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Statistical Computational of Volatility in Financial Time Series Data

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Abstract**—**It is well known that during the developments in the economic sector and through the financial crises occur everywhere in the whole world, volatility measurement is the most important concept in financial time series. Therefore in this paper we discuss the volatility for Amman stocks market (Jordan) for certain period of time. Since wavelet transform is one of the most famous filtering methods and grows up very quickly in the last decade, we compare this method with the traditional technique, Fast Fourier transform to decide the best method for analyzing the volatility. The comparison will be done on some of the statistical properties by using Matlab program.

Keywords**—**Fast Fourier transforms, Haar wavelet transform, Matlab (Wavelet tools), stocks market, Volatility.

I. INTRODUCTION

MOST of the financial researchers suppose that the probability theorems effect on the current values of the option only, then probably the stocks go up. But the recent rules and the financial practical show that there are two affections on the current value; the rate of the risk free borrowing as well as stock price volatility.

The volatility is directly related with the risk management, describe it specifically and it consider as one of the most method to measure the risk in the real financial data. As well as the volatility for stocks return becomes very common and important in the practice. If the data has a very high volatility, then it will be very risky data. Furthermore, the opposite is correct, in the other hand, volatility implies to the uncertainty or risky data. So that, the high volatility means that the security's value can be spread out over the whole range of values. However, if the security's value does not fluctuate dramatically, then the volatility is very low. Consequently, a lot of models was established to evaluate and estimate the volatility such that; autoregressive conditional heteroskedasticity, stochastic and GARCH model which is considered as generalization for the autoregressive conditional heteroskedasticity. For more examples and details refer to [1, 2, and 3].

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Recently, wavelet analysis and Fourier analysis are two methods that are used for filtering time series representing. Wavelet analysis has grows very quickly in the recent years and more recently Wall Street analysts have started using wavelets as a mathematical models to analyze their financial data. Although Fourier analysis has been quite popular in the scientific world, wavelet analysis also has been used in signal processing, pattern recognition, decomposition, approximation techniques and quantum field [6], it has been shown that wavelet transform are more efficient than Fourier transform [7-11]. This is because wavelet transform can be used to analyzed nonlinear and nonstationary time series signals. Therefore, to show the efficiency of wavelet transform, this paper will introduce the discrete wavelet transform (DWT) and fast Fourier transforms (FFT) and then demonstrates that Haar wavelet transform is better than fast Fourier transform in decomposing or analyzing the volatility financial time series data. We will use MATLAB to obtain some of numerical and statistical results. (The Haar wavelet is the simplest example of DWT which is Daubechies-1 transformation as discussed in Siddiqi (2004) [13] and Daubechies (1992) [14]).

II. DEFINITIONS AND CONCEPTS

Volatility**:** the researchers improve a lot of the definitions about the volatility. Firstly, the most important popular one is the standard deviation as a traditional technique. Secondly, the difference in the price's result between up and down. Practically, both of them have the same result. Therefore, in this paper we use the second definition since consider as a modern technique. In the other words, it defined as the absolute value of the daily return. Mathematically expressed as: [10]

$$r_t = \left|\log(x_t) - \log(\mathrm{x}_{t-1})\right|.$$

Fourier transform: mathematically is an operation to transfer the complex valued function to other function; this function is known as frequency domain. Consequently, the Fourier transforms look like the other operation in mathematical science. So that, we discuss one case of Fourier transform which is the discrete Fourier transform (DFT). [4]

Definition: [4] discrete Fourier transforms (DFT) was defined for discrete points N as

$$X(K) = \sum_{n=0}^{N-1} X(n) W_n^{kn}. \qquad K = 0,1,...,N-1.$$

Where $X(n)$ represents the time series data.

And

$$W_n = e^{\frac{-j\pi 2}{N}}.$$

More generally, for a vector matrix

$$X = F_x$$

Where

$$x = \left[x(0), x(1), ..., x(N-1)\right]^T.$$

And

$$X = \left[X(0), X(1), ..., X(N-1)\right]^T.$$

F : Presents $N \times N$ with elements

$$\left[F\right]_{i,L} = W_N^{i,L}.$$

The inverse discrete Fourier transform (IDFT) was defined by this equation:

$$X(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) W_N^{-kN}, \qquad n = 0,1,...,N-1.$$

FFT and IFFT are two algorithms which designed from the previous equations DFT and IDFT respectively. Consequently, FFT and IFFT directly depend on the DFT and IDFT respectively, we use other concept related to the Fourier series which is called Gabour transform or short time Fourier transform, this method was introduced by Gabour (1946) and attempts to make a balance between time and frequency. [10].

Haar wavelet transform: consider the following function:

$$\psi_{j,k}(t) = 2^{\frac{j}{2}}\psi(2^j t - k), \; j,k \in Z; \; z = \{0,1,2,....\}.$$

Where; ψ are a real valued function having compactly supported, and $\int_{-\infty}^{\infty}\psi(t)dt = 0$. For examples and details explanation refers to [15, 16]. The oldest and simplest example of ψ is the Haar wavelet, defined as:

$$\psi^H(t) = \begin{cases} 1, & 0 \le t \le \frac{1}{2} \\ -1, & \frac{1}{2} \le t \le 1 \\ 0, & Otherwise \end{cases}$$

To calculate the wavelet function, we use the dilation equations, given as:

$$\phi(t) = \sqrt{2}\sum_k l_k \phi(2t-k), \qquad (1)$$

$$\psi(t) = \sqrt{2}\sum_k h_k \phi(2t-k). \qquad (2)$$

Previously, we defined father and mother wavelets in (6) and (7) where $\phi(2t-k)$ is the father wavelet, $\psi(t)$ is the mother wavelet. Father is used to represent the high scale smooth components of the signal, while the mother wavelets display the deviations from the smooth components. Since the father wavelet generates the scaling coefficients and mother wavelet gives the differencing coefficients. We define the father wavelets as lower pass filter coefficients (h_k) and the mother wavelets as high pass filters coefficients (l_k) [14].

$$l_k = \sqrt{2}\int_{-\infty}^{\infty}\phi(t)\phi(2t-k)dt$$

$$h_k = \sqrt{2}\int_{-\infty}^{\infty}\psi(t)\psi(2t-k)dt.$$

However, quadrature mirror filters (QMF) is other concept related to the wavelet transform, it has some a good ability to reconstruct the signals perfectly without aliasing effects, [17].

For the Haar wavelet:

$$l_k = \sqrt{2}\int_{-\infty}^{\infty}\phi(t)\phi(2t-k)dt = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

And for N = 2.

$$l_k = \{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\} \quad \text{and} \quad h_k = \{\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\}.$$

Note: the mother wavelet satisfies the following conditions:

$$\int_{-\infty}^{\infty}\psi(t)dt = 0, \quad \int_{-\infty}^{\infty}\left|\psi(t)\right| < \infty. \qquad \text{And}$$

$$\int_{-\infty}^{\infty}\frac{\left|\psi_1(\omega)\right|^2}{\left|\omega\right|}d\omega < \infty.$$ Where $\psi_1(\omega)$ is the Fourier transform.

3-ANALYZING THE VOLATILITY DATA:

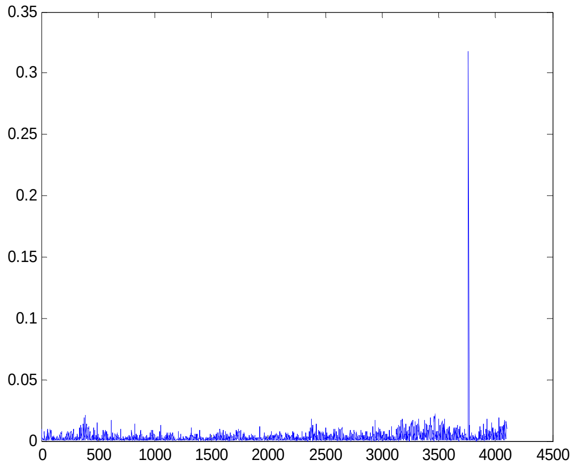

Fig. 1 Volatility data for Amman Stocks Market closing price "between" January 1992- October 2008

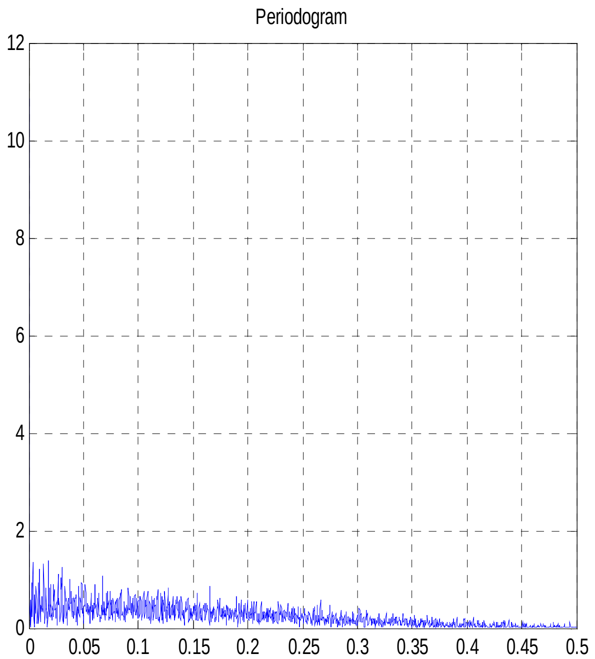

Fig. 2 The Power Spectrum Estimation or the magnitude of the complex vector square. Since the result of this transform is a complex vector, which represented by $z = x + iy$, where $i = \sqrt{-1}$, x, y are real numbers. Furthermore, z can be represented by using polar form; $z = Ae^{i\theta}$, where A shows the magnitudes for θ and z, θ is the angle and z is the phase

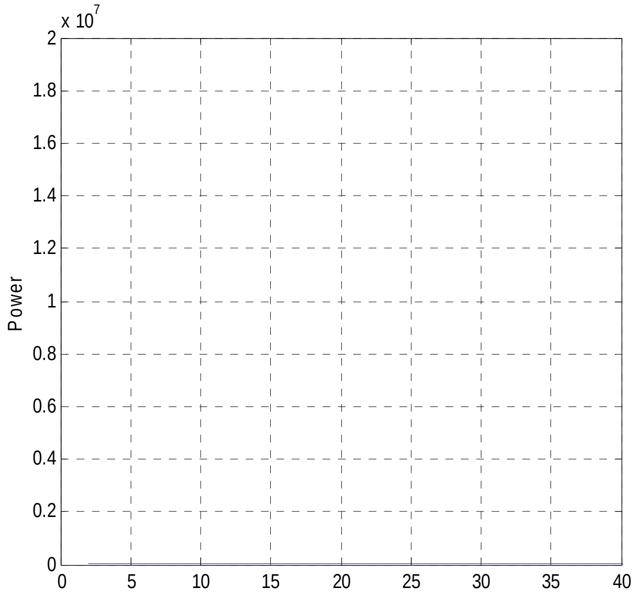

Fig. 3 The magnitude of the frequency plot precisely for the previous transformation. Since Fig. 2 is inconvenient to present all of the magnitudes very well. So that, we shift the transformation to appear the fluctuations and magnitude more precisely. However, it is still very weak transformation

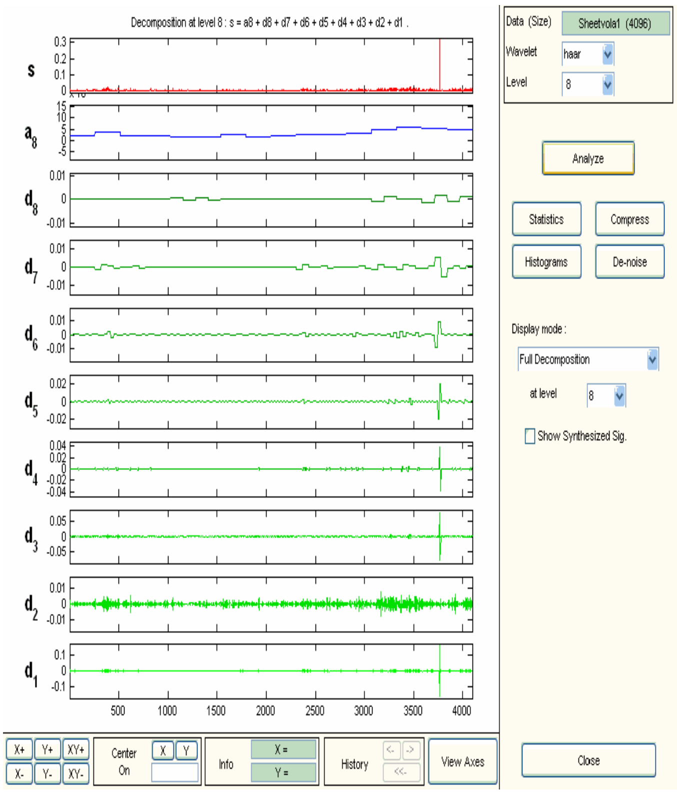

Fig. 4 Data analysis: using MATLAB_ Haar wavelet transform level 8. Which represents the analysis for the volatility data for level 8; it represents the fluctuations, magnitudes and phases for Amman stocks markets for certain period of time

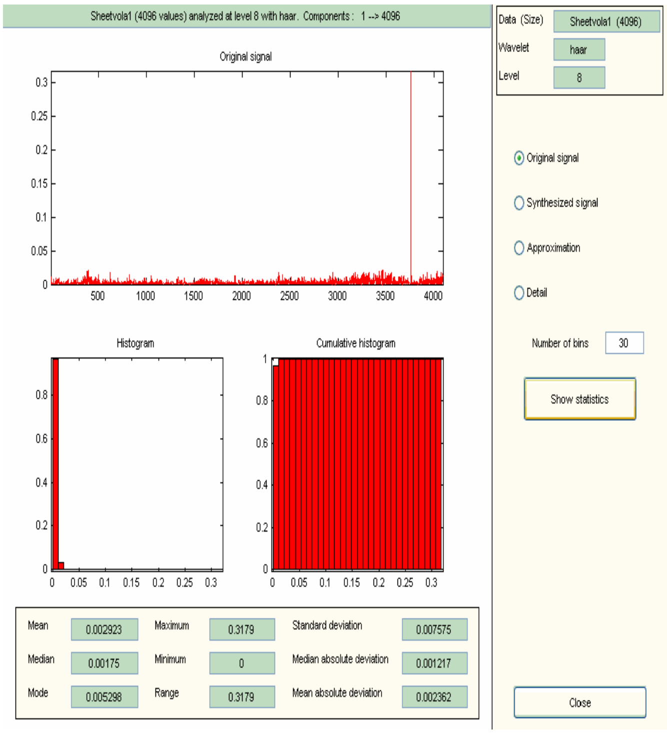

Fig. 5 Statistical analysis: using MATLAB_ Haar wavelet transform level 8. It shows the most important statistical results for volatility data which is very useful to understand the data features

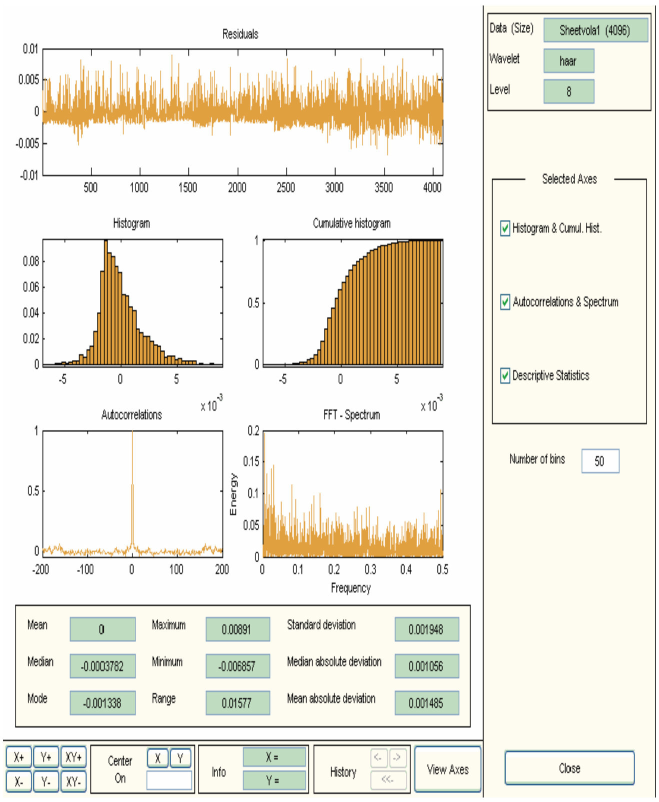

Fig. 6 Data analysis: using MATLAB_ Haar wavelet transform level 8. It shows the residual signals with its observations. The residual is very important in many cases. Especially in time series and numerical methods which represented in the time series model as ε_t .

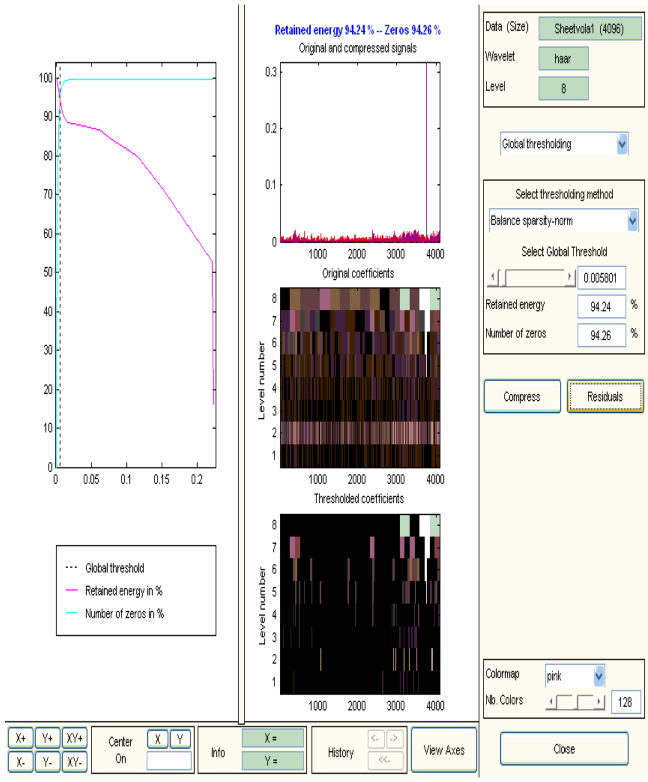

Fig. 7 Data analysis: using MATLAB_ Haar wavelet transform level 8. It shows the compressed time series observations with its details. Firstly, it shows the insurance time series data with its compressed signals, and then it represents its coefficients details. Secondly, it shows the threshold coefficients which are the simplest method to present the de-noising data. It usually produces a smaller estimate since the wavelet coefficients are pushed to be zero, and shows the numbers of zeros

4-DISCUSSION AND CONCLUSION

The Figs. (1-7) display the distributions for the volatility time series data by using the spectral filtering method fast Fourier transform and Haar Wavelet transform. Fig. 1 gives the original distribution for the volatility data, Fig. 2 shows the periodogram which is the plot of the estimation power spectrum versus frequently, and we notice that this transformation is convenient since it appears all of the fluctuations filtering the data from some features like the outlier data. However, in Fig. 3 after we do some shifting for the data we get that fast Fourier transform becomes unsuitable since after the shifting all of data adherent and approach on the x –axis, it means that hardly to capture and detect the fluctuations for the stocks market during the whole time . So that, we consider the transform technique FFT is inconvenient method since the magnitudes are very necessary to determine and explain more information about the Niquest frequency.

At the same time, if we focus in the Figs. 4, 5, 6 and 7 we notice a wide differences in the decompositions since it is more meaningful and convenient to capture and understand the stocks market behavior. Fig. 4 shows the analysis for the volatility time series data until level 8. According to the Figs. 5, 6 and 7, they show the statistical analysis, residuals and the compressed time series data respectively. However, these results cannot display by using FFT. Moreover, these results are very necessary and important in the analysis.

Wavelet compression is very fit for the image compression which is aimed to store the image data inside a little space, as well as, in any time series model we have to evaluate the residual data which is represented by any model as ε_t.

Finally, as a conclusion, we have compared the distribution of the volatility for two methods FFT and HWT for a certain period of time, after we test the methods we conclude this result HWT is the most appropriate model to decompose nonlinear and non stationary time series data. It provides a helpful, meaningful and powerful method for estimation and captures the features.

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