

# On the Quantizer Design for Base Station Cooperation Systems with SC-FDE Techniques

K. Firsanov, S. Gritsutenko, and R. Dinis

**Abstract**—By employing BS (Base Station) cooperation we can increase substantially the spectral efficiency and capacity of cellular systems. The signals received at each BS are sent to a central unit that performs the separation of the different MT (Mobile Terminal) using the same physical channel. However, we need accurate sampling and quantization of those signals so as to reduce the backhaul communication requirements.

In this paper we consider the optimization of the quantizers for BS cooperation systems. Four different quantizer types are analyzed and optimized to allow better SQNR (Signal-to-Quantization Noise Ratio) and BER (Bit Error Rate) performance.

**Keywords**—Base Stations cooperation scheme, Bit Error Rate (BER), Quantizer, Signal to Quantization Noise Ratio (SQNR), SC-FDE.

## I. INTRODUCTION

BASE STATION (BS) cooperation techniques are under consideration for the LTE standard (Long Term Evolution) for high speed data transfer in wireless systems. Since the LTE technology is adopted in many countries of the world and is constantly developing, this technique can be used in future standard releases (see [1], [9] and references within). The application of the BS cooperation schemes for the uplink transmission is designated to decrease frequency reuse factor value up to 1. Frequency reuse factor is the rate at which frequencies can be used in cells of wireless network. Wireless system, operating with smaller frequency reuse factor, provides higher capacity and specter efficiency. Also the BS cooperation architecture provides macro-diversity effects and can compensate the high interference effects inherent to systems operating with universal frequency reuse factor (this interference is especially high at cell edges). The general idea of the base station cooperation technique is that signal, transmitted from MT (mobile terminal) is received by BS and then is sent to the CPU (central processing unit), that performs detection and separation of signals, corresponding to different MTs. The channel capacity between BS and CPU is limited, so signal samples, that are sent from BS and are processed in the CPU has to be quantized in the optimal way. It is

K. Firsanov and S. Gritsutenko are with the Omsk State Transport University, Omsk, 644046 Russia (e-mail: Konstantin.firsanov@gmail.com st256@mail.ru).

R. Dinis is with IT (Instituto de Telecomunicações) and FCT-UNL (Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa), Lisbon, Portugal e-mail: rdinis@fct.unl.pt).

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necessary to represent signal samples with the number of bits large enough to avoid BER performance degradation.

The efficient detection and quantization requirements have been determined in [1]. According to this work results, even coarse quantization 4-5 bits in in-phase and quadrature components of the complex envelope provides efficient separation of signals, transmitted by MTs and close to optimum macro diversity gain. But quantization as an operation can be done in a numerous ways. And for different signals at the quantizer input, different quantizer types, having the same number of quantization levels, provide different performance. So in order to select optimal quantizer, it's necessary to estimate performance of every quantizer type.

In this paper we consider the uplink of the broadband wireless communication system that consists of mobile terminals, base stations and CPU. MTs are regarded as employing SC-FDE schemes. Each BS receives signals, performs sampling and quantization and sends signal samples to the CPU, where user detection is done. The separation of signals is done with IB-DFE application, as shown in [9]. In this paper four quantizer types are analyzed: uniform quantizer, non-uniform floating point quantizer, non-uniform optimal Lloyd Max quantizer and non-uniform optimized quantizer, which preserves maximum information on its output. The signal is considered to be complex. The representation of complex signal is regarded as values of its in-phase and quadrature components. All quantizer types are investigated for SQNR and for the system performance in the terms of BER of the detector. The paper is organized as follows: Section II reviews every quantizer type investigated in the paper, Section III estimates each quantizer upon the SQNR term, Section IV contains evaluation of the performance of every quantizer type in the terms of the BER. Section V concludes the paper.

## II. REVIEW OF QUANTIZATION DEVICES

Fig. 1 shows the structure of quantizer, that operates on the complex valued samples. This quantizer can be considered as two identical "I-Q" memoryless nonlinearities, which operate separately on the real and imaginary parts of the complex samples  $x$ , applied to the quantizer input. The equation, that characterizes this class of quantizers, by the relation between input and output samples, can be recorded as:

$$x_q = Q(\text{Re}\{x\}) + jQ(\text{Im}\{x\}) \quad (1)$$

In (1), Q denotes quantizer function. In this paper four types of quantizers, according to their quantizer function Q are investigated.

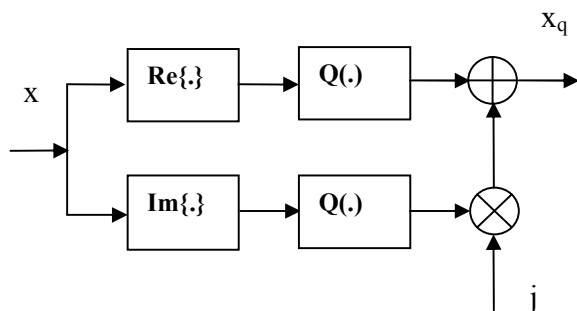


Fig. 1 Quantizer model, operating on the real and imaginary parts of complex envelope

According [2] quantizer function Q can be written as:

$$Q(x) = \sum_{i=0}^{i=N-1} q_i L_i(x, x_i, x_{i+1}) \quad (2)$$

In (2),  $L_i(x, x_i, x_{i+1}) = 1$ , if  $x_i \leq x < x_{i+1}$  and  $L_i(x) = 0$  otherwise. Values  $x_i$  are quantizer decision levels and  $q_i$  values are its representation levels, so that  $x_i \leq q_i < x_{i+1}$ .  $N$  - is number of quantizer steps. Quantizer has a dynamic range, so all N quantizer steps are distributed between quantizer saturation levels  $\pm Sl$ , for quantizers, that operate on in-phase and quadrature components. The relationship between decision and representation levels is shown on the Fig. 2.

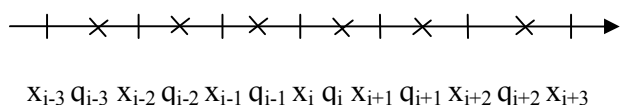


Fig. 2 Quantizer decision ( $x_i$ ) and representation ( $q_i$ ) levels

The decision levels are adjacent intervals and representation levels are values inside of these intervals. Quantizers can be classified according to the selection of representation and decision levels. The quantizer that has equal distance between decision levels and between representation levels is uniform quantizer. Otherwise, the quantizer is called as non-uniform. Decision and representation levels of non-uniform quantizer are usually calculated, according to some criteria. In this paper uniform and non-uniform quantizer types are investigated. Among non-uniform quantizers, we explore non-uniform floating point quantizer, Lloyd Max quantizer and quantizer, which levels are selected so, that to preserve maximum quantity of the information on the quantizer output. Fig. 3 shows Q function of the uniform quantizer. For the mid-raiser

uniform quantizer Q function decision and representation levels can be calculated as:

$$x_i = -Sl + i * \Delta \text{ if } i < N \quad (3)$$

$$q_i = -Sl + i * \Delta + \Delta / 2 \quad (4)$$

In (3) and in (4)  $\Delta = 2Sl / N$  is quantizer step size,  $Sl$  is quantizer saturation level.

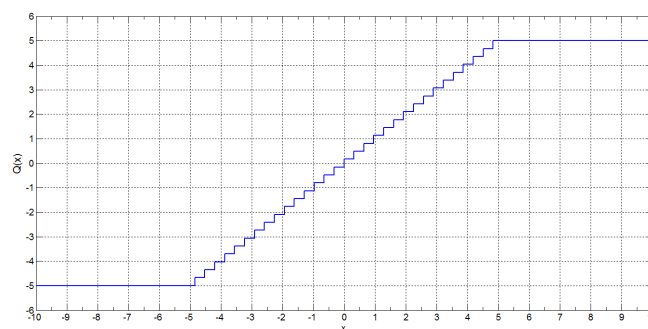


Fig. 3 Q-function of the uniform quantizer

The floating point non-uniform quantizer can be represented as function of number of quantization levels in mantissa and order:  $Q(M, N)$ , where  $M$  is number of bits in mantissa and  $N$  is number of bits in order of floating point representation. The order of the floating point quantizer can be calculated as:

$$n = \text{floor}(\log_2(2^{N-1}|x|/Sl)) \quad (5)$$

The mantissa of floating point number is quantized uniformly. The decision and representation levels of the mantissa are calculated like for the uniform quantizer and distributed in the range  $\left[ \frac{Sl}{2^{N-1}}, \frac{Sl}{2^{N-2}} \right)$ :

$$x_{im} = \text{sign}(x) \left( \frac{Sl}{2^{N-1}} + \frac{i}{2^{N+M-2}} \right) \quad (6)$$

$$q_{im} = \text{sign}(x) \left( \frac{Sl}{2^{N-1}} + \frac{Sl(2i+1)}{2^{N+M-1}} \right) \quad (7)$$

Using (5), (6) and (7) the quantized value  $x_q$  of input value  $x$  can be calculated as:

$$x_q = q_{im} 2^n \quad (8)$$

Fig. 4 shows Q function of the floating point non-uniform quantizer. The Q function of this quantizer type represents  $2^N$  intervals quantized uniformly. Quantizer step in each interval is different.

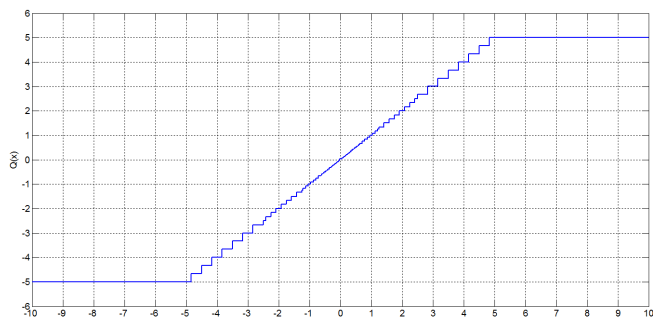


Fig. 4 Q-function of the floating point non-uniform quantizer

Decision and representation levels of the Lloyd Max quantizer and quantizer, that preserves maximum information are calculated on the base of statistical characteristics of the signal on the quantizer input. The decision and representation levels of the Lloyd Max optimal quantizer are calculated so, that to minimize mean square error between quantizer input and output signals, according to [10]. In order to perform calculation it's necessary to have analytical description of input signal probability density function (PDF). According to [4], decision and representation levels of the Lloyd Max quantizer are calculated by the solution of the non-linear system of equations:

$$q_i = \frac{\int_{x_i}^{x_{i+1}} xf(x)dx}{\int_{x_i}^{x_{i+1}} f(x)dx} \quad (9)$$

$$x_i = (q_i + q_{i-1}) / 2 \quad (10)$$

In (9)  $f(x)$  is probability density function of the signal  $x$  on the quantizer input. For calculation of the system of (9) and (10) above, an iteration algorithm is applied. On every iteration, the mean square error (MSE) is checked. Fig. 5 displays relation between PDF of the input signal  $x$  and representation levels of the Lloyd Max quantizer. In can be seen, the more value PDF takes, the less distance between quantizer representation levels is provided. And from the other side, the distance between representation levels is more in the areas, where PDF has smaller values.

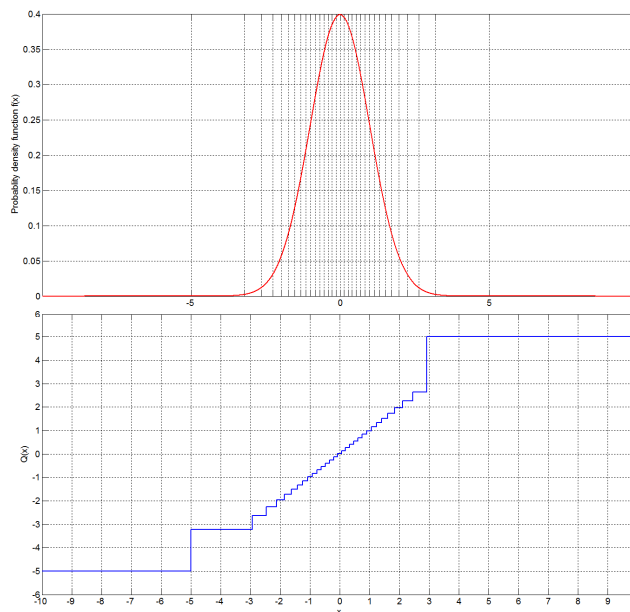


Fig. 5 Selection quantizer decision and representation levels, according to Gaussian PDF, and Lloyd Max quantizer Q-function

Decision and representation levels of the quantizer, that preserves maximum information, are calculated so, that to maximize information of the input signal in the quantized signal. As it was proved in [5] it can be done only under fulfillment of the conditions (11) and (12). In (11)  $F^{-1}$  is cumulative distribution function of the input values  $x$ . In (12)  $f(x)$  is PDF of the input values  $x$ .  $N$  is number of quantizer intervals.

$$x_i = F^{-1}(i / (N + 1)) \quad (11)$$

$$q_i = \frac{\int_{x_i}^{x_{i+1}} xf(x)dx}{\int_{x_i}^{x_{i+1}} f(x)dx} \quad (12)$$

Fig. 6 shows  $F^{-1}$  function and relation between decision and representation levels of this quantizer. The decision levels are marked with dotted lines, going out of abscissa and representation levels are shown with dashed lines.

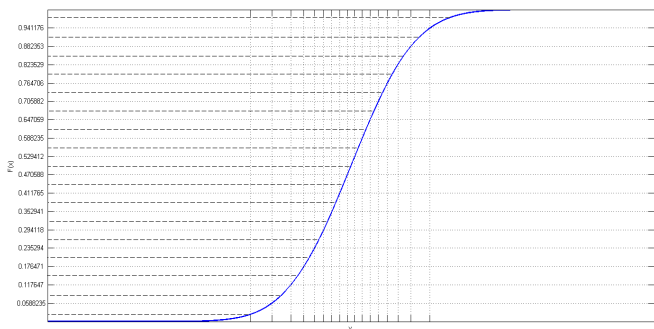


Fig. 6 Selection decision and representation levels of the quantizer, that preserves maximum information, according to  $F^{-1}$  function of the input signal  $x$

### III. ESTIMATION OF THE QUANTIZER SQNR

Quantization is a non-linear operation. The side effect of this operation is quantization noise. In literature quantization noise is often analyzed as an additive, uniformly distributed, uncorrelated with input signal noise, having white spectrum. But in fact, as it is shown in [2] and the quantization noise is

correlated with input signal. But if input signal PDF satisfies to quantizing theorem 1 and 2, proved by Widrow, the signal at the output of quantizer can be modeled as sum of input signal and uncorrelated uniformly distributed noise, as it is shown in [8]. In this paper, we consider signal at quantizer input as Gaussian, having not limited characteristic function, so the model with uniformly distributed noise can't be taken for analysis. Another issue, that complicates analysis of quantizers is clipping, that takes place because of the limited dynamic range of quantizer device. Despite SC-FDE signal is characterized by a low peak to average power ratio, the effect of clipping has to be taken into account as well. In works [3] and [6] quantizer is analyzed as memoryless non-linearity, the Gaussian input signal is applied to. Both works give estimation of the SQNR of the quantizer, with the Gaussian signal on its input. According to [7], the signal  $y(t)$  at the output of the memoryless non-linearity  $Q$  with Gaussian input  $x(t)$  can be written as:

$$y(t) = \alpha x(t) + d(t) \quad (13)$$

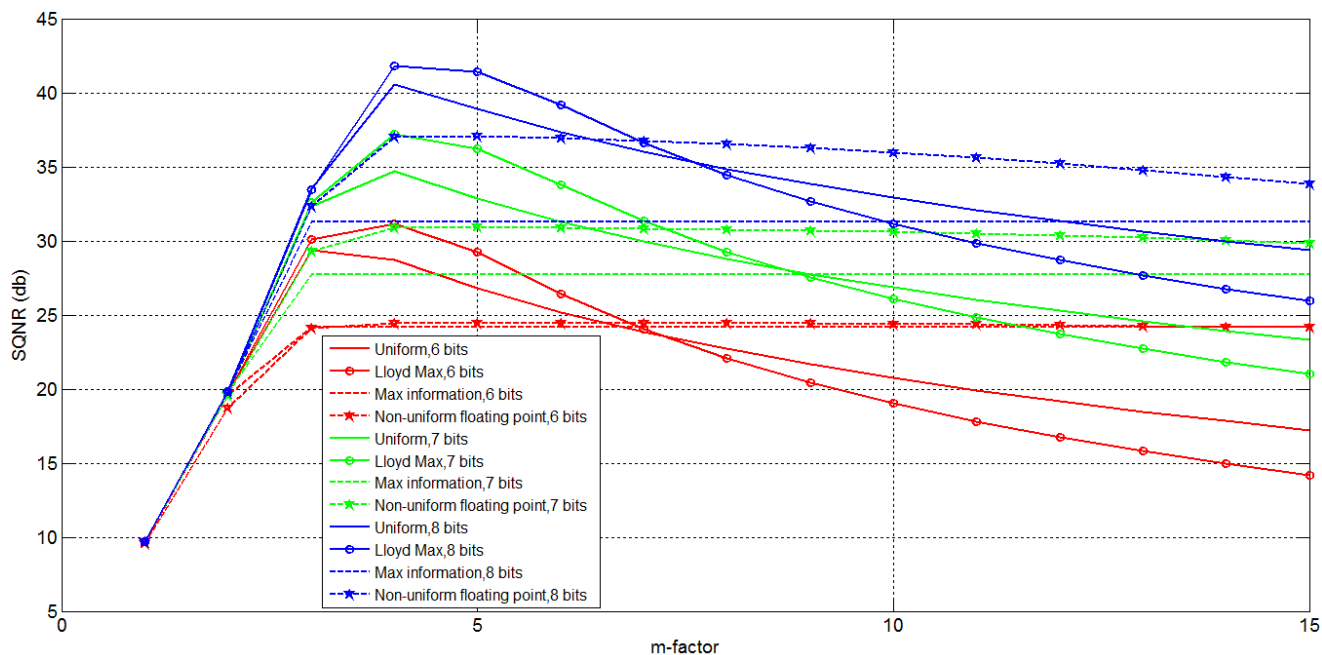


Fig. 7 SQNR as a function of  $\mu$ -factor for quantizer types, with accuracy of both in-phase and quadrature components 6, 7 and 8 bits

In (13),  $d(t)$  - distortion component, that is not correlated with  $x(t)$ .  $\alpha$  - is constant, expressed as:

$$\alpha = \frac{1}{\sigma^3 \sqrt{2\pi}} \int_{-\infty}^{\infty} x Q(x) \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \quad (14)$$

Using results of works [3], [6] it's easy to get SQNR charts for analyzed quantizer types with Gaussian input. For

quantizers that operate with in-phase and quadrature components the application is straightforward. The Fig. 7 shows SQNR dependency from clipping factor  $\mu$ :

$$\mu = Sl / \sigma \quad (15)$$

In (15),  $Sl$  - quantizer saturation level,  $\sigma$  - signal variance. It can be seen on the figure, that uniform quantizer shows comparable SQNR values with Lloyd Max quantizer, that provides MSE less than  $1e-6$  and quantizer, that preserves

maximum information shows worse performance, than uniform quantizer. As to optimal Max Lloyd quantizer, the considerable superiority over uniform quantizer can be reached, if quantizer decision and representation levels are calculated with more accuracy, so that MSE would be as small as possible. Both Lloyd Max and uniform quantizers show better performance, than floating point non-uniform quantizer. For each chart the order of the floating point non-uniform quantizer is presented with 3 bits accuracy. The mantissa is represented with  $n - 3$  bits, where  $n = 6, 7, 8$ , according to charts. The positive moment in floating point quantizer and quantizer, which preserves maximum information is that they are more robust, than Lloyd Max and uniform quantizers. According to Fig. 7, the SQNR chart dies slowly for floating point quantizer and quantizer, which preserve maximum information, comparing to uniform and Lloyd Max quantizers.

#### IV. QUANTIZERS PERFORMANCE EVALUATION

This section presents performance evaluation results. Model with 2 MTs and 2 cooperative BS is considered for the uplink transmission, employing SC-FDE. Channels between MTs are uncorrelated and time dispersive. Each channel has rich multipath propagation and uncorrelated Rayleigh fading for different multipath components. Each MT performs transmission in blocks, having 256 symbols, selected from QPSK constellation. BSs receive signals, perform quantization in the time domain and send quantized samples to CPU, which

performs DFT and after that makes separation of the different signals, using iterative frequency domain receivers, based on the IB-DFE schemes. Generally SC-FDE signal doesn't have Gaussian distribution. However, taking in account large number of multipath components, signal distribution can be regarded as Gaussian. Fig. 8 contains charts showing BER as function of SNR for different quantizer types, used in BS, for optimal  $\mu$ -factor ratio. The accuracy of the real and imaginary parts is 8 bits. Optimal  $\mu$ -factor is calculated for each quantizer type, according to accuracy in bits. The chart shows BER after the first and 4-th iteration in the IB-DFE receiver. The number of bits in mantissa of floating point quantizer is 5, order is presented with 3 bits. If accuracy of the signal representation is 8 bits all quantizers show comparable results after the 4-th iteration. In results of [1] optimal number of bits in quantized signal was determined as 4-5. Fig. 9 contains charts showing BER as function of SNR for different quantizer types, used in BS, for optimal  $\mu$ -factor ratio for the accuracy of the real and imaginary parts of the complex envelope 5 bits. For the case of floating point quantizer mantissa was represented with 3 bits and order with 2 bits. It can be seen from the Fig. 9, that uniform and Lloyd Max quantizers show comparable results, better than quantizer, that preserves maximum information. Floating point quantizer shows poor performance if total number of bits in mantissa and order is 5.

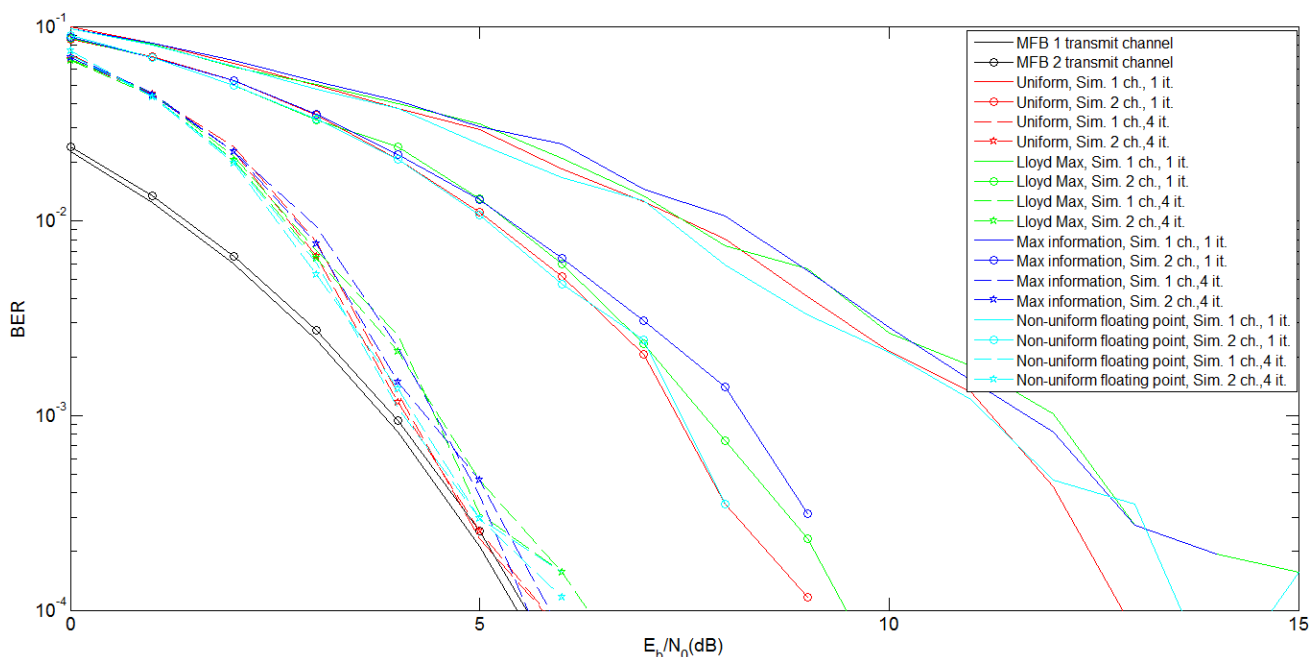


Fig. 8 Performance of different quantizer types. Accuracy of the representation of real and imaginary parts of signal is 8 bits. Signal modulation: QPSK Channel type: XTAP. Saturation level: optimal Number of iterations: 4. Signal quantized in the time domain

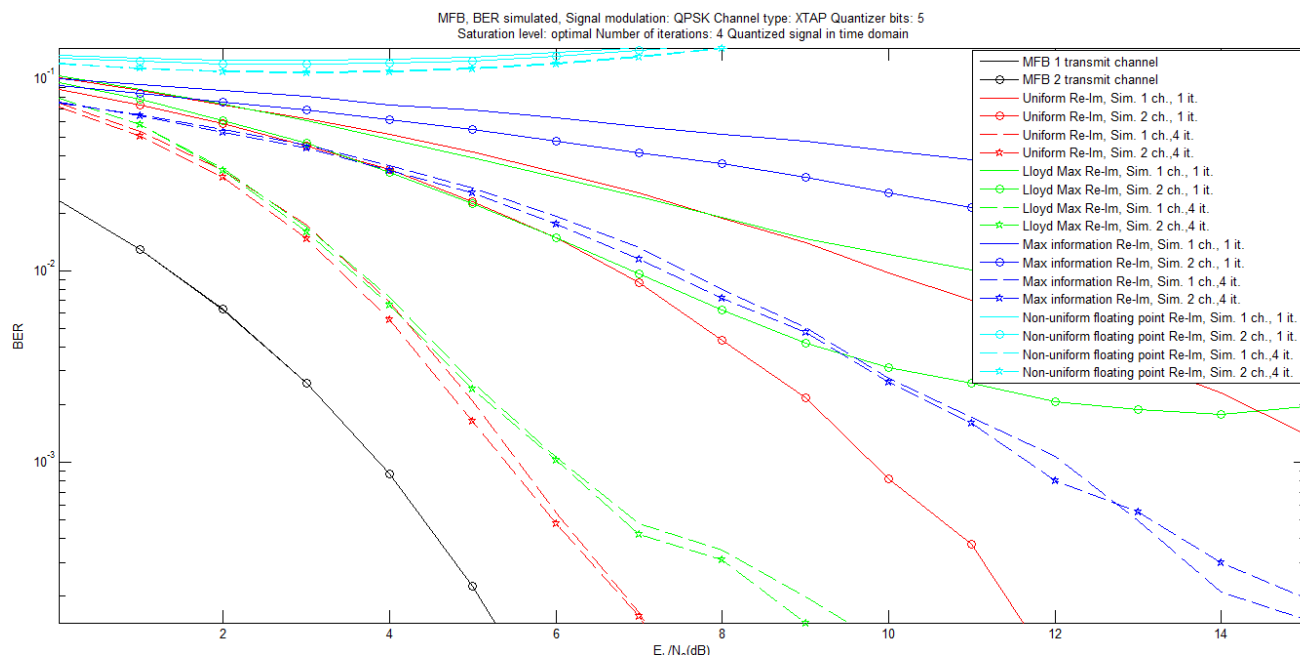


Fig. 9 Performance of different quantizer types. Accuracy of the representation of real and imaginary parts of signal is 5 bits. Signal modulation: QPSK Channel type: XTAP. Saturation level: optimal Number of iterations: 4. Signal quantized in the time domain

#### V. CONCLUSION

In this paper the uplink of the BS cooperation scheme based on the SC-FDE modulations has been considered. Four quantizer types have been investigated for the application in the BS of the considered cooperation system. Quantizers have been estimated for SQNR and for the BER. It has been shown, that with equal bit representation, optimum Max Lloyd quantizer and uniform quantizer show better SQNR from  $\mu$ -factor, than optimal quantizer, that preserves maximum information and floating point quantizer. Max Lloyd quantizer provides even better SQNR, comparing to the uniform quantizer, if its decision and representation levels are calculated to provide MSE less than  $1e-6$ . Floating point quantizer shows better SQNR, than quantizer, that preserves maximum information. It has been shown, that floating point quantizer and quantizer, that preserves maximum information are more robust to high  $\mu$  ratios, than uniform and Max Lloyd quantizers. For signal accuracy 8 bits and more all quantizers provide comparable performance in BER term. If signal is quantized with small number of bits (4-6), uniform and Lloyd Max quantizers provide nearly equal performance in BER, which is better than performance of quantizer, that preserves maximum information and floating point quantizer. The poor performance of the floating point quantizer, comparing to others if signal resolution in bits is equal, can be explained by the less accuracy, comparing to other quantizers. In floating point quantizer some bits are reserved for order representation. So the mantissa is represented with less accuracy, comparing to other quantizers.

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