World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:7, No:2, 2013

Soft connected spaces and soft paracompact spaces

Fucai Lin

Abstract—Soft topological spaces are considered as mathematical tools for dealing with uncertainties, and a fuzzy topological space is a special case of the soft topological space. The purpose of this paper is to study soft topological spaces. We introduce some new concepts in soft topological spaces such as soft closed mapping, soft open mappings, soft connected spaces and soft paracompact spaces. We also redefine the concept of soft points such that it is reasonable in soft topological spaces. Moreover, some basic properties of these concepts are explored.

Keywords—soft sets, soft open mappings, soft closed mappings, soft connected spaces, soft paracompact spaces.

I. Introduction

THE real world is too complex for our immediate and direct understanding, for example, many disciplines, including medicine, economics, engineering and sociology, are highly dependent on the task of modeling uncertain data.Since the uncertainty is highly complicated and difficult to characterize, classical mathematical approaches are often insufficient to useful models or derive effective. There are some theories: the theory of rough sets [10], the theory of vague sets [2] and the theory of fuzzy sets [13], which can be regarded as mathematical tools for dealing with uncertainties. However, all these theories have their own difficulties. The main reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory as it was mentioned by Molodtsov in [6]. In [6], Molodtsov introduced the concept of a soft set in order to solve complicated problems, and then Molodtsov presented the fundamental results of the new theory and successfully applied it to several directions such as operations research, game theory, Riemann-integration, theory of probability, smoothness of functions, Perron integration etc.

A soft set is a collection of approximate descriptions of an object. In [6], Molodtsov also proved how soft set theory is free from the parametrization inadequacy syndrome of probability theory, rough set theory, game theory and fuzzy set theory. Soft systems provide a very general framework with the involvement of parameters. Hence research works on soft set theory and its applications in various fields are progressing rapidly. Recently, Shabir and Naz [11] have introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Then some authors has began to study some of basic concepts and properties of soft topological spaces, see [3], [5], [11], [8], [12]. In particular, Akdag, Zorlutuna, Min and Atmaca [12] proved that an ordinary topological space can be considered a soft topological space, and that a fuzzy topological space is a special case of the soft topological space. Of course, a soft topological space is not certain an ordinary topological space.

In the present study, we introduce some new concepts in soft topological spaces such as soft closed mapping, soft open mappings, soft connected spaces and soft paracompact spaces. We also redefine the concept of soft points such that it is reasonable in soft topological spaces.

Fucai Lin is with the Department of Mathematics and Information Science, Zhangzhou Normal University, Zhangzhou 363000, P. R. China e-mail: linfucai2008@yahoo.com.cn

Supported by the NSFC (No. 11201414) and the Natural Science Foundation of Fujian Province (No. 2012J05013) of China.

II. Preliminaries

Definition 2.1: [6] Let U be an initial universe and E be a set of parameters. Suppose that $\mathcal{P}(U)$ denotes the power set of U and A is a non-empty subset of E. A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$.

In indeed, a soft set over U is a parameterized family of subsets of the universe U. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A).

Definition 2.2: [7] For two soft sets (F, A) and (G, B) over a common universe U, $(F; A)$ is a *soft subset* of $(G; B)$, denoted by $(F, A)\widetilde{\subseteq}(G, B)$, if $A \subset B$ and $e \in A$, $F(e) \subseteq G(e)$.

(F, A) is called a *soft superset* of (G, B), if (G, B) is a soft subset of (F, A), $(F, A)\widetilde{\supseteq}(G, B)$.

Definition 2.3: [7] Two soft sets (F, A) and (G, B) over a common universe U are said to be *soft equal* if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4: [4] The complement of a soft set (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$, $F^c : A \to \mathcal{P}(U)$ is a mapping given by $F^c(e) = U - F(e)$ for arbitrary $e \in A$. F^c is called the *soft complement function* of F. Obviously, $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.

Definition 2.5: [7] A soft set (F, A) over U is said to be a *NULL soft set* denoted by $\emptyset$ if for each $e \in A$, $F(e) = \emptyset$(null set).

Definition 2.6: [7] A soft set (F, A) over U is said to be an *absolute soft set*, denoted by U_A, if $e \in A$, $F(e) = U$.

Obviously, ones have $U_A^c = \emptyset_A$ and $\emptyset_A^c = U_A$.

Definition 2.7: [7] The union of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for arbitrary $e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A \setminus B, \\ G(e), & \text{if } e \in B \setminus A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

Definition 2.8: [9] The intersection of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cap B$ and for each $e \in C$, $H(e) = F(e) \cap G(e)$.

Note In order to efficiently discuss, we consider only soft sets (F, E) over a universe U in which all the parameter set E are same. We denote the family of these soft sets by $\mathcal{SS}(U)_E$.

Definition 2.9: [12] Let I be an arbitrary index set, and let $\{(F_i, E)\}i \in I$ be a subfamily of $\mathcal{SS}(U)_E$.

1) The union of these soft sets is the soft set (H, E), where $H(e) = \cup_{i\in I} F_i(e)$ for every $e \in E$. We write $\widetilde{\cup}_{i\in I}(F_i, E) = (H, E)$.
2) The intersection of these soft sets is the soft set (M, E), where $M(e) = \cap_{i\in I} F_i(e)$ for every $e \in E$. We write $\widetilde{\cap}_{i\in I}(F_i, E) = (M, E)$.

Definition 2.10: [11] Let τ be a collection of soft sets over a universe U with a fixed set E of parameters, then $\tau \subseteq \mathcal{SS}(V)_E$ is called a *soft topology* on U with a fixed set E if

1) $\emptyset_E, U_E$ belong to τ;
2) the union of arbitrary number of soft sets in τ belongs to τ;
3) the intersection of arbitrary two soft sets in τ belongs to τ.

Definition 2.11: Let $(U, \mathcal{F}_1, E)$ and $(U, \mathcal{F}_2, E)$ be two soft topological spaces over U. If $\mathcal{F}_1 \subseteq \mathcal{F}_2$, then we say that $(U, \mathcal{F}_2, E)$ is finer than $(U, \mathcal{F}_1, E)$ or $(U, \mathcal{F}_1, E)$ is coarser than $(U, \mathcal{F}_2, E)$. If $\mathcal{F}_1 \subseteq \mathcal{F}_2$ and $\mathcal{F}_1 \neq \mathcal{F}_2$, then we say that $(U, \mathcal{F}_2, E)$ is strict finer than $(U, \mathcal{F}_1, E)$ or $(U, \mathcal{F}_1, E)$ is strict coarser than $(U, \mathcal{F}_2, E)$.

Note: The soft indiscrete space is the coarsest soft topology, and the soft discrete pace is the finest soft topology.

Definition 2.12: [12] A soft set (G, E) in a soft topological space (U, τ, E) is called a *soft neighborhood* of the soft set (F, E) if there is a soft open set (H, E) such that $(F, E)\widetilde{\subseteq}(H, E)\widetilde{\subseteq}(G, E)$.

Definition 2.13: Let (U, τ, E) be a soft topological space, and let (G, E) be a soft set over U.

1) The *soft closure* [11] of (G, E) is the soft set $\overline{(G, E)} = \widetilde{\cap}\{(S, E) : (S, E)$ is soft closed and $(G, E)\widetilde{\subseteq}\underline{(S, E)}\}$;
2) The *soft interior* [12] of (G, E) is the soft set $\widetilde{(G, E)} = \widetilde{\cup}\{(S, E) : (S, E)$ is soft open and $(S, E)\widetilde{\subseteq}(G, E)\}$.

Definition 2.14: The soft set $(F, E) \in \mathcal{SS}(V)_E$ is called a *soft point* in U_E if there exist $x \in U$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \emptyset$ for each $e' \in E - \{e\}$, and the soft point (F, E) is denoted by e_x.

Theorem 2.15: Let (U, τ, E) be a soft topological space. A soft point $e_x\widetilde{\in}\overline{(A, E)}$ if and only if each soft neighborhood of e_x intersects (A, E).

Proof: Necessity. Let a soft neighborhood (B, E) of e_x disjoints from (A, E). Without loss of generality, we may assume that (B, E) is soft open. Then $(B, E)^c$ is soft closed and contains (A, E), and hence $\overline{(A, E)}\widetilde{\subseteq}(B, E)^c$. Since $e_x\widetilde{\in}(B, E)^c$, we have $e_x\widetilde{\in}\overline{(A, E)}$.

Sufficiency. Let $e_x\widetilde{\notin}\overline{(A, E)}$. Then $\overline{(A, E)}^c$ is a soft open neighborhood of e_x and disjoints from (A, E), which is a contradiction. ■

Readers may refer to [7], [9], [11], [12] for notations and terminology not explicitly given here.

III. Soft Open and Soft Closed Mappings

Definition 3.1: [5] Let $\mathcal{SS}(U)_A$ and $\mathcal{SS}(V)_B$ be families of soft sets. Let $u : U \to V$ and $p : A \to B$ be mappings. Then a mapping $f_{pu} : \mathcal{SS}(U)_A \to \mathcal{SS}(V)_B$ is defined as:

(1) Let (F, A) be a soft set in $\mathcal{SS}(U)_A$. The image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $\mathcal{SS}(V)_B$ such that

$$f_{pu}(F)(y) \in \begin{cases} \cup_{x\in p^{-1}(y)\cap A} u(F(x)), & p^{-1}(y) \cap A \neq \emptyset, \\ \emptyset, & \text{otherwise} . \end{cases}$$

for all $y \in B$.

(2) Let (G, B) be a soft set in $\mathcal{SS}(V)_B$. Then the inverse image of (G, B) under f_{pu}, written as $f^{-1}_{pu}(G, B) = (f^{-1}_{pu}(G), p^{-1}(B))$, is a soft set in $\mathcal{SS}(U)_A$ such that

$$f^{-1}_{pu}(G)(x) \in \begin{cases} u^{-1}(G(p(x))), & p(x) \in B, \\ \emptyset, & \text{otherwise} . \end{cases}$$

for all $x \in A$.

Definition 3.2: Let (U, τ, A) and (V, τ^*, B) soft topological spaces. Let $u : U \to V$ and $p : A \to B$ be mappings. Let $f_{pu} : \mathcal{SS}(U)_A \to \mathcal{SS}(V)_B$ be a function.

(1) The function f_{pu} is *soft continuous* [12] if $f^{-1}_{pu}(H, B) \in \tau$ for each $(H, B) \in \tau^*$.

(2) The function f_{pu} is *soft open* if $f_{pu}(G, A) \in \tau^*$ for each $(G, A) \in \tau$.

(3) The function f_{pu} is *soft closed* if $f_{pu}(G, A)$ is soft closed in (U, τ, A) for each soft closed set (G, A) in (V, τ^*, B).

(4) The function f_{pu} is *soft homeomorphism* if if f_{pu} is an onto, one to one, soft continuous and soft open mapping.

Remark 3.3: (1) The soft continuous mappings may not soft closed and soft open.

(2) The soft closed and soft open mappings may not soft continuous.

Example 3.4: Let $U = V$ be an initial universe set, and A be the set of parameters. Suppose that (U, τ, A) is a non-discrete soft topological space, and that (V, τ^*, A) is the soft discrete topology. Then the identical mapping i_U from (U, τ, A) to (V, τ^*, A) is soft open and soft closed. However, it is easy to see that i_X is not soft continuous. Moreover, the identical mapping i_V from (V, τ^*, A) to (U, τ, A) is soft continuous. However, it is easy to see that i_X is not soft open and soft closed.

Theorem 3.5: Let (U, τ, A) and (V, τ^*, B) be soft topological spaces. Let $u : U \to V$ and $p : A \to B$ be onto mappings. Let $f_{pu} : \mathcal{SS}(U)_A \to \mathcal{SS}(V)_B$ be a function. Then the following are equivalent:

1) f_{pu} is soft open;
2) For each soft set (G, A) over U, we have $f_{pu}((G, A)^\circ)\widetilde{\subseteq}(f_{pu}(G, A))^\circ$;
3) For each soft set (F, B) over V, we have $f^{-1}_{pu}(\overline{(F, B)})\widetilde{\subseteq}\overline{f^{-1}_{pu}(F, B)}$;
4) For each soft point $e_x\widetilde{\in}U_A$ and each soft neighborhood (U, E) at e_x over U, $f_{pu}(U, E)$ is a soft neighborhood at soft point $f_{pu}(e_x)$ over V.

Proof: (1) $\Rightarrow$ (2). Obvious, $f_{pu}((A, E)^\circ)\widetilde{\subseteq}f_{pu}(A, E)$, and since $f_{pu}((A, E)^\circ)$ is soft open by (1), we have $f_{pu}((A, E)^\circ)\widetilde{\subseteq}(f(A, E))^\circ$.

(2) $\Rightarrow$ (4). Let (U, E) be a soft neighborhood at a soft point e_x. Then we have $e_x\widetilde{\in}(U, E)^\circ$. By (2), we have $f_{pu}(e_x)\widetilde{\in}(f_{pu}(U, E))^\circ$. Therefore, $f_{pu}(U, E)$ is a soft neighborhood at point $f_{pu}(e_x)$ over V.

(4) ⇒ (3). Let $e_x\widetilde{\in}f_{pu}^{-1}(\overline{(F,B)})$. Then $f_{pu}(e_x)\widetilde{\in}\overline{(F,B)}$. Let (H,A) be an arbitrary soft neighborhood at soft point e_x. By (4), $f_{pu}((H,A))$ is a soft neighborhood of $f_{pu}(e_x)$, and hence $f_{pu}((H,A))\widetilde{\cap}(F,B)\neq\emptyset$ by Theorem 2.15. Then there exists a soft point $e_{x'}\widetilde{\in}(H,A)$ such that $f_{pu}(e_{x'})\widetilde{\in}(F,B)$, and thus $e_{x'}\widetilde{\in}f_{pu}^{-1}(F,B)$. Then $(H,A)\widetilde{\cap}f_{pu}^{-1}(F,B)\neq\emptyset$, and therefore, $e_x\widetilde{\in}\overline{f_{pu}^{-1}(F,B)}$.

(3) ⇒ (2). Since $(G,A)^\circ\widetilde{\subseteq}(G,A)\widetilde{\subseteq}f_{pu}^{-1}(f_{pu}(G,A))$ and $(G,A)^\circ$ is soft open, we have $(G,A)^\circ\widetilde{\subseteq}[f_{pu}^{-1}(f_{pu}(G,A))]^\circ$. Obvious, we have

$$[f_{pu}^{-1}(f_{pu}(G,A))]^\circ=\overline{[f_{pu}^{-1}(f_{pu}(G,A))]^c}^c,$$

and

$$[f_{pu}^{-1}(f_{pu}(G,A))]^\circ=[\overline{f_{pu}^{-1}([f_{pu}(G,A)]^c)}]^c$$

since $[f_{pu}^{-1}(f_{pu}(G,A))]^c=f_{pu}^{-1}([f_{pu}(G,A)]^c)$. Then

$$[f_{pu}^{-1}(f_{pu}(G,A))]^\circ\widetilde{\subseteq}[f_{pu}^{-1}\overline{([f_{pu}(G,A)]^c)}]^c.$$

Therefore, it is easy to see that

$$\begin{aligned} f_{pu}((G,A)^\circ)&\widetilde{\subseteq}f_{pu}([f_{pu}^{-1}\overline{([f_{pu}(G,A)]^c)}]^c)\\ &=f_{pu}(f_{pu}^{-1}([f_{pu}(G,A)]^c)^c)\\ &\widetilde{\subseteq}[f_{pu}(G,A)]^c)^c\\ &=(f_{pu}(G,A))^\circ. \end{aligned}$$

(2) ⇒ (1). Let (G,A) be a soft open set. Then $(G,A)=(G,A)^\circ$. By (2), we have $f_{pu}(G,A)=f_{pu}((G,A)^\circ)\widetilde{\subseteq}(f_{pu}(G,A))^\circ$, that is, $f_{pu}(G,A)$ is soft open. Therefore, f_{pu} is a soft open mapping. ■

Theorem 3.6: Let (U,τ,A) and (V,τ^*,B) soft topological spaces. Let $u:U\to V$ and $p:A\to B$ be mappings. Let $f_{pu}:\mathcal{SS}(U)_A\to\mathcal{SS}(V)_B$ be a function. Then the following are equivalent:

1) f_{pu} is soft closed;
2) For each soft set (G,A) over U, $\overline{f_{pu}(G,A)}\widetilde{\subseteq}f_{pu}(\overline{(G,A)})$.

Proof: (1) ⇒ (2). Since $f_{pu}(G,A)\widetilde{\subseteq}f_{pu}(\overline{(G,A)})$ and $f_{pu}(\overline{(G,A)})$ is soft closed over V, we have $\overline{f_{pu}(G,A)}\widetilde{\subseteq}f_{pu}(\overline{(G,A)})$.

(2) ⇒ (1). Let (G,A) be a soft closed set over V. By (2), we have

$$\overline{f_{pu}(G,A)}\widetilde{\subseteq}f_{pu}(\overline{(G,A)}).$$

Since $\overline{(G,A)}=(G,A)$, $f(\overline{(G,A)})\widetilde{\subseteq}f(G,A)$, and thus $f(G,A)$ is a soft closed set over V. ■

The proof of the following proposition is an easy exercise.

Proposition 3.7: Let (U,τ,A) and (V,τ^*,B) soft topological spaces. Let $u:U\to V$ and $p:A\to B$ be mappings. Let $f_{pu}:\mathcal{SS}(U)_A\to\mathcal{SS}(V)_B$ be a function. Then $f_{pu}^{-1}(B,E)\widetilde{\subseteq}(A,E)$ if and only if $(B,E)\widetilde{\subseteq}(f_{pu}((A,E)^c))^c$.

Theorem 3.8: Let (U,τ,A) and (V,τ^*,B) soft topological spaces. Let $u:U\to V$ and $p:A\to B$ be mappings. Let $f_{pu}:\mathcal{SS}(U)_A\to\mathcal{SS}(V)_B$ be a function. Then f_{pu} is soft closed if and only if, for each soft point e_y in V_B and each soft open set (F,E) with $f_{pu}^{-1}(e_y)\widetilde{\subseteq}(F,E)$ in U_E, there exists a soft open set (W,B) in V_B such that $e_y\widetilde{\in}(W,B)$ and $f_{pu}^{-1}(W,B)\widetilde{\subseteq}(F,E)$.

Proof: Necessity. Let f_{pu} be a soft closed mapping. For each soft point e_y in V_B and each soft open set (F,E) with $f_{pu}^{-1}(e_y)\widetilde{\subseteq}(F,E)$ in U_E, we put $(W,B)=(f_{pu}((F,E)^c))^c$. Then (W,B) is soft open. By Proposition 3.7, we have $e_y\widetilde{\in}(W,B)$ and $f_{pu}^{-1}(W,B)\widetilde{\subseteq}(F,E)$.

Sufficiency. Let (G,E) be a soft closed set in U_E. Take arbitrary soft point $e_y\in(f_{pu}(G,E))^c$. Then $f_{pu}^{-1}(e_y)\widetilde{\subseteq}(G,E)^c$. By the assumption, there exists a soft open set (W,B) such that $e_y\widetilde{\in}(W,B)$ and $f_{pu}^{-1}(W,B)\widetilde{\subseteq}(G,E)^c$, and hence (W,B) is a soft neighborhood of e_y. By Proposition 3.7, we have $(W,B)\widetilde{\cap}f_{pu}(G,E)=\emptyset$, and thus $f_{pu}(G,E)$ is soft closed. ■

IV. Soft Connected Spaces

Definition 4.1: Let (U,τ,E) be a soft topological space, and $(F_1,E),(F_2,E)$ be two soft sets over U. The soft sets (F_1,E) and (F_2,E) are said to *soft separated* if $(F_1,E)\widetilde{\cap}\overline{(F_2,E)}=\emptyset$ and $\overline{(F_1,E)}\widetilde{\cap}(F_2,E)=\emptyset$.

Remark 4.2: Two disjoint soft open sets over U may not be a soft separated.

Example 4.3: Let $U=\{h_1,h_2,h_3\}$, $E=\{e_1,e_2\}$ and

$$\tau=\{\emptyset,U_E,(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E)\}$$

are soft sets over U, defined as follows:

$F_1(e_1)=\{h_2\}$, $F_1(e_2)=\{h_1\}$;
$F_2(e_1)=\{h_3\}$, $F_2(e_2)=\{h_2\}$;
$F_3(e_1)=\{h_2,h_3\}$, $F_3(e_2)=\{h_1,h_2\}$;
$F_4(e_1)=\{h_1,h_2\}$, $F_4(e_2)=U$;
$F_5(e_1)=\{h_1,h_2\}$, $F_5(e_2)=\{h_1,h_3\}$;
$F_6(e_1)=\emptyset$, $F_6(e_2)=\{h_2\}$.

Then τ defines a soft topology on U and hence (U,τ,E) is a soft topological space over U. It is easy to see that $(F_1,E)\widetilde{\cap}\overline{(F_2,E)}=\emptyset$. However, $\overline{(F_1,E)}=(F_6,E)'$, and hence $\overline{(F_1,E)}\widetilde{\cap}(F_2,E)\neq\emptyset$.

Definition 4.4: Let (U,τ,E) be a soft topological space. If there exist two non-empty soft separated sets (F_1,E) and (F_2,E) such that $(F_1,E)\widetilde{\cup}(F_2,E)=U_E$, then (F_1,E) and (F_2,E) are said to be a *soft division* for soft topological space (U,τ,E). (U,τ,E) is said to be *soft disconnected* if (U,τ,E) has a soft division. Otherwise, (U,τ,E) is said to be *soft connected*.

It is easy to see that each soft indiscrete space is soft connected, and that each soft discrete non-trivial space is not soft connected.

Theorem 4.5: Let (U,τ,E) be a soft topological space. Then the following conditions are equivalent:

1) (U,τ,E) has a soft division;
2) There exist two disjoint soft closed sets (F_1,E) and (F_2,E) such that
$$(F_1,E)\widetilde{\cup}(F_2,E)=U_E;$$
3) There exist two disjoint soft open sets (F_1,E) and (F_2,E) such that
$$(F_1,E)\widetilde{\cup}(F_2,E)=U_E;$$

4) (U, τ, E) has a proper soft open and soft closed set in U.

Proof: (1)⇒(2). Let (U, τ, E) have a soft division (F_1, E) and (F_2, E). Then

$$(F_1, E)\widetilde{\cap}(F_2, E) = \emptyset$$

and

$$\begin{aligned}\overline{(F_1, E)} &= \overline{(F_1, E)}\widetilde{\cap}((F_1, E)\widetilde{\cup}(F_2, E)) \\ &= (\overline{(F_1, E)}\widetilde{\cap}(F_1, E))\widetilde{\cup}(\overline{(F_1, E)}\widetilde{\cap}(F_2, E)) \\ &= (F_1, E).\end{aligned}$$

Therefore, (F_1, E) is a soft closed set in U_E. Similar, we can see that (F_2, E) is also a soft closed set in U_E.

(2)⇒(3). Let (U, τ, E) has a soft division (F_1, E) and (F_2, E) such that (F_1, E) and (F_2, E) are soft closed. Then $(F_1, E)'$ and $(F_2, E)'$ are soft open sets in U_E. Then it is easy to see that $(F_1, E)'\widetilde{\cap}(F_2, E)' = \emptyset$ and $(F_1, E)'\widetilde{\cup}(F_2, E)' = U_E$.

(3)⇒(4). Let (U, τ, E) have a soft division (F_1, E) and (F_2, E) such that (F_1, E) and (F_2, E) are soft open in U_E. Then (F_1, E) and (F_2, E) are also soft closed in U_E.

(4)⇒(1). Let (U, τ, E) has a proper soft open and soft closed set (F, E) in U_E. Put $(H, E) = (F, E)'$. Then (H, E) and (F, E) are non-empty soft closed set in U_E, $(H, E)\widetilde{\cap}(F, E) = \emptyset$ and $(H, E)\widetilde{\cup}(F, E) = U_E$. Therefore, (H, E) and (F, E) is a soft division of U_E. ■

Theorem 4.6: Let (U, τ, E) be a soft topological space. Then the following conditions are equivalent:

1) (U, τ, E) is soft connected;
2) There exist two disjoint soft closed sets (F_1, E) and (F_2, E) such that
$$(F_1, E)\widetilde{\cup}(F_2, E) = U_E;$$
3) There exist two disjoint soft open sets (F_1, E) and (F_2, E) such that
$$(F_1, E)\widetilde{\cup}(F_2, E) = U_E;$$
4) (U, τ, E) at most has two soft closed and soft open sets in U_E, that is, $\emptyset$ and U_E.

Note: By Theorem 4.6, the soft topological space in Example 4.15 is a soft disconnected space since the soft set (F_2, E) is soft open and soft closed in U_E.

Lemma 4.7: Let (U, τ, E) be a soft topological space over U and V be a non-empty subset of U_E. If (F_1, E) and (F_2, E) are soft sets in V_E, then (F_1, E) and (F_2, E) are a soft separation of V_E if and only if (F_1, E) and (F_2, E) are a soft separation of U_E.

Proof: We have

$$\overline{(F_1, E)}^V\widetilde{\cap}(F_2, E) = (\overline{(F_1, E)}\widetilde{\cap}V_E)\widetilde{\cap}(F_2, E) = \overline{(F_1, E)}\widetilde{\cap}(F_2, E).$$

Similar, we have

$$\overline{(F_2, E)}^V\widetilde{\cap}(F_1, E) = \overline{(F_2, E)}\widetilde{\cap}(F_1, E).$$

Therefore, the lemma holds. ■

Lemma 4.8: Suppose that (U, τ, E) is a soft topological space over U_E, and that V is a non-empty subset of U such that (V, τ_V, E) is soft connected. If (F_1, E) and (F_2, E) are a soft separation of U_E such that $V_E\widetilde{\subseteq}(F_1, E)\widetilde{\cup}(F_2, E)$, then $V_E\widetilde{\subseteq}(F_1, E)$ or $V_E\widetilde{\subseteq}(F_2, E)$.

Proof: Since $V_E \subseteq (F_1, E)\widetilde{\cup}(F_2, E)$, we have $V_E = (V_E\widetilde{\cap}(F_1, E))\widetilde{\cup}(V_E\widetilde{\cap}(F_2, E))$. By Lemma 4.7, $V_E\widetilde{\cap}(F_1, E)$ and $V_E\widetilde{\cap}(F_2, E)$ are a soft separation of V_E. Since (V, τ_V, E) is soft connected, we have $V_E\widetilde{\cap}(F_1, E) = \emptyset$ or $V_E\widetilde{\cap}(F_2, E) = \emptyset$. Therefore, $V_E\widetilde{\subseteq}(F_1, E)$ or $V_E\widetilde{\subseteq}(F_2, E)$. ■

Lemma 4.9: Let $\{(U_\alpha, \tau_{U_\alpha}, E) : \alpha \in J\}$ be a family non-empty soft connected subspaces of soft topological space (U, τ, E). If $\cap_{\alpha\in J}U_\alpha \neq \emptyset$, then $(\cup_{\alpha\in J}U_\alpha, \tau_{\cup_{\alpha\in J}U_\alpha}, E)$ is a soft connected subspace of (U, τ, E).

Proof: Let $V = \cup_{\alpha\in J}U_\alpha$. Choose a soft point $e_x\widetilde{\in}V_E$. Let (C, E) and (D, E) be a soft division of $(\cup_{\alpha\in J}U_\alpha, \tau_{\cup_{\alpha\in J}U_\alpha}, E)$. Then $e_x\widetilde{\in}(C, E)$ or $e_x\widetilde{\in}(D, E)$. Without loss of generality, we may assume that $e_x\widetilde{\in}(C, E)$. For each $\alpha \in J$, since $(U_\alpha, \tau_{U_\alpha}, E)$ is soft connected, it follows from Lemma 4.8 that $(U_\alpha)_E\widetilde{\subseteq}(C, E)$ or $(U_\alpha)_E\widetilde{\subseteq}(D, E)$. Therefore, we have $V_E\widetilde{\subseteq}(C, E)$ since $e_x\widetilde{\in}(C, E)$, and then $(D, E) = \emptyset$, which is a contradiction. Thus $(\cup_{\alpha\in J}U_\alpha, \tau_{\cup_{\alpha\in J}U_\alpha}, E)$ is a soft connected subspace of (U, τ, E). ■

Theorem 4.10: Let $\{(U_\alpha, \tau_{U_\alpha}, E) : \alpha \in J\}$ be a family non-empty soft connected subspaces of soft topological space (U, τ, E). If $U_\alpha \cap U_\beta \neq \emptyset$ for arbitrary $\alpha, \beta \in J$, then $(\cup_{\alpha\in J}U_\alpha, \tau_{\cup_{\alpha\in J}U_\alpha}, E)$ is a soft connected subspace of (U, τ, E).

Proof: Fix an $\alpha_0 \in J$. For arbitrary $\beta \in J$, put $A_\beta = U_{\alpha_0}\cup U_\beta$. By Lemma 4.9, each $(A_\beta, \tau_{A_\beta}, E)$ is soft connected. Then $\{(A_\beta, \tau_{A_\beta}, E) : \beta \in J\}$ is a family non-empty soft connected subspaces of soft topological space (U, τ, E), and $\cap_{\beta\in J}A_\beta = U_{\alpha_0} \neq \emptyset$. Obvious, we have $\cup_{\alpha\in J}U_\alpha = \cup_{\beta\in J}A_\beta$. It follows from Lemma 4.9 that $(\cup_{\alpha\in J}U_\alpha, \tau_{\cup_{\alpha\in J}U_\alpha}, E)$ is a soft connected subspace of (U, τ, E). ■

Theorem 4.11: Let (U, τ, E) be a soft topological space over X and (V, τ_V, E) be a soft connected subspace of (U, τ, E). If $V_E\widetilde{\subseteq}A_E\widetilde{\subseteq}\overline{(Y, E))}$, then (A, τ_A, E) is a soft connected subspace of (U, τ, E). In particular, $\overline{(Y, E))}$ is a soft connected subspace of (U, τ, E).

Proof: Let (C, E) and (D, E) be a soft division of (A, τ_A, E). By Lemma 4.8, we have $A_E\widetilde{\subseteq}(C, E)$ or $A_E\widetilde{\subseteq}(D, E)$. Without loss of generality, we may assume that $A_E\widetilde{\subseteq}(C, E)$. By Lemma 4.7, we have $\overline{(C, E)}\widetilde{\cap}(D, E) = \emptyset$, and hence $A_E\widetilde{\cap}(D, E) = \emptyset$, which is a contradiction. ■

Theorem 4.12: The image of soft connected spaces under a soft continuous map are soft connected.

Proof: Let (U, τ, A) and $(V, \mathcal{G}, B)$ be two soft topological spaces, where (U, τ, A) is soft connected, and let f be a soft pu-continuous mapping from U to V. Obvious, the restricted mapping is soft continuous, and without loss of generality, we may assume that $u(U) = u(V)$ and $p(A) = B$. Suppose that $(V, \mathcal{G}, B)$ is soft disconnected. By Theorem 4.6, there exists a proper soft open and soft closed set (F, E) in V. Since f is soft continuous, $f^{-1}((F, E))$ is a proper soft open and soft closed set in U by [12, Theorem 6.3], which is a contradiction. ■

Proposition 4.13: [11] Let (U, τ, E) be a soft topological space. Then the collection $\tau_\alpha = \{F(\alpha) : (F, E) \in \tau\}$ for each $\alpha \in E$, defines a topology on U.

Remark 4.14: There exists a soft connected soft topological space (U, τ, E) such that (U, τ_α) is a disconnected topological space for some $\alpha \in E$.

Example 4.15: Let $U = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\emptyset, U_E, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E)\}$ are soft sets over U, defined as follows:
$F_1(e_1) = \{h_1, h_2\}$, $F_1(e_2) = U$;
$F_2(e_1) = \{h_1, h_3\}$, $F_2(e_2) = U$;
$F_3(e_1) = \{h_1\}$, $F_3(e_2) = U$;
$F_4(e_1) = \{h_2, h_3\}$, $F_4(e_2) = U$;
$F_5(e_1) = \{h_2\}$, $F_5(e_2) = U$;
$F_6(e_1) = \{h_3\}$, $F_6(e_2) = U$;
$F_7(e_1) = \emptyset$, $F_5(e_2) = U$. (U, τ, E) Then τ defines a soft topology on U and hence (U, τ, E) is a soft topological space over U. Obvious, (U, τ, E) is soft connected. However, (U, τ_1) is discrete space, and hence (U, τ_1) is disconnected.

Definition 4.16: [1] Let (U, τ, E) be a soft topological space. A subcollection γ of τ is said to be a *base* for τ if every member of τ can be expressed as a union of members of γ.

Definition 4.17: Let $\{(U^\alpha, \tau_\alpha, E_\alpha)\}_{\alpha\in I}$ be a family a soft topological spaces. Let us take as a basis for a soft topology on the product space $(\prod_{\alpha\in I} U^\alpha, \prod_{\alpha\in I} \tau_\alpha, \prod_{\alpha\in I} E_\alpha)$ the collection of all soft sets $\{(\prod_{\alpha\in I} F_\alpha, \prod_{\alpha\in I} E_\alpha)$: There is a finit set $J \subset I$ such that $(F_\alpha, E_\alpha) = U^\alpha_{E_\alpha}$ for each $\alpha \in I \setminus J\}$.

Theorem 4.18: A finite product of soft connected spaces is soft connected.

Proof: We prove the theorem first for the product of two soft connected spaces (U, τ_1, A) and (V, τ_2, B). Choose a fix point $a \times b \in U \times V$. Obvious, $(U \times b, \tau_1 \times \tau_2|_{U\times b}, A \times B)$ is soft connected. For each $u \in U$, $(u\times V, \tau_1 \times \tau_2|_{u\times V}, A\times B)$ is also soft connected, and put $T_u = (U\times b)\cup(u\times V)$. Then each $(T_u, \tau_1 \times \tau_2|_{T_u}, A \times B)$ is soft connected Lemma 4.9. Since $a\times b \in T_u$ for each $u \in U$, it follows from Theorem 4.10 that $(\cup_{u\in U}T_u, \tau_1 \times \tau_2|_{\cup_{u\in U}T_u}, A \times B)$ is soft connected.

The proof for any finite product of soft connected spaces follows by induction, using the fact that $(\prod_{i=1}^n U_i, \prod_{i=1}^n \tau_i, \prod_{i=1}^n A_i)$ is soft homeomorphic with $((\prod_{i=1}^{n-1} U_i) \times U_n, (\prod_{i=1}^{n-1} \tau_i) \times \tau_n, (\prod_{i=1}^{n-1} A_i) \times A_n)$. ■

Theorem 4.19: The product of a family of soft connected spaces is soft connected.

Proof: Let $\{(U_\alpha, \tau_\alpha, E_\alpha) : \alpha \in J\}$ be a family of soft connected spaces, and put $U = \prod_{\alpha\in J} U_\alpha$. Take a fixed point $a = (a_\alpha)$ of U. Take arbitrary finite subset K of J, let U_K denote the subset of U consisting of all points $u = (u_\alpha)$ such that $x_\alpha = a_\alpha$ for $\alpha \notin K$. By Theorem 4.18, it is easy to see that $(U_K, \prod_{\alpha\in J} \tau_\alpha|_{U_K}, \prod_{\alpha\in J} E_\alpha)$ is soft connected. Let $V = \cup\{U_K : K$ is finite subset of $J\}$. It is easy to see that $\overline{V_{\prod_{\alpha\in J} E_\alpha}} = U_{\prod_{\alpha\in J} E_\alpha}$. By Theorem 4.11, $(U, \prod_{\alpha\in J} \tau_\alpha, \prod_{\alpha\in J} E_\alpha)$ is soft connected. ■

Definition 4.20: Given a soft topological space (U, τ, E), define an equivalence relation on U_E by setting $e_x \sim e_y$ if there exists a soft connected subspace of (U, τ, E) containing both soft points e_x and e_y. The equivalence classes are called the *soft components* (or the "*soft connected components*") of U_E.

Reflexivity and symmetry of the relation are obvious. Transitivity follows by noting that if A_E is a soft connected subspace containing soft points e_x and e_y, and if B_E is a soft connected subspace containing soft points e_y and e_z, then $A_E\widetilde{\cup}B_E$ is a subspace containing soft points e_x and e_z that is soft connected because A_E and B_E have the soft point e_y in common.

Theorem 4.21: The soft components of soft topological space (U, τ, E) are soft connected disjoint soft subspace of U_E whose union is U_E, such that each non-empty soft connected subspace of U_E intersects only one of them.

Proof: Being equivalence classes, the soft components of U_E are disjoint and their union is U_E. Let A_E be an arbitrary soft connected subspace. Then A_E intersects only one of them. For if A_E intersects the soft components C_E and D_E of U_E, say in soft points e_x and e_y, respectively, then $e_x \sim e_y$ by definition; this cannot happen unless $C_E = D_E$. Next we shall show the soft component C_E is soft connected. Choose a soft point e_z of C_E. For each soft point e_x of C_E, we know that $e_z \sim e_x$, hence there exists a soft connected subspace $L^{e_x}_E$ containing e_z and e_x. Obvious, each $L^{e_x}_E \widetilde{\subseteq} C_E$. Therefore, $C_E = \widetilde{\cup}_{e_x\in C_F} L^{e_x}_E$. Since the soft subspace $L^{e_x}_E$ are soft connected and have the soft point e_z in common, C_E is soft connected by Theorem 4.10. ■

V. SOFT PARACOMPACT SPACES

Definition 5.1: Let (U, τ, E) be a soft topological space, and $\mathcal{A}$ be a collection of soft sets of U_E. Then

(1) The collection $\mathcal{A}$ is said to be *locally finite* in U_E if each soft point of U_E has a soft neighborhood that intersects only finitely many elements of $\mathcal{A}$;

(2) A collection $\mathcal{B}$ of soft sets of U_E is said to be a *refinement* of $\mathcal{A}$ if for each element $B \in \mathcal{B}$, there exists an element $A \in \mathcal{A}$ containing B. If the elements of $\mathcal{B}$ are soft open sets, we call $\mathcal{B}$ a *soft open refinement* of $\mathcal{A}$; if they are soft closed, we call $\mathcal{B}$ a *soft closed refinement*.

Proposition 5.2: Let $\mathcal{A}$ be a locally finite collection of soft subsets of U_E. Then

1) Any subcollection of $\mathcal{A}$ is locally finite;
2) The collection $\mathcal{B} = \{\overline{(F, E)} : (F, E) \in \mathcal{A}\}$ is locally finite;
3) $\overline{\widetilde{\cup}_{(F,E)\in\mathcal{A}}(F, E)} = \widetilde{\cup}_{(F,E)\in\mathcal{A}}\overline{(F, E)}$.

Proof: Statement (1) is trivial.

(2) Note that any soft open set (G, E) that intersects the soft set $\overline{(F, E)}$ necessarily intersects (F, E). Thus if (G, E) is a soft neighborhood of soft point e_x that intersects only finitely many elements (F, E) of $\mathcal{A}$, then (F, E) can intersect at most the same number of soft sets of the collection $\mathcal{B}$.

(3) Let $\widetilde{\cup}_{(F,E)\in\mathcal{A}}(F, E) = (Y, E)$. Obvious, $\widetilde{\cup}_{(F,E)\in\mathcal{A}}\overline{(F, E)} = \overline{(Y, E)}$. We prove the reverse inclusion, under the assumption of local finiteness. Let $e_x \in \overline{(Y, E)}$; let (G, E) be a soft neighborhood of e_x that intersects only finitely many elements of $\mathcal{A}$, say $\overline{(F_1, E)}, \cdots, \overline{(F_k, E)}$. Then e_x belongs to one of the soft sets $\overline{(F_1, E)}, \cdots, \overline{(F_k, E)}$. For otherwise, the soft set $(G, E)\widetilde{\cap}(\widetilde{\cup}\{\overline{(F_1, E)}, \cdots, \overline{(F_k, E)}\})^c$ would be a soft neighborhood of e_x that intersects no element

of $\mathcal{A}$, and therefore it does not intersect (Y, E), which is a contradiction with $e_x \in \overline{(Y, E)}$. ■

Definition 5.3: A soft topological space (U, τ, E) is *soft paracompact* if each soft open covering $\mathcal{A}$ of U_E has a locally finite soft open refinement $\mathcal{B}$ that covers U_E.

Definition 5.4: [12] A soft topological space (U, τ, E) is *soft compact* if each soft open covering $\mathcal{A}$ of U_E has a finite subcover.

Definition 5.5: A soft topological space (U, τ, E) is *soft Lindelöf* if each soft open covering $\mathcal{A}$ of U_E has a countable subcover.

Proposition 5.6: Each soft compact space is soft Lindelöf and each soft Lindelöf space is soft paracompact.

Proposition 5.7: Let (U, τ, E) be a soft paracompact space. If $E = \{e\}$, then (U, τ, E) is soft paracompact if and only if the collection $\eta = \{F(e) : (F, E) \in \tau\}$ is a paracompact topology on U.

It is well known that a Lindelöf space may not compact and a paracompact space may not Lindelöf. Therefore, it follows from Proposition 5.7 that a soft Lindelöf space may not soft compact and a soft paracompact space may not soft Lindelöf.

Theorem 5.8: Each soft paracompact soft T_2 space is soft normal.

Proof: Let (U, τ, E) be a soft paracompact soft T_2 space. First one proves soft regularity. Let e_x be a soft point of U_E and let (A, E) be a soft closed set of U_E disjoint from e_x. The soft T_2 condition enable us to take, for each soft point e_y in (A, E), an soft open set (B^{e_y}, E) about e_y whose soft closure is disjoint from e_x. Let $\mathcal{A} = \{(B^{e_y}, E) : e_y \in (A, E)\} \cup \{(A, E)^c\}$. Then $\mathcal{A}$ is a soft open covering of U_E. Since (U, τ, E) is soft paracompact, there exists a locally finite soft open refinement $\mathcal{B}$ that covers U_E. Form the subcollection $\mathcal{C}$ of $\mathcal{B}$ consisting of each element of $\mathcal{B}$ that intersects (A, E). Then $\mathcal{C}$ covers (A, E). Moreover, if $C \in \mathcal{C}$, then the soft closure of C is disjoint from e_x. Since C intersects (A, E), it lies in some soft open set (B^{e_y}, E), whose soft closure is disjoint from e_x. Let $(V, E) = \widetilde{\cup}_{C \in \mathcal{C}} C$. Obvious, (V, E) is soft open in U_E containing (A, E). Since $\mathcal{C}$ is locally finite, $\overline{(V, E)} = \widetilde{\cup}_{C \in \mathcal{C}} \overline{C}$ by Proposition 5.2. Then $\overline{(V, E)}$ is disjoint from e_x. Thus soft regularity is proved.

To prove soft normality, one only repeats the same argument, replacing e_x by a soft closed set throughout and replacing the soft T_2 condition by soft regularity. ■

Theorem 5.9: Each soft closed subspace of a soft paracompact is soft paracompact.

Proof: Let (U, τ, E) be a soft paracompact space, and $Y \subseteq U$ such that Y_E is soft closed in U_E. Let $\mathcal{A}$ be a soft covering of Y_E by soft open in Y_E. For every $(A, E) \in \mathcal{A}$, take a soft open set (A', E) of U_E such that $(A', E) \widetilde{\cap} Y_E = (A, E)$. Cover U_E by the soft open (A', E), along with the soft open set Y_E^c. Suppose that $\mathcal{B}$ is a locally finite soft open refinement of this soft covering that covers U_E. Then the collection

$$\mathcal{C} = \{(B, E) \widetilde{\cap} Y_E : (B, E) \in \mathcal{B}\}$$

is the required locally finite soft open refinement of $\mathcal{A}$. ■

Remark 5.10: By Proposition 5.7, it is easy to see the following two facts:

(1) A soft paracompact subspace of a soft Hausdorff space (U, τ, E) need do not be soft closed in U_E;

(2) A soft subspace of a soft paracompact need not be soft paracompact.

Lemma 5.11: Let (U, τ, E) be a soft topological space. If each soft open covering of (U, τ, E) has a locally finite soft closed refinement, then every soft open covering of (U, τ, E) has a locally finite soft open refinement.

Proof: Let $\mathcal{A}$ be a soft open covering of (U, τ, E), and let $\mathcal{B} = \{(F_s, E) : s \in \mathcal{S}\}$ be a locally finite soft closed refinement of $\mathcal{A}$. For each soft point $e_x \in U_E$, choose a soft open neighborhood (V_{e_x}, E) of e_x such that (V_{e_x}, E) intersects finitely many elements of $\mathcal{B}$. Let $\mathcal{C} = \{(V_{e_x}, E) : e_x \in E_E\}$, and let $\mathcal{D}$ be a locally finite soft closed refinement of $\mathcal{C}$. For each $s \in \mathcal{S}$, put

$$(W_s, E) = (\widetilde{\cup}\{(D, E) : (D, E) \in \mathcal{D}, (D, E) \widetilde{\cap} (F_s, E) = \emptyset\})^c.$$

Obvious, each (W_s, E) is soft open and contains (F_s, E). Moreover, for each $s \in \mathcal{S}$ and each $(D, E) \in \mathcal{D}$, we have

$$(W_s, E) \widetilde{\cap} (D, E) \neq \emptyset \text{ if and only if } (F_s, E) \widetilde{\cap} (D, E) \neq \emptyset.$$

For each $s \in \mathcal{S}$, choose a $(A_s, E) \in \mathcal{A}$ such that $(F_s, E) \widetilde{\subseteq} (A_s, E)$, and let $(G_s, E) = (A_s, E) \widetilde{\cap} (W_s, E)$. Then $\{(G_s, E) : s \in \mathcal{S}\}$ is a soft open covering and refines $\mathcal{A}$. It is easy to see that each element of $\mathcal{D}$ intersects only finitely many (G_s, E). Therefore, $\{(G_s, E) : s \in \mathcal{S}\}$ is locally finite. ■

Lemma 5.12: Each σ-locally finite soft open covering has a locally finite refinement.

Proof: Let $\mathcal{U} = \cup_{n \in \mathbf{N}} \mathcal{U}_n$ be a σ-locally finite soft open covering for some soft topological space, where each $\mathcal{U}_n$ is locally finite. Put $\mathcal{V}_1 = \mathcal{U}_1, \mathcal{V}_n = \{(F, E) \widetilde{\cap} (\widetilde{\cup}_{k<n} \mathcal{U}_k^*)^c : (F, E) \widetilde{\in} \mathcal{U}_n\}$, where $\mathcal{U}_k^* = \widetilde{\cup}\{(F, E) : (F, E) \widetilde{\in} \mathcal{U}_k\}$. Then it is easy to see that $\mathcal{V} = \cup_{n \in \mathbf{N}} \mathcal{V}_n$ is a locally finite soft open covering and refines $\mathcal{U}$. ■

Lemma 5.13: Let (U, τ, E) be soft regular. if each soft open covering of (U, τ, E) has a locally finite refinement, then it has a locally finite soft closed refinement.

Proof: Let $\mathcal{U} = \{(F_\alpha, E) : \alpha \in A\}$ be an arbitrary soft open covering. Then, for each soft point $e_x \in U_E$, there exists some $(F_\alpha, E) \in \mathcal{U}$ such that $e_x \widetilde{\in} (F_\alpha, E)$. By soft regularity, there is an soft neighborhood (V_{e_x}, E) such that $e_x \widetilde{\in} (V_{e_x}, E) \widetilde{\subseteq} \overline{(V_{e_x}, E)} \widetilde{\subseteq} (F_\alpha, E)$. Put $\mathcal{V} = \{(V_{e_x}, E) : e_x \in U_E\}$. Then $\mathcal{V}$ is a soft open covering and refines $\mathcal{U}$. By the assumption, there is a locally finite soft covering $\mathcal{W} = \{(W_\beta, E) : \beta \in B\}$ such that $\mathcal{W}$ refines $\mathcal{V}$. Then $\{\overline{(W_\beta, E)} : \beta \in B\}$ is a locally finite soft closed covering and refines $\mathcal{U}$. ■

By Lemmas 5.11, 5.12 and 5.13, we have the following theorem.

Theorem 5.14: Let (U, τ, E) be soft regular. Then the following conditions on U_E are equivalent:

1) (U, τ, E) is soft paracompact;
2) Every soft open covering has a σ-locally finite soft open refinement;
3) Every soft open covering has a locally finite refinement;
4) Every soft open covering has a locally finite soft closed refinement.

VI. Conclusion

In the present work, we have continued to study the properties of soft topological spaces. We mainly introduce soft open mappings, soft closed mappings, soft connected spaces and soft paracompact spaces. Moreover, we have also established several interesting results and presented its fundamental properties with the help of some examples. We hope that the findings in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

References

[1] Abdükadir Aygüoğlu and Halis Aygün, *Some notes on soft topological spaces*, Neural Comput & Applic, DOI 10.1007/s00521-011-0722-3.
[2] W.L. Gau and D.J. Buehrer, *Vague sets*, IEEE Transactions on Systems, Man, and Cybernetics, 23(2)(1993)610–614.
[3] S. Hussain and B. Ahmadc, *Some properties of soft topological spaces*, Comput. Math. Appl., 62(2011)4058–4067.
[4] M. Irfan Ali, F. Feng, X. Liu, W. K.'Min and M. Shabir, *On some new operations in soft set theory*, Comput. Math. Appl., 57(2009)1547–1553.
[5] A. Kharal and B. Ahmad, *Mappings of soft classes*, New Math. Nat. Comput, 7(3)(2011)471–481.
[6] D. Molodtsov, *Soft set theoryłfirst results*, Comput. Math. Appl., 37(1999)19–31.
[7] P.K. Maji, R. Biswas and R. Roy, *Soft set theory*, Comput. Math. Appl., 45(2003)555–562.
[8] W.K. Min, *A note on soft topological spaces*, Comput. Math. Appl., 62(2011)3524–3528.
[9] D. Pei and D. Miao, *From soft sets to information systems*, in Proceedings of the IEEE International Conference on Granular Computing, 2(2005)617–621.
[10] Z. Pawlak, *Rough sets*, International Journal of Information and Computer Science, 11(1982)341–356.
[11] M. Shabir and M. Naz, *On soft topological spaces*, Comput. Math. Appl., 61 (2011)1786–1799.
[12] I. Zorlutuna, M. Akdag, W.K. Min and S. Atmaca, *Remarks on soft topological spaces*, Annals of Fuzzy Mathematics and Informatics, 3(2)(2012)171–185.
[13] L.A. Zadeh, *Fuzzy sets*, Information and Control, 8(1965)338–353.