $w_j (j = 1, 2, \ldots, n)$ is calculated by

$$w_j = \frac{1 - E_j}{\sum_{j=1}^n (1 - E_j)}.$$

That is, if the entropy value for an attribute is smaller across alternatives, it should provide decision-maker with the useful information. Therefore, the attribute should be assigned a bigger weight; otherwise, such an attribute will be judged unimportant by decision-maker. In other words, such an attribute should be assigned a very small weight[21].

Step 3. Based on the attribute weights in Step 2, we obtain the weighted arithmetic average value expressed by $\alpha_i = ([a_i, b_i], [c_i, d_i])$ for $A_i (i = 1, 2, \ldots, m)$ using the interval-valued intuitionistic fuzzy weighted averaging(IIFWA) operator[19]:

$$\begin{aligned} \alpha_i &= \text{IIFWA}_w(\tilde{r}_{i1}, \tilde{r}_{i2}, \ldots, \tilde{r}_{in}) \\ &= w_1 \tilde{r}_{i1} \oplus w_2 \tilde{r}_{i2} \oplus \cdots w_n \tilde{r}_{in} \\ &= ([1 - \prod_{j=1}^n (1 - \tilde{\mu}_{ij}^-)^{w_j}, 1 - \prod_{j=1}^n (1 - \tilde{\mu}_{ij}^+)^{w_j}], \\ &\quad [\prod_{j=1}^n (\tilde{v}_{ij}^-)^{w_j}, \prod_{j=1}^n (\tilde{v}_{ij}^+)^{w_j}]). \end{aligned}$$

Step 4. Calculate the scores $S(\tilde{\alpha}_i)(i = 1, 2, \ldots, m)$ and the accuracy $H(\tilde{\alpha}_i)(i = 1, 2, \ldots, m)$ of the collective overall intuitionistic fuzzy preference values $\tilde{\alpha}_i(i = 1, 2, \ldots, m)$, where $S(\tilde{\alpha}_i)$ and $H(\tilde{\alpha}_i)$ are defined as follows:

$$S(\tilde{\alpha}_i) = \frac{1}{2}(a_i - c_i + b_i - d_i),$$

and

$$H(\tilde{\alpha}_i) = \frac{1}{2}(a_i + c_i + b_i + d_i).$$

Step 5. Rank all the alternative $A_i (i = 1, 2, \ldots, m)$ and then to select the best one(s) in accordance with $S(\tilde{\alpha}_i)$ and $H(\tilde{\alpha}_i)(i = 1, 2, \ldots, m)$.

Step 6. End.

IV. ILLUSTRATIVE EXAMPLE

In this section, we utilize a practical multiple attribute decision making problem to illustrate the application of the developed approach.

Suppose an organization plans to implement ERP system(adapted from [20]). The first step is to form a project team that consists of CIO and two senior representatives from user departments. By collecting all possible information about ERP vendors and systems, project team choose five potential ERP systems $A_i (i = 1, 2, \ldots, 5)$ as candidates. The company employs some external professional organizations (or experts) to aid this decision-making. The project team selects four attributes to evaluate the alternatives: (1)function and technology G_1; (2) strategic fitness G_2; (3) vendor's ability G_3; (4)vendor's reputation G_4. The five possible suppliers $A_i (i = 1, 2, \ldots, 5)$ are to be evaluated using the interval-valued intuitionistic fuzzy numbers under the above four attributes. The decision matrix is listed in the following matrices $D = (a_{ij})_{5 \times 4}$ as follows:

$$D = \begin{bmatrix} ([0.5, 0.6], [0.2, 0.3]) & ([0.4, 0.7], [0.2, 0.3]) \\ ([0.3, 0.5], [0.2, 0.4]) & ([0.2, 0.4], [0.4, 0.5]) \\ ([0.5, 0.7], [0.1, 0.2]) & ([0.1, 0.4], [0.5, 0.6]) \\ ([0.5, 0.6], [0.3, 0.4]) & ([0.0, 0.1], [0.7, 0.8]) \\ ([0.4, 0.5], [0.4, 0.5]) & ([0.5, 0.6], [0.1, 0.2]) \end{bmatrix}$$

$$\begin{bmatrix} ([0.3, 0.6], [0.3, 0.4]) & ([0.4, 0.5], [0.2, 0.4]) \\ ([0.4, 0.7], [0.2, 0.3]) & ([0.0, 0.3], [0.5, 0.7]) \\ ([0.6, 0.8], [0.1, 0.2]) & ([0.2, 0.4], [0.3, 0.6]) \\ ([0.2, 0.4], [0.3, 0.6]) & ([0.5, 0.7], [0.1, 0.2]) \\ ([0.2, 0.5], [0.3, 0.4]) & ([0.6, 0.8], [0.1, 0.2]) \end{bmatrix}$$

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Then, we utilize the developed approach in Section 3 to get the most desirable alternative(s).

Step 1. Firstly, calculate the normalized decision matrix R, and the result is shown as follows:

$$R = \begin{bmatrix} ([0.16, 0.19], [0.65, 0.70]) & ([0.15, 0.26], [0.57, 0.62]) \\ ([0.09, 0.16], [0.65, 0.74]) & ([0.07, 0.15], [0.67, 0.73]) \\ ([0.16, 0.22], [0.61, 0.65]) & ([0.04, 0.15], [0.73, 0.78]) \\ ([0.16, 0.19], [0.70, 0.74]) & ([0.00, 0.04], [0.84, 0.89]) \\ ([0.13, 0.16], [0.74, 0.78]) & ([0.18, 0.22], [0.51, 0.57]) \end{bmatrix}$$

$$\begin{bmatrix} ([0.10, 0.19], [0.68, 0.73]) & ([0.13, 0.16], [0.63, 0.72]) \\ ([0.13, 0.22], [0.64, 0.68]) & ([0.00, 0.10], [0.77, 0.86]) \\ ([0.19, 0.26], [0.59, 0.64]) & ([0.06, 0.13], [0.68, 0.81]) \\ ([0.06, 0.13], [0.68, 0.82]) & ([0.16, 0.23], [0.58, 0.63]) \\ ([0.06, 0.16], [0.68, 0.73]) & ([0.19, 0.26], [0.58, 0.63]) \end{bmatrix}$$

Step 2. Set $\lambda = 0.5$, then the entropy matrix e of the normalized decision matrix R is

$$e = \begin{bmatrix} 0.3333 & 0.4388 & 0.2821 & 0.3072 \\ 0.2739 & 0.2579 & 0.3468 & 0.1331 \\ 0.3889 & 0.2048 & 0.4388 & 0.2121 \\ 0.2945 & 0.0840 & 0.2085 & 0.4184 \\ 0.2384 & 0.4925 & 0.2539 & 0.4493 \end{bmatrix}$$

and the entropy vector of attribute $G_j(j = 1, 2, \ldots, 4)$ is

$$E = [0.3058, 0.2956, 0.3060, 0.3040].$$

Thus the attribute weight vector is

$$w = (0.2489, 0.2526, 0.2489, 0.2496).$$

If we set $\lambda = 1$, then the entropy matrix e of the normalized decision matrix R is

$$e = \begin{bmatrix} 0.3939 & 0.5094 & 0.3450 & 0.3801 \\ 0.3486 & 0.3371 & 0.4121 & 0.1947 \\ 0.4568 & 0.2707 & 0.4968 & 0.2818 \\ 0.3394 & 0.1378 & 0.2762 & 0.4907 \\ 0.2807 & 0.5750 & 0.3315 & 0.5097 \end{bmatrix}$$

and the entropy vector of attribute $G_j(j = 1, 2, 3, 4)$ is

$$E = [0.3639, 0.3660, 0.3723, 0.3714].$$

Thus the attribute weight vector is

$$w = (0.2518, 0.2509, 0.2484, 0.2488).$$

Step 3. Using the interval-valued intuitionistic fuzzy weighted averaging(IIFWA) operator, we obtain the weighted arithmetic average value expressed by $\alpha_i = ([a_i, b_i], [c_i, d_i])$ for $A_i(i = 1, 2, \ldots, 5)$:

$$\alpha_1 = ([0.1354, 0.2011], [0.6310, 0.6908]),$$

$$\alpha_2 = ([0.0736, 0.1585], [0.6807, 0.7497]),$$

$$\alpha_3 = ([0.1145, 0.1915], [0.6504, 0.7163]),$$

$$\alpha_4 = ([0.0973, 0.1501], [0.6944, 0.7641]),$$

$$\alpha_5 = ([0.1417, 0.2013], [0.6207, 0.6720])$$

for $\lambda = 0.5$, and

$$\alpha_1 = ([0.1354, 0.2009], [0.6311, 0.6910]),$$

$$\alpha_2 = ([0.0737, 0.1585], [0.6806, 0.7497]),$$

$$\alpha_3 = ([0.1147, 0.1917], [0.6502, 0.7160]),$$

$$\alpha_4 = ([0.0976, 0.1503], [0.6943, 0.7639]),$$

$$\alpha_5 = ([0.1416, 0.2011], [0.6212, 0.6725])$$

for $\lambda = 1$.

Step 4. Calculate the scores $S(\alpha_i)(i = 1, 2, \ldots, 5)$ of the collective overall preference values $\alpha_i(i = 1, 2, 3, 4, 5)$:

$$S(\alpha_1) = -0.4926, S(\alpha_2) = -0.5992,$$

$$S(\alpha_3) = -0.5303, S(\alpha_4) = -0.6056, S(\alpha_5) = -0.4749$$

for $\lambda = 0.5$, and

$$S(\alpha_1) = -0.4929, S(\alpha_2) = -0.5990,$$

$$S(\alpha_3) = -0.5299, S(\alpha_4) = -0.6052, S(\alpha_5) = -0.4755$$

for $\lambda = 1$.

Step 5. Rank all the alternatives $A_i(i = 1, 2, \ldots, 5)$ in accordance with the scores $S(\alpha_i)(i = 1, 2, \ldots, 5)$ of the overall preference values $\alpha_i(i = 1, 2, \ldots, 5)$, and we have the same ranking order for $\lambda = 0.5$ and 1: $A_5 \succ A_1 \succ A_3 \succ A_2 \succ A_4$, and thus the most desirable alternative is A_5.

V. CONCLUSION

In this paper, we have proposed a family of entropies on interval-valued intuitionistic fuzzy sets, which depend on a parameter and include some well-known entropies as special cases. We have studied some desirable properties of the proposed entropies, and give an approach based on the proposed entropies. Finally, an illustrative example has been given to show the efficiency of the developed method.

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