# New EEM/BEM Hybrid Method for Electric Field Calculation in Cable Joints

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**Abstract**—A power cable is widely used for power supply in power distributing networks and power transmission lines. Due to limitations in the production, delivery and setting up power cables, they are produced and delivered in several separate lengths. Cable itself, consists of two cable terminations and arbitrary number of cable joints, depending on the cable route length. Electrical stress control is needed to prevent a dielectric breakdown at the end of the insulation shield in both the air and cable insulation. Reliability of cable joint depends on its materials, design, installation and operating environment. The paper describes design and performance results for new modeled cable joints. Design concepts, based on numerical calculations, must be correct. An Equivalent Electrodes Method/Boundary Elements Method-hybrid approach that allows electromagnetic field calculations in multilayer dielectric media, including inhomogeneous regions, is presented.

*Keywords*—Cable joints, deflector's cones, equivalent electrode method, electric field distribution

#### I. INTRODUCTION

GENERALLY, the power cable is a conductor made of a copper or aluminum material, clad with a multi-layer electric shield and an insulating layer, made of a rubber-plastic material, and further with a metal shielding sheath. The power cable is designed to transfer an electric power, the voltage of which ranges from 1000 V to 1000 kV.

Due to limitations in the production, power cables are produced and delivered in several separate lengths. They are typically produced up to 1 km long sections, wound on the cable drum. There are many techniques in the practice for jointing and terminating the power cables, but the most preferable is heat shrinkable (HS) one.

From the aspect of failures, the most important parts of cable lines are the cable joints and terminations. The functions of a typical cable accessories are to provide a cable end seal, electrical stress control, and external insulation covering. The majority of cable failures on distribution system are caused by

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This paper is partially supported by the Ministry of Science, Republic of Serbia, through the scientific project No. 33008.

defects in the cable joints. For that reason any improvement in their construction is of interest. The design and choice of materials of a cable joints are vital to ensure its adequate performance during a long service life. Good cable joints reduce power loss and in the same time limit a possible electromagnetic field influence on the environment. This work is an effort to combine geometric and a high permittivity regulation of the electric field.

Electric field control and rigorous technological process are important for cable joints reliability. Hot spots very often coincide with maximum electric field. In order to optimize the cable joints, two criteria were monitored – total electric field magnitude,  $E_{\rm max}$ , and magnitude of the tangential component,  $E_t$ .

Without control, the high stress can lead to partial discharges in the dielectric, ionization and breakdown in the air or dielectrics, rapid aging of the insulation, leading to a dielectric puncture and failure. Appropriate choice of dielectrics and shape of deflectors is the most important. The field enhancement leads to a non-linear distribution of the potential on the insulation surface.

There are several methods for the solution of electric field distribution. These can be summarised as analytical, experimental, free-hand field mapping, analogue methods and numerical methods. Numerical calculations are the most powerful design tools. An accurate computation of this distribution can be made only numerically, using numerical methods and computing systems.

The boundary element method (BEM) is one of a variety of numerical methods for the solution of problems in applied science and engineering. It is an alternative to the domain methods of analysis in electromagnetics, such as the finite difference method (FDM) or the FEM. This paper presents a hybrid numerical method [1]-[3], based on Equivalent Electrode Method (EEM) ([4]-[8], [16]) and BEM, where the basic idea of successful application of EEM is that an arbitrary shaped electrode can be replaced by Equivalent Electrodes (EEs). Using the EEM it is possible to determine electric scalar potential and electric field in arbitrarily chosen point of cable joint region [10]-[12]. Similar procedure can be applied on determination of cable terminations [9]. EEs which replace various segments of cable conductor ends have toroidal shapes. Far away from the cable termination, charge distribution is continuous on its conductors.

Numerical calculations for electric field distribution at cable

joints are presented to further demonstrate the efficiency, accuracy and potentials of the hybrid EEM/BEM. Application of this hybrid numerical method is easier than the finite elements calculations ([13], [14]) in complex regions, and that the high degree of accuracy is achieved along with saving the computational time.

## II. OUTLINE OF THE METHOD

The basic idea of the Hybrid Boundary Elements Method (HBEM) is that an arbitrary shaped boundary between two dielectrics can be replaced by equivalent charges (ECH), where ECH are located at the dielectrics boundary. It is possible, by using condition for the normal component of polarization vector:

$$\mathbf{P}_{1n} - \mathbf{P}_{2n} = \boldsymbol{\eta}_{p} \,, \tag{1}$$

to form a system of linear equations, where equivalent polarized charges,  $\eta_p$ , are unknown. By solving this system, the unknown charges can be determined.

Similar procedure can be applied on arbitrary shaped perfect conducting electrodes, where electrodes are replaced by finite system of Equivalent Electrodes. In contrast to Charge Simulation Method, where the fictitious sources are placed inside the electrodes volume, the EEs are located on the body surface. The radius of EEs is equal to equivalent radius of electrode part, which is substituted.

This consideration can be applied on magnetic materials [2]. Boundary between two magnetic materials can be replaced by equivalent currents (ECU) where ECU are located on the boundary surface of the magnetic layers, having different magnetic permeability. A system of linear equations can be formed again, and surface density of Ampere's microscopic currents,  $J_s$ , are now unknown. Condition for different tangential components of magnetization vector on boundary of two magnetic materials is applied.

Metamaterial media and surfaces (with unconventional electromagnetic properties) have attracted a great deal of attention and interest in recent years. Various ideas involving double-negative (DNG) media, single-negative (SNG) materials, electromagnetic band gap (EBG) structures, and artificial very thin magnetic conductors (AMC) have been explored by many researchers over the past few years. New EEM/BEM hybrid method can be applied for metamaterial structures determination [17]-[18].

#### III. ELECTRIC FIELD DETERMINATION AT NON-MODELLED CABLE JOINTS

Due to the symmetry of the problem, adopting a cylindrical coordinate system with the z-axis coincident with the axis of the conductor, an equivalent two-dimensional problem can be studied (Fig.1).



Fig. 1 Non-modeled cable joint

When the boundary conditions are axisymmetric, the problem is completely two-dimensional in a single meridian plane. However, when the boundary conditions are nonaxisymmetric, the problem also depends on the azimuthal angular direction and is analysed as a sequence of twodimensional problems.

Charge density per unit surface is constant in the distant regions from the cable breaks. Appropriate electrical field is:

$$E_{\rm hom} = \frac{a E_0}{r \log \frac{b}{a}},\tag{2}$$

where  $E_0 = \frac{U}{a}$ , and U is supply voltage of the coaxial cable.

If it is presumed that such charge distribution is also in the surroundings of the cable break, and:

$$g(C, L_u) = L_u + \sqrt{C^2 + L_u^2},$$
 (3)

$$A^{2} = r^{2} + a^{2} - 2ar\cos\theta'; \ B^{2} = r^{2} + b^{2} - 2br\cos\theta', \quad (4)$$

$$I(r) = \begin{cases} \ln \frac{b}{a}, & 0 \le r \le a \\ \frac{1}{a}, & \frac{b}{a} \le r \le b, \\ \frac{1}{a}, & \frac{1}{a} \le r \le b, \\ 0, & r \ge b \end{cases}$$
(5)

where  $r, \theta$  and z are cylindrical coordinates, the

approximate expressions for potential, in different cable joint regions, are:

$$\frac{\varphi_{\rm apr}(r,z)}{U} = \frac{1}{2\pi \ln \frac{b}{a}} \int_{0}^{\pi} \ln \left( \frac{g(B, L_2 - z)}{g(A, L_1 - z)} \right) \mathrm{d} \, \theta' \tag{6}$$

for 
$$z \leq L_1 \leq L_2$$
,

$$\frac{\Phi_{\rm apr}(r,z)}{U} = \frac{1}{2\pi \ln \frac{b}{a}} \int_{0}^{\pi} \ln\left(\frac{g(B, L_2 - z) g(A, z - L_1)}{A^2}\right) d\theta'$$
(7)

for 
$$L_1 \le z \le L_2$$
,  

$$\frac{\varphi_{apr}(r,z)}{U} = \frac{1}{2\pi \ln \frac{b}{a}} \int_0^{\pi} \ln \left( \frac{B^2}{g(A, L_1 - z) g(B, z - L_2)} \right) d\theta'$$
(8)  
for  $L_2 \le z \le L_1$ , and:

$$\frac{\varphi_{\rm apr}(r,z)}{U} = \frac{1}{2\pi \ln \frac{b}{a}} \int_{0}^{\pi} \ln\left(\frac{g(A,z-L_1)}{g(B,z-L_2)}\right) \mathrm{d}\,\theta' + 2\pi I(r) \tag{9}$$

for  $L_1 \leq L_2 \leq z$ .

On the basis of the expressions for potential (6-9), and for  $L_1 = L_2 = L$ , approximate expressions for electric field's radial and axial components are determined.

For numerical calculations, the boundary region between two dielectrics, with different dielectric permittivities, can be divided into many little parts, replaced by polarised charges. Real expressions are superposition of approximate expressions (6-9) and additional terms that originate from equivalent electrodes (at boundary electrode-air (dielectric)) or equivalent charges (at boundary dielectric- dielectric).

35 kV cable (XHE 49-A,  $1x150/25mm^2$ , 20/35), is analyzed. Radius of inner conductor is a=7.62mn, outer is

$$b = 17.5 \, mm$$
, and  $E_0 = \frac{O}{a} = 4.58 \, \text{MV/m}$ 

The electric field distribution in cable joints is strongly nonuniform. 3D electric field distribution, at the place where two parts of this cable is jointed, is shown in Fig. 2.



Fig. 2 Tangential component  $(E_t)$  of electrical field distribution

# IV. ELECTRIC FIELD DETERMINATION AT MODELLED CABLE JOINTS

There are two main methods for electrical stress control: one being a geometry electrical stress control means and the other being a capacitance electrical stress control means. At present, the commonly used geometry means refers to an electrical stress cone.

# A. Geometric Method for Cable Joints Modelling

Stress distribution control is usually based on geometrical regulation with the stress relief cones, special materials of high relative dielectric constant, or embedded electrodes system application. There is no universal cable joint. There is a variety of different types of joints, each with advantages and disadvantages. The optimization of cable joints is achieved by considering various constructions [15].

Geometric stress control involves an extension of the shielding (Fig. 3) that expands the joint's diameter and reduces the stress at the discontinuity. Deflectors, which border lines have elliptical shape, the shape of polynomial or exponential function, as well as the combination of those shapes are mostly applied.



Equivalent sources (Fig. 4), which replace various segments of cable conductor ends and boundary between dielectrics, have toroidal shape. Axial cross-section of equipotential surfaces, where deflector has elliptical shape is presented in Fig. 5.



Fig. 5 Axial cross-section of equipotential surfaces

Multilayer dielectric systems can be applied on electric field reduction at cable joints. Cable joint, modelled by four layer dielectric system and very thin deflector's cones (conventional stress relief cones) is shown in Fig. 6.



Fig. 6 Modelled cable joint by using five - layer dielectric system and very thin deflector's cones, having polynomial form.

Electric scalar potential and intensity of electric field at jointing region are shown in Fig. 7 and Fig. 8.



Fig. 7 Equipotential curves at modelled cable joint by using fivelayer dielectric system and very thin deflector's cones



Fig. 8 Electric field distribution, E (MV/m), in radial direction for r/a = 3.0 (curve 1), r/a = 4.0 (curve 2), r/a = 5.0 (curve 3)

#### B. Refractive Method for Cable Joints Modelling

Although the stress relief cones offer good solution for the electric stress reduction, the high relative dielectric constant extruded tubes have many advantages in fabrication and installation. The tube is easily made and can be fabricated in shape of strips or tapes with a thick layer of increased relative dielectric constant. Numerical program allowed the study of optimal relative dielectric constant and thickness.

Some microscopic air bubbles necessarily could remain in the interface between dielectric layers, causing the local discharges under the both electric and thermal field. Partial discharges are allowed up to the determined level, which must not be overcome.

One of our proposals for cable termination construction is shown in Fig.9. The relative permittivities of insulation rubber and polyethylene were assumed to be 2.5 and 2.3, respectively.



Charge distribution at outer conductor (1), dielectrics

TABLE I Electric field distribution			
Distance from	Electric Field Strength (V/m)		
cable shield (mm)	Measured values	Hybrid EEM/BEM	FEM
0	34	37.489021	37.003298
5	26	25.974018	25.288782
10	7	5.1192010	5.1043127
15	29	27.251391	27.000296
20	14	13.840133	13.199352
25	10	9.7778921	9.7712175
50	4.5	4.4832913	4.483200
75	3	2.9923074	2.9923071
100	1.5	1.4050387	1.4067385
125	1	1.0565571	1.0526554
150	2.5	1.0004896	1.0002276





Fig. 10 Charge distribution at outer conductor (1), dielectrics boundary (2) and deflector (3)

Electric field distribution along dielectric layer, starting from cable shield, is shown In Table I. Normalized DC cable voltage (1 V) is applied. Experimental results, values obtained by using our hybrid EEM/BEM, and calculated values obtained by FEM were compared.Due to good convergence of series and integrals it is enough to take 15-20 terms in the series to get an accuracy not worse than 4%. The results of this work can form a basis for further investigations of hybrid cable joints construction with complex dielectrics.Axial component of electric field distribution,  $E_z$ , in radial direction, r/a, for z/a = 0.5 (curve 1), z/a = 2.0 (curve 2), z/a = 10.0 (curve 3), is presented in Fig. 11.



Fig. 11 Axial component of electric field distribution,  $E_z$  (MV/m), in radial direction, r/a for z/a = 0.5 (curve 1), z/a = 2.0 (curve 2), z/a = 10.0 (curve 3)

## C. Complex Method for Cable Joints Modelling

It is possible to construct geometrically modelled cable joint where deflector's ends are placed in dielectric with very high relative dielectric permittivity (Fig. 12). Deflector's end is ellipse's centre, made from "strong" dielectric.



Fig. 12 Modelled cable joint where deflector's ends are placed in dielectric with very high dielectric permittivity

Electric scalar potential and axial component of electric field at jointing region are shown in Fig. 13 and Fig. 14.

Lines of electrical flux are regulated to equalize the electrical stresses in a controlled manner along the entire area where the shielding has been removed.



Relative dielectric permittivity of applied dielectric, placed

in ellipse, is 50. The effects of electro-thermal and mechanical stresses can be enhanced in the presence of interfaces that may, thus, become the weakest points of the insulation system, both in AC and DC. Special attention must be paid to the interface between cable and joint body. The electrical field along this interface (part of the field parallel to the interface) is always a critical issue as the dielectric strength of this interface is practically lower than the strength of an insulating body. Therefore the stress control elements must be designed that way, which the field along this interface stays within the permissible limits. Interfaces can act as a trigger for partial discharges (PD) when the contact between surfaces is not well made, and such activity should be strictly avoided for cable and accessories. The purpose of corona testing is to determine whether all properly installed terminations operate corona-free at a minimum of 150% of their operating voltage.



Fig. 15 Refractive+geometrically modelled cable joint

Equipotential lines for this modelled cable joint are shown in Fig. 16.



Fig. 17 Axial component of electrical field distribution

The spacing of the electric flux lines and the corresponding equipotential lines is closer in the vicinity of the conductor than at the shield, indicating a higher electric stress on the insulation at the conductor. This stress increase, or concentration, is a direct result of the geometry of the conductor and shield in the cable section and is accommodated in practical cables by insulation thickness sufficient to keep the stress within acceptable values.

Electric field distribution (3D) at the cable joint, using refractive modeling and deflector with exponential shape, is presented in Fig. 17.

Magnitude of the tangential component of electric field is reduced from 3.27 MV/m (Fig. 2) to 0.96 MV/m (Fig. 17).

The most complex construction of cable joint, but cable joint with best electric field reduction, is shown in Fig. 18.



Modeled cable joint, except base construction, contains the other embedded (grounded or "floating") electrodes.

Axial component of electric field distribution,  $E_{z}$ , in axial

direction, z/a, for r/a = 1.5 (curve 1), r/a = 2.5 (curve 2), r/a = 4.5 (curve 3), is presented in Fig. 19.



Fig. 19 Axial component of electric field distribution in axial direction, for different distance from *z* axis

## V.CONCLUSION

It was presented a method, which allows computing the electric potential and field in 2D insulating structures, in presence of thin layers. Design could reduce the manufacturing cost, lower the difficulty of manufacturing, and reduce the time for installation.

The optimized stress control tube application technique possibly with non linear characteristic material or multiple stress control layers are suggested.

The method is applicable to a wide class of problems, and can be easily and efficiently implemented on modern material design [17]-[18].

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