World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:7, No:7, 2013

Model Order Reduction of Discrete-Time Systems Using Fuzzy C-Means Clustering

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***Abstract*—**A computationally simple approach of model order reduction for single input single output (SISO) and linear time-invariant discrete systems modeled in frequency domain is proposed in this paper. Denominator of the reduced order model is determined using fuzzy C-means clustering while the numerator parameters are found by matching time moments and Markov parameters of high order system.

***Keywords*—**Model Order reduction, Discrete-time system, Fuzzy C-Means Clustering, Padé approximation.

I. Introduction

HIGHER order models are difficult to handle due to computational complexities and implementation difficulties and they are too complicated to be used in real time problems. It is therefore, desirable that a higher model is replaced with lower model. A large variety of methods of model order reduction are available in literature [1]-[13]. In spite of several methods available, no approach gives best result for all systems.

A very powerful method that involves simple algebraic calculations comprises continued fraction, moments matching and Padé approximation [2], [3]. However, the Padé method has a drawback that it may produce an unstable approximant for a given stable original system. Many methods such as Routh-approximation and Routh-Padé approximants [4]-[6] for continuous-time and [26], [27], [30] for discrete-time system have been suggested to obtain stable model. Model-order reduction by matching Markov parameters, time-moments and impulse energy approximation [7]; [8] have also suggested to ensure the stability of the reduced order model. Krylov subspace methods [9] is also found to be quite popular tool for obtaining reduced order model of very high order linear time-invariant systems which is relatively simple and cheap that can handle the system with a few thousand degrees of freedom but stability of the reduced model is not guaranteed. Another important group of reduction algorithm is the eigen value preservation technique [10], [11] where important eigen values of the system are retained to find stable lower order model. Several other methods [12], [13] are available to find stable reduced order model of a stable high order systems (HOS). Recently, pole clustering techniques [14]-[16], [21] has become quite popular in the area of model order reduction that is conceptually simple and easy to implement.

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Many methods are available for model order reduction of continuous systems but very few are extended to discrete-time systems. The techniques for discrete-time systems may be classified into two groups. The first group contains the methods which exploit the already existing continuous-time algorithm, G(z) into another one, $G_1(w)$, using the bilinear transformation $z = (1+w)/(1-w)$ [17] or other transformations such as $z = w/(Aw + B)$, $z = w+1$ [18]. Then one of the known techniques for continuous-time systems is applied to obtain a reduced approximant $R_1(w)$ of $G_1(w)$. Finally the corresponding inverse transformation $w= \varphi(z)$ yields from $R_1(w)$ the required approximant R(z). The second group contains so called direct method [19] that derive R(z) directly from G(z) without using the transformation. Model order reduction of discrete-time systems using canonical expansion of z-transfer function and stable optimal method is discussed in [24], [25]. Methods like power decomposition and system identification [28], and order reduction using multipoint step response matching [31] give simple and quite effective method of model-order reduction of discrete-time systems. Modern heuristic optimization techniques like genetic algorithm [22] and particle swarm optimization [32] are also used for reducing the order of discrete-time systems.

The proposed method is a mixed method for model order reduction which combines pole clustering and Padé approximation. In this paper, technique of pole clustering called Fuzzy C- Mean (FCM) clustering [20] is used to find the desired number of pole clusters. Once denominator polynomial of reduced order model is determined, the numerator coefficients are obtained by Padé approximation. This paper is organized as follows: Problem formulation is given in Section II. Numerical examples and comparison of proposed method with other well known techniques is shown in Section III and conclusion in Section IV.

II. Problem Formulation

Consider, a stable SISO discrete-time system described by the transfer function

$$G(z)=\frac{a_1z^{n-1}+a_2z^{n-2}+a_3z^{n-3}+...+a_n}{z^n+b_1z^{n-1}+b_2z^{n-2}+b_3z^{n-3}+...+b_n} \tag{1}$$

Using discrete-time to continuous-time transformation algorithm of MATLAB on (1) an equivalent continuous-time system obtained as

$$G(s)=\frac{a_1 s^{n-1}+a_2 s^{n-2}+a_3 s^{n-3}+.....+a_n}{s^n+b_1 s^{n-1}+b_2 s^{n-2}+.....+b_n} \tag{2}$$

The problem is to determine its stable reduced-order (rth-order) approximant.

$$G_r(s)=\frac{\hat{a}_1 s^{r-1}+\hat{a}_2 s^{r-2}+\hat{a}_3 s^{r-3}+...+\hat{a}_r}{s^r+\hat{b}_1 s^{r-1}+\hat{b}_2 s^{r-2}+\hat{b}_3 s^{r-3}+...+\hat{b}_r} \tag{3}$$

Once (3) is found using continuous-time to discrete-time transformation algorithm of MATLAB the discrete-time reduced order model of (1) is found as

$$G_r(z)=\frac{\hat{c}_1 z^{r-1}+\hat{c}_2 z^{r-2}+\hat{c}_3 z^{r-3}+...+\hat{c}_r}{z^r+\hat{d}_1 z^{r-1}+\hat{d}_2 z^{r-2}+\hat{d}_3 z^{r-3}+...+\hat{d}_r} \tag{4}$$

A. Pole Clustering

Clustering of data is a process by which large amount of data is grouped into a smaller number of groups to facilitate its meaningful analysis by lowering dimensionality (two or three maximum). Clustering methods are generally used for organizing and categorizing data to solve classification and pattern recognition problems [16], [20]. It can also be useful for data compression and model order reduction. A number of clustering techniques are available in literature. K-Means and Fuzzy C-Means clustering are the types that can be used if numbers of clusters are known apriori as is required in the case of model order reduction. These techniques are used in conjunction with radial basis function networks (RBFNs) and fuzzy modeling.

K-means or Hard C-means (HCM) clustering [16] algorithm relies on finding the cluster centers by minimizing a cost function (or an objective function) of dissimilarity (or distance) measure [20]. In most cases dissimilarity measure is chosen as the Euclidean distance and the cost function based on the Euclidean distance between a vector $\boldsymbol{x}_k$ in group j and corresponding cluster centre $\mathbf{c_i}$, can be defined by

$$J=\sum_{i=1}^{c} J_i=\sum_{i=1}^{c}\left(\sum_{k,x_k \hat{I} G_i}\left\|x_k-c_i\right\|^2\right) \tag{5}$$

where: J_i is the cost function within group *i*.

The partitioned groups are defined by a c×n binary membership matrix **U,** where element u_{ij} is 1 if the jth data point $\mathbf{x}_j$ belongs to group *i*, and 0 otherwise. Once the cluster centers $\boldsymbol{c}_i$ are fixed, minimizing u_{ij} for (5) can be derived as

$$u_{ij}=\begin{cases} 1 \text{ if } \left\|x_j-c_i\right\|^2 \le \left\|x_j-c_k\right\|^2, \text{for each } k \ne i \\ \text{o otherwise} \end{cases} \tag{6}$$

If the membership matrix u_{ij} is fixed, then the optimal center $\mathbf{c_i}$ that minimize (5) is the mean of all vectors in group *i*:

$$c_i=\frac{1}{\left|G_i\right|}\sum_{k,x_k \hat{I} G_i} x_k \tag{7}$$

Fuzzy C-Means clustering [20] is an improvement over HCM clustering. In FCM-clustering each data point belongs to a cluster to a degree specified by a membership grade and it allows one piece of data to belong to two or more clusters. The membership matrix **U** is allowed to have elements with values between 0 and 1, while, the sum of degrees of belongingness of the data point to all clusters is always equal to unity:

$$\sum_{i=1}^{c} u_{ij}=1 \qquad \text{where } j=1,2,...,n \tag{8}$$

The cost function for FCM is generalization of (5)

$$J=\sum_{i=1}^{c} J_i=\sum_{i=1}^{c}\sum_{j=1}^{n} u_{ij}^m d_{ij}^2 \tag{9}$$

where u_{ij} is between 0 and 1, $\mathbf{c_i}$ is the cluster center of fuzzy group i; $d_{ij}=\left\|c_i - x_j\right\|$ is the Euclidean distance between the i^{th} cluster center and the j^{th} data point; and m Є [1,∞].

The necessary conditions for (9) to reach to its minimum are:

$$c_i=\frac{\sum_{j=1}^{n} u_{ij}^m x_j}{\sum_{j=1}^{n} u_{ij}^m} \tag{10}$$

and

$$u_{ij}=\frac{1}{\sum_{k=1}^{c}\left[\frac{\left\|c_i-x_j\right\|}{\left\|c_k-x_j\right\|}\right]^{2/(m-1)}} \tag{11}$$

The FCM Algorithm [20] works iteratively through the preceding two conditions until no more improvement is noticed. In a batch mode operation, FCM determines the cluster centers $\mathbf{c_i}$ and the membership matrix **U** using the following steps:

Step 1. Initialize the membership matrix **U** with random values between 0 and 1 such that the constraints of (8) are satisfied.

Step 2. Calculate c fuzzy cluster centers $\mathbf{c_i}$, i=1,…,c using (10)

Step 3. Compute the cost function according to (9). Stop if it is below a certain tolerance value or its improvement over previous iteration is below a certain threshold.

Step 4. Compute a new **U** according to (11) and go to step 2.

FCM algorithm starts by assigning random values to the membership matrix **U**, therefore several iterations have to be conducted for getting good performance and it is possible to have different degree of membership function to each cluster. The algorithm must be run several times, each starting with different values of membership grades of data points.

B. Determination of Denominator of Reduced Order Model

The r^th order denominator of reduced model is obtained as follows:

The most dominant poles $\lambda'_{d1}, \lambda'_{d2,\dots,} \lambda'_{dm}$ of HOS are retained and the following rules are used for clustering the poles.

i. Separate clusters should be made for real poles and complex poles.
ii. Poles on the jw-axis have to be retained in reduced-order model.

The desired number of clustered centers for the denominator poles of the HOS is determined using FCM algorithm as discussed above. If all the poles to be clustered are real, desired number of cluster centers λ'_{ci} i=1, 2,…, (*r-m*).

For complex conjugate poles $\left[(\alpha_1 \pm j\beta_1),\ (\alpha_2 \pm j\beta_2)\dots,\ \right]$ using FCM algorithm for both real and imaginary parts separately the cluster pole centers are obtained as $\Phi_{cj} = A_{cj} \pm jB_{cj}$ where A_{cj} and B_{cj} are cluster centers of real poles and imaginary poles respectively.

For synthesizing r^{th} order denominator polynomial, one of the following cases may occur:

Case 1. If all the cluster centers are real, then the denominator polynomial of r^{th} order model can be obtained as:

$$D_r(s) = (s-\lambda'_{d1})\dots(s-\lambda'_{dm})(s-\lambda'_{c1})\dots(s-\lambda'_{c(r-m)}) \qquad (12)$$

Case 2. If all the cluster centers are complex conjugate, the cluster centers are found using FCM algorithm. The denominator of ROM is realized using the dominant poles and cluster centers (poles) with its conjugate pairs.

Case 3. If system has both real and complex conjugate poles, the denominator polynomial is synthesized using the combination of dominant pole as well as cluster centers in its real and complex conjugate form. From the above cases, the r^{th} order denominator of ROM is obtained as

$$D_r(s) = s^r + \hat{b}_1 s^{r-1} + \hat{b}_2 s^{r-2} + \hat{b}_3 s^{r-3} + \dots + \hat{b}_r \qquad (13)$$

C. Determination of Numerator of Reduced Order Model

The numerator of ROM is determined by using Padé Approximation. G(s) (1) is expanded around s=0 and s=∞ as

$$\begin{aligned} G(s) &= t_1 + t_2 s + \dots + t_n s^{n-1} + \dots \\ &= M_1 s^{-1} + M_2 s^{-2} + \dots + M_n s^{-n} + \dots \end{aligned} \qquad (14)$$

where t_is are the time moments and M_is are the Markov parameters $(i = 1, 2, 3, \dots)$ of HOS

Similarly $G_r(s)$ is expanded around s=0 and s=∞ as

$$\begin{aligned} G_r(s) &= \hat{t}_1 + \hat{t}_1 s + \dots + \hat{t}_r s + \dots \\ &= \hat{M}_1 s^{-1} + \hat{M}_2 s^{-2} + \dots + \hat{M}_r s^{-r} + \dots \end{aligned} \qquad (15)$$

where $\hat{t}_i$s are the time moments and $\hat{M}_i s$ are Markov parameters $(i=1,2,3,\dots)$ of ROM. The matching equations of time moments and Markov parameters of system and ROM are found as follows

Case 1. r even

We seek a stable model for which following r equations are to be satisfied

$$\hat{t}_i - t_i = 0 \quad \text{and} \quad \hat{M}_i - M_i = 0 \quad \text{for } i=1, 2, \dots, r/2 \qquad (16)$$

The time moments and Markov parameters are related with numerator & denominator coefficients as

$$\hat{a}_{r+1-i} = \sum_{j=1}^{i} t_j \hat{b}_{r-i+j} \text{ and } \hat{a}_i = \sum_{j=1}^{i} M_j \hat{b}_{i-j} \quad \text{for } i=1,2,3,\dots,r/2 \qquad (17)$$

Case 2. r odd

In this case, with the requirement that $\hat{t}_1, \hat{t}_2, \dots, \hat{t}_{\frac{r-1}{2}}, \hat{t}_{\frac{r+1}{2}}$ and $\hat{M}_1, \hat{M}_2, \dots, \hat{M}_{\frac{r-3}{2}}, \hat{M}_{\frac{r-1}{2}}$ are exactly matched.

With $t_1, t_2, \dots, t_{\frac{r-1}{2}}, t_{\frac{r+1}{2}}$ and $M_1, M_2, \dots, M_{\frac{r-3}{2}}, M_{\frac{r-1}{2}}$ respectively, we have

$$\hat{a}_{r+1-i} = \sum_{j=1}^{i} t_j \hat{b}_{r-i+j}, i=1,2,\dots,\frac{r+1}{2} \text{ and } \hat{a}_i = \sum_{j=1}^{i} M_j \hat{b}_{i-j} \quad \text{for } i=1,2,\dots,\frac{r-1}{2} \qquad (18)$$

Using above equations and selecting the desired number of time moments and Markov parameters of the original and reduced order model to be matched, the modified numerator coefficients $\hat{a}_1, \hat{a}_2 \dots\dots \hat{a}_r$ of the reduced order model is computed. Thus, the numerator of the r^{th} order model comes out to be

$$N_r(s) = s^r + \hat{a}_1 s^{r-1} + \hat{a}_2 s^{r-2} + \hat{a}_3 s^{r-3} + \dots\dots + \hat{a}_r \qquad (19)$$

D. Calculations of Time Moments [23] and Markov Parameters of Discrete Time System

Putting z = p+1 in (1) and expanding about p=0, (1) becomes

$$G(p)=\frac{a_1(p+1)^{n-1}+a_2(p+1)^{n-2}+a_3(p+1)^{n-3}+...+a_n}{(p+1)^n+b_1(p+1)^{n-1}+b_2(p+1)^{n-2}+b_3(p+1)^{n-3}+...+b_n} \quad (20)$$

$$=\frac{A_1p^{n-1}+A_2p^{n-2}+A_3p^{n-3}+...+A_n}{p^n+B_1p^{n-1}+B_2p^{n-2}+B_3p^{n-3}+...+B_n} \quad (21)$$

$$=t_0+t_1p+t_2p^2+...$$

$$=t_0+t_1(z-1)+t_2(z-1)^2+... \quad (22)$$

The parameters t_i's are given by

$$t_0=\frac{A_n}{B_n} \text{ and } t_i=(A_{n-1}-\sum_{j=0}^{i-1}B_{n+j-i}t_j)/B_{n,}\ i=1,2,... \quad (23)$$

where it is understood that $A_i=0 \text{ for } i\le 0$; $B_0=1; B_i=0 \text{ for } i\le -1$. Hence the time moments of discrete time system becomes

$$T_i=\begin{cases} t_i & \text{for } i=0 \\ (-)^i\sum_{j=1}^{i}\frac{1}{j!}(T_s)^j w_{ij}t_j & \text{for } i=1,2,... \end{cases} \quad (24)$$

where T_S is the sampling frequency and w_{ij} is defined as

$$w_{ij}=\begin{cases} w_{i-1,j-1}+jw_{i-1,j} & \text{for } i>j \\ 0 & \text{for } i<j \end{cases} \quad \text{with } w_{ii}=w_{i1}=1 \quad (25)$$

For the reduced-order model represented by (4) the respective time moments T_i's take the form

$$\hat{T}_i=\begin{cases} \hat{t}_i & \text{for } i=0 \\ (-)^i\sum_{j=1}^{i}\frac{1}{j!}(T_s)^j w_{ij}\hat{t}_j & \text{for } i=1,2,... \end{cases} \quad (26)$$

where $\hat{t}_i$'s are given by

$$\hat{t}_0=\frac{\hat{A}_r}{\hat{B}_r} \text{ and } \hat{t}_i=(\hat{A}_{r-1}-\sum_{j=0}^{i-1}\hat{B}_{r+j-i}t_j)/\hat{B}_{r,}\ i=1,2,... \quad (27)$$

and it is understood that $\hat{A}_i=0 \text{ for } i\le 0$; $\hat{B}_0=1; \hat{B}_i=0 \text{ for } i\le -1$. Note that $\hat{A}_i$'s, $\hat{B}_i$'s are obtained for the reduced model in same manner as A_i's, B_i's are obtained from the high order system.

The Markov parameters of the system (1) is determined by expanding (1) around z=0. Thus

$$G(z)=M_1z^{-1}+M_2z^{-2}+M_3z^{-3}+... \quad (28)$$

$M_1=a_1$ and $M_i=a_i-\sum_{j=1}^{i-1}b_{i-j}M_j, i=2,3,...$ where it is understood that $a_i=b_i=0 \text{ for } i=n+1,\ n+2,...$

Expanding model (4) around z=0, one has

$$G_r(z)=\hat{M}_1z^{-1}+\hat{M}_2z^{-2}+\hat{M}_3z^{-3}+... \quad (29)$$

$\hat{M}_1=\hat{a}_1$ and $\hat{M}_i=\hat{a}_i-\sum_{j=1}^{i-1}\hat{b}_{i-j}\hat{M}_j, i=2,3,...$ where it is understood that $\hat{a}_i=\hat{b}_i=0 \text{ for } i=n+1,n+2,...$

III. Numerical Examples

Example 1: Consider a stable 6th order discrete-time system [29]

$$G(z)=\frac{0.3277z^6+0.9195z^5+1.038z^4+0.5962z^3+0.1618z^2+0.006986z-0.005308}{z^6+1.129z^5+0.2889z^4-0.08251z^3-0.04444z^2-0.00476z+0.0000} \quad (30)$$

(T_0=1.3320, T_1=-2.4904, M_1=0.3277, M_2=0.5498)

As the given discrete-time system have a pole at origin, zero-order hold (ZOH) command of continuous to discrete transformation cannot be used. Using the Tustin transformation G(z) is transformed to G(s) taking sampling time 1.0s.

$$G(s)=\frac{s^5+15.6s^4+124.2s^3+510.3s^2+1166s+959.3}{s^6+21s^5+175s^4+735s^3+1624s^2+1764s+720} \quad (31)$$

(t_1=1.3320, t_2=-1.6448, t_3=1.7334; M_1=1, M_2=-5.4, M_3=62.6)

The poles of G(s) are
$\lambda_1=-1, \lambda_2=-2, \lambda_3=-3, \lambda_4=-4, \lambda_5=-5, \text{and } \lambda_6=-6$

It is desired to obtain a third order model of the form

$$G_3(s)=\frac{\hat{a}_1s^2+\hat{a}_2s+\hat{a}_3}{s^3+\hat{b}_1s^2+\hat{b}_2s+\hat{b}_3} \quad (32)$$

The dominant pole '-1' is retained and using FCM algorithm (see section A) from remaining poles two clustered poles are found to be -2.6907 and -5.3114 and the denominator of ROM is obtained as

$$D_3(s) = s^3 + 9.002s^2 + 22.29s + 14.29 \tag{33}$$

Matching two time-moments and Markov parameter and using (19) the numerator of ROM is found to be

$$N_3(s) = s^2 + 6.186s + 19.0343 \tag{34}$$

Thus the ROM of the proposed method turns out to be

$$G_3(s) = \frac{s^2 + 6.186s + 19.0343}{s^3 + 9.002s^2 + 22.29s + 14.29} \tag{35}$$

(t_1=1.3320, t_2=-1.6448; M_1=1, M_2=-2.8160)

Third-order approximant using Inverse Distance Measure (IDM) method of pole clustering [14] is given as

$$G_3(s) = \frac{s^2 - 0.328s + 33.22}{s^3 + 10.22s^2 + 30.55s + 24.93} \tag{36}$$

(t_1=1.3325, t_2= -1.6461; M_1=1, M_2= -10.5480)

Step responses of continuous time model of original system (31), proposed model (35) and IDM model (36) are plotted in Fig. 1. It is found that the response of the proposed model is identical to that of HOS (31) while the IDM model shows deviation from the original model.

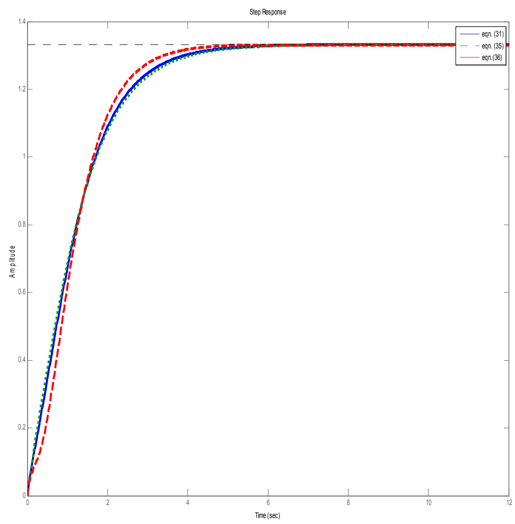

Fig. 1 Step response of continuous-time (transformed) original systems and ROMs

Transforming continuous-time third-order proposed ROM (35) to discrete time system using Tustin transformation in MATLAB and taking sampling time 1 s. The proposed model in z-domain comes out to be

$$G_3(z) = \frac{0.3442z^3 + 0.6364z^2 + 0.3959z + 0.1036}{z^3 + 0.2667z^2 - 0.1333z - 0.02218} \tag{37}$$

(T_0=1.3321, T_1=-2.5745, M_1=0.3442, M_2=0.5445)

Similarly transforming continuous-time third-order of IDM model (36) to discrete-time system using Tustin transformation and taking sampling time 1s comes out to be

$$G_3(z) = \frac{0.271z^3 + 0.7039z^2 + 0.7134z + 0.2805}{z^3 + 0.5264z^2 - 0.02365z - 0.02439} \tag{38}$$

(T_0=1.3322, T_1=-2.6052, M_1=0.2710, M_2=0.5615)

Step responses of discrete-time model of original system (30), proposed model (37) and IDM model (38) are plotted in Fig. 2. The response of the proposed model is identical to the HOS (30). These findings are also confirmed by examining the ISE (Integral Square Error) given in Table I.

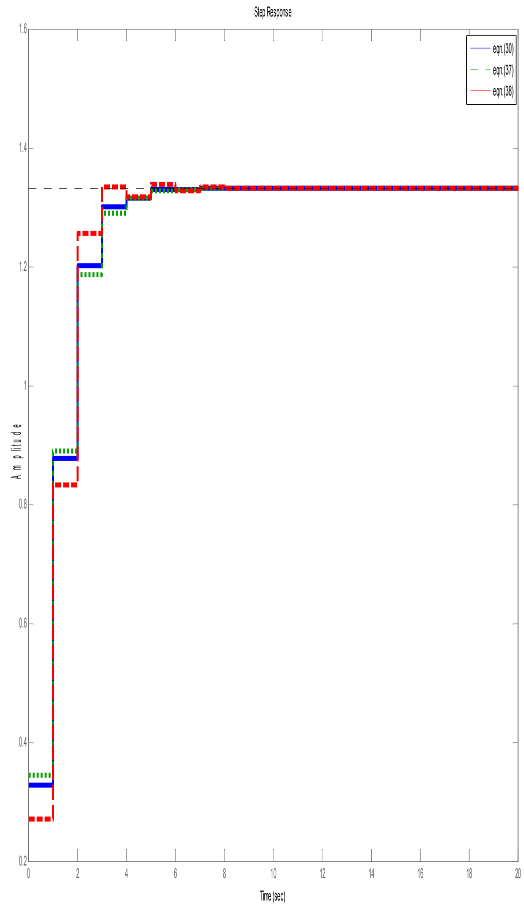

Fig. 2 Step response of discrete-time original system and ROMs

TABLE I
COMPARISON OF PROPOSED METHOD AND IDM METHOD

Reduction method	Reduced model	ISE
Proposed method (37)	$G_3(z)=\dfrac{0.3442z^3+0.6364z^2+0.3959z+0.1036}{z^3+0.2667z^2-0.1333z-0.02218}$	0.0010
IDM method (38)	$G_3(z)=\dfrac{0.271z^3+0.7039z^2+0.7134z+0.2805}{z^3+0.5264z^2-0.02365z-0.02439}$	0.0154

Example 2: Consider a stable 8th order discrete-time system [32]

$$G(z)=\frac{0.165z^7+0.125z^6-0.0025z^5+0.00525z^4-0.02263z^3-0.00088z^2+0.003z-0.000413}{z^8-0.6208z^7-0.416z^6+0.07613z^5-0.05915z^4+0.1906z^3+0.09737z^2-0.01635z+0.002226} \quad (39)$$

(T_0=1.0701, T_1=1.3774, T_2=-6.0620; M_1=0.1650, M_2=0.2274, M_3=0.2073)

G(z) is transformed to continuous-time system G(s) taking sampling time (T_S) 0.1s.

$$G(s)=\frac{0.2618s^7+26.64s^6+2603s^5+1.018e05s^4+2.331e06s^3+3.599e07s^2+3.236e08s+1.186e09}{s^8+61.08s^7+2671s^6+7.765e04s^5+1.502e06s^4+1.926e07s^3+1.747e08s^2+3.896e08s+1.108e09} \quad (40)$$

(t_1=1.0704, t_2=-0.0843, t_3=-0.1066; M_1=0.262, M_2=10.65, M_3=1253.3)

Using proposed method the 6th order reduced model of the original HOS (39) by matching 3-time moments and 3-Markov parameters is found to be

$$G_6(s)=\frac{0.262s^5+24.2s^4+2317.7s^3+47401.5s^2+505355.5s+2158996.8}{s^6+51.7s^5+1961s^4+2.365e04s^3+2.947e05s^2+6.291e05s+2.017e06} \quad (41)$$

(t_1=1.0704, t_2=-0.0833, t_3=-0.1069; M_1=0.262, M_2=10.65, M_3=1253.0)

Using IDM [14] method 6th order approximant of HOS (39) comes out to be

$$G_6(s)=\frac{0.262s^5+17.68s^4+1738.5s^3+17318.8s^2+172443.8s+717596.16}{s^6+26.83s^5+761.4s^4+8684s^3+9.979e04s^2+2.139e05s+6.704e05} \quad (42)$$

(t_1=1.0704, t_2=-0.0843, t_3=-0.1066; M_1=0.2620, M_2=10.65, M_3=1253.25)

Step responses of continuous time model of original system HOS (40), proposed model (41) and IDM model (42) are plotted in Fig. 3 and found that the response of the proposed model is almost identical to that of HOS.

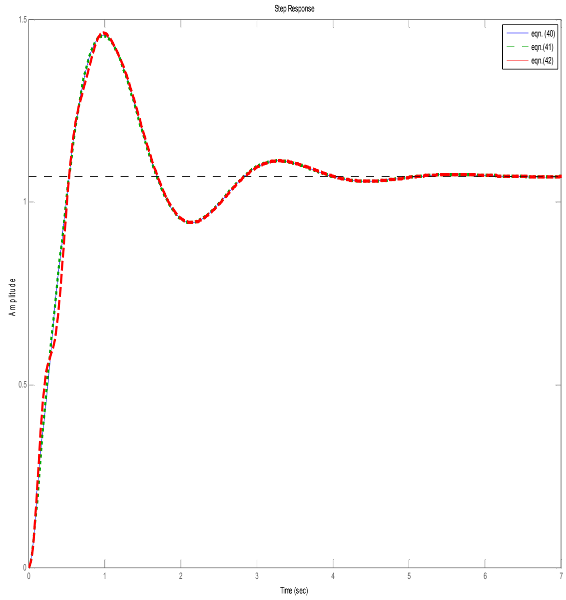

Fig. 3 Step response of continuous-time (transformed) original systems and ROMs

Transforming continuous-time sixth-order proposed ROM (41) to discrete time system using Tustin transformation and taking sampling time 0.1 s discrete-time model comes out to be

$$G_6(z)=\frac{0.1453z^5-0.01645z^4-0.04762z^3+0.03367z^2-0.006161z-0.001431}{z^6-1.841z^5+1.303z^4-0.5537z^3+0.09981z^2+0.06726z+0.005685} \quad (43)$$

(T_0=1.0613, T_1=1.3359, T_2=-5.9920; M_1=0.1453, M_2=0.2510, M_3=0.2252)

Similarly transforming continuous-time sixth-order of IDM model ROM (42) to discrete-time system using Tustin transformation and taking sampling time 0.1s it's discrete-time model comes out to be

$$G_6(z)=\frac{0.178z^5-0.00544z^4-0.1796z^3+0.2027z^2-0.044z-0.01755}{z^6-1.755z^5+1.446z^4-1.103z^3+0.6373z^2-0.1685z+0.06836} \quad (44)$$

(T_0=1.0711, T_1=1.3786, T_2=-6.0800; M_1=0.1780, M_2=0.3069, M_3=0.1017)

Fig. 4 shows that the step responses of discrete-time model of original system (39), proposed model (43) and IDM model (44) and found that the response of proposed model is identical to HOS. These findings are also confirmed by examining the ISE given in Table II.

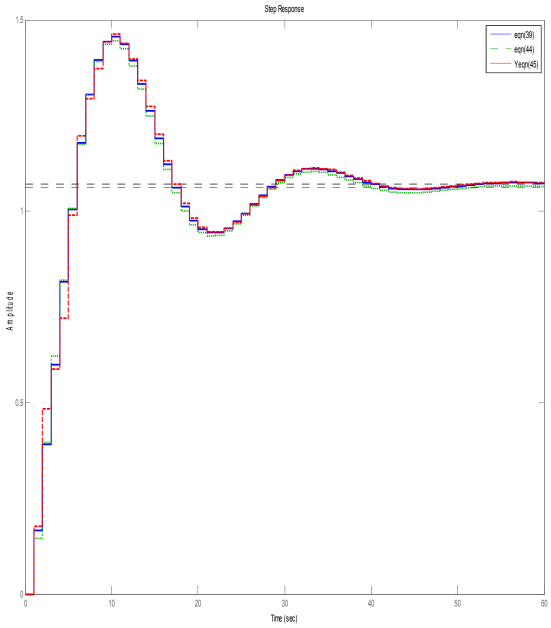

Fig. 4 Step response of discrete-time original system and ROMs

TABLE II
COMPARISON OF PROPOSED METHOD WITH OTHER METHOD

Reduction method	Reduced model	ISE
Proposed method (43)	$G_6(z)=\frac{0.1453z^5-0.01645z^4-0.04762z^3+0.03367z^2-0.006161z-0.001431}{z^6-1.841z^5+1.303z^4-0.5537z^3+0.09981z^2+0.06726z+0.005685}$	0.0017
IDM method (44)	$G_6(z)=\frac{0.178z^5-0.00544z^4-0.1796z^3+0.2027z^2-0.044z-0.01755}{z^6-1.755z^5+1.446z^4-1.103z^3+0.6373z^2-0.1685z+0.06836}$	0.0333

IV. CONCLUSION

In this paper, the original high-order discrete-time system is transformed to continuous-time system and its stable ROM is obtained using pole clustering- Padé method. The denominator is obtained by retaining dominant pole and clustering the remaining poles of the original HOS using Fuzzy C-means clustering technique. Having obtained the denominator, the numerators parameters are calculated by fully retaining the first r time moments/ Markov parameters of the system. Once the reduced-order model is obtained, its discrete-time model is derived. The proposed approach, therefore leads to improved approximants. It is worth mentioning that for r ≤ 4 with complex conjugate poles, the problem of identifying clusters of poles may possibly surface. In this situation pole clustering techniques [14], [15] may be used. However, this problem is open to investigation.

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