A Centroid Ranking Approach Based Fuzzy MCDM Model

T. C. Chu and S.H. Wu

***Abstract*—**This paper suggests ranking alternatives under fuzzy MCDM (multiple criteria decision making) via an centroid based ranking approach, where criteria are classified to benefit qualitative, benefit quantitative and cost quantitative ones. The ratings of alternatives versus qualitative criteria and the importance weights of all criteria are assessed in linguistic values represented by fuzzy numbers. The membership function for the final fuzzy evaluation value of each alternative can be developed through α-cuts and interval arithmetic of fuzzy numbers. The distance between the original point and the relative centroid is applied to defuzzify the final fuzzy evaluation values in order to rank alternatives. Finally a numerical example demonstrates the computation procedure of the proposed model.

***Keywords*—**Fuzzy MCDM, Criteria, Fuzzy number, Ranking, Relative centroid.

I. Introduction

FUZZY MCDM (multiple criteria decision making) [6] has been widely applied to resolve many problems under uncertain environment. A fuzzy MCDM model is to assess alternatives versus selected criteria through a committee of decision makers, where suitability of alternatives versus criteria and the importance weights of criteria can be evaluated in linguistic values represented by fuzzy numbers. Fuzzy set theory, initially proposed by Zadeh [23], has been extensively applied to objectively resolve the uncertainties in human judgment and effectively reflect the ambiguities in the available information in an ill-defined multiple criteria decision making environment.

Areview of fuzzy MCDM methods can be found in Carlsson and Fullér [5]. Some recent applications can be found in [2, 4, 7, 9, 12, 16, 21].The final evaluation values of alternatives in most of the above fuzzy MCDM problems are usually still fuzzy numbers and these fuzzy numbers need a proper ranking approach to defuzzify them into crisp values for decision making. Thus a ranking method is needed.A comparison of many of fuzzy number ranking methods can be seen in [18].

Some recent works can be found in [1, 3, 11, 13, 15, 17, 19]. However, in spite of the merits, some methods are computational complex and others are difficult to implement the connection between the ranking procedure and the final fuzzy evaluation values, limiting the applicability of the fuzzy MCDM model. To resolve the above limitations, this paper suggests an centroid ranking approach based fuzzy MCDM model.

In the suggested model, criteria are classified to benefit qualitative, benefit quantitative and cost quantitative ones. Benefit criteria have the characteristics of the larger the better, while cost criteria have the characteristics of the smaller the better. In addition, quantitative criteria can be objectively evaluated, such as numerical values, while qualitative criteria can only be subjectively evaluated, such as linguistic values. Furthermore, in the suggested model, ratings of alternatives versus qualitative criteria and the importance weights of all criteria are assessed in linguistic values [22] represented by triangular fuzzy numbers. Through α-cuts and interval arithmetic of fuzzy numbers, membership function of the final fuzzy evaluation value of each alternative can be developed. These final fuzzy evaluation values need to be defuzzified to determine their ranking order. Herein this paper suggests applying the distance between the original point and the relative centroid to defuzzify the final fuzzy evaluation values in order to rank alternatives. The relative centroid [20] is determined by the centroid developed on x-axis and the centroid developed on y-axis. Formulas of ranking procedure can be developed. Finally a numerical example demonstrates feasibility of the proposed model.

The rest of this work is organized as follows. Section II briefly introduces fuzzy set theory. Section III introduces the suggested model. Meanwhile, an example is presented in Section IV to demonstrate the computational process of the proposed model and conclusions are finally made in Section V.

II. Fuzzy Set Theory

A. Fuzzy Sets

$A = \{(x, f_A(x)) \mid x \in U\}$, where$U$ is the universe of discourse, x is an element in U, A is a fuzzy set in U, $f_A(x)$ is the membership function of A at x [14]. The larger $f_A(x)$, the stronger the grade of membership for x in A.

T. C. Chu isprofessor of the Graduate Institute of Industrial Management, Graduate Institute of Technology Management, Southern Taiwan University of Science and Technology, No.1, Nantai St., Yungkang Dist., Tainan, Taiwan(e-mail:tcchu@mail.stust.edu.tw).

S. H. Wu was a graduate student of theGraduate Institute of Technology Management, Southern Taiwan University of Science and Technology, No.1, Nantai St., Yungkang Dist., Tainan, Taiwan (e-mail:tcchu@mail.stust.edu.tw). Currently he works for a private company in Taiwan.

B. Fuzzy Numbers

A real fuzzy number A is described as any fuzzy subset of the real line R with membership function f_A which possesses the following properties [10]:

(a) f_A is a continuous mapping from R to [0,1];

(b) $f_A(x) = 0, \forall x \in (-\infty, a]$;

(c) f_A is strictly increasing on $[a, b]$;

(d) $f_A(x) = 1, x \in [b, c]$;

(e) f_A is strictly decreasing on $[c, d]$;

(f) $f_A(x) = 0, \forall x \in [d, \infty)$;

where $a \le b \le c \le d$, A can be denoted as $[a, b, c, d]$. The membership function f_A of the fuzzy number A can also be expressed as:

$$f_A(x) = \begin{cases} f_A^L(x), & a \le x \le b \\ 1, & b \le x \le c \\ f_A^R(x), & c \le x \le d \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where $f_A^L(x)$ and $f_A^R(x)$ are left and right membership functions of A, respectively [14]. A fuzzy triangular number can be denoted as (a, b, c).

C. α-Cuts

The α-cuts of fuzzy number A can be defined as $A^\alpha = \{x \mid f_A(x) \ge \alpha\}, \alpha \in [0,1]$, where A^α is a non-empty bounded closed interval contained in R and can be denoted by $A^\alpha = [A_l^\alpha, A_u^\alpha]$, where A_l^α and A_u^α are its lower and upper bounds, respectively [14].

D. Arithmetic Operations on Fuzzy Numbers

Given fuzzy numbers A and B, $A, B \in R^+$, the α-cuts of A and B are $A^\alpha = [A_l^\alpha, A_u^\alpha]$ and $B^\alpha = [B_l^\alpha, B_u^\alpha]$, respectively. By the interval arithmetic, some main operations of A and B can be expressed as follows [14]:

$$(A \oplus B)^\alpha = [A_l^\alpha + B_l^\alpha, A_u^\alpha + B_u^\alpha] \quad (2)$$

$$(A \ominus B)^\alpha = [A_l^\alpha - B_u^\alpha, A_u^\alpha - B_l^\alpha] \quad (3)$$

$$(A \otimes B)^\alpha = [A_l^\alpha \cdot B_l^\alpha, A_u^\alpha \cdot B_u^\alpha] \quad (4)$$

$$(A \otimes r)^\alpha = [A_l^\alpha \cdot r, A_u^\alpha \cdot r], r \in R^+ \quad (5)$$

E. Linguistic Values

A linguistic variable is a variable whose values are expressed in linguistic terms. Linguistic variable is a very helpful concept for dealing with situations which are too complex or not well-defined to be reasonably described by traditional quantitative expressions [22]. For example, “importance” is a linguistic variable whose values include UI (unimportant), OI (ordinary important), I (important), VI (very important) and AI (absolutely important). These linguistic values can be further represented by trapezoidal fuzzy numbers such as UI=(0.0,0.1,0.3), OI=(0.1,0.3,0.5), I=(0.3,0.5,0.7), VI=(0.5,0.7,0.9) and AI=(0.7,0.9,1.0).

III. Model Development

Suppose there are k decision-makers (i.e. D_t, t=1~k) who are responsible for evaluating m alternative (i.e. A_i, i=1~m) under n criteria (C_j, j=1~n). Criteria are divided into benefit qualitative $C_j, j = 1,...,g$; benefit quantitative $C_j, j = g+1,...,h$; cost quantitative $C_j, j = h+1,...,n$.

A. Rating Values of Qualitative Criteria

Assume $x_{ijt} = (o_{ijt}, p_{ijt}, q_{ijt})$, $i = 1,...,m$, $j = 1,...,g$, $t = 1,...,k$. x_{ijt} denotes rating assigned by each decision maker for each alternative versus each qualitative criterion.

B. Rating Values of Quantitative Criteria

Assume $x_{ij} = (o_{ij}, p_{ij}, q_{ij})$, $i = 1,...,m$, $j = g+1,...,n$. x_{ij} denotes ratings assigned by each projects for each alternative versus each quantitative criterion.

C. Weights of Criteria

Assume $w_{jt} = (a_{jt}, b_{jt}, c_{jt})$, $w_{jt} \in R^+$, $j = 1,...,n$, $t = 1,...,k$. w_{jt} represents the weight assigned by each decision maker for each criterion.

D. Normalize Values of Alternatives Versus Quantitative Criteria

In this work, objective criteria can be classified to benefit (B) and cost (C). Benefit criterion has the characteristics: the larger the better; while the cost criterion has the characteristics: the smaller the better. Values under both benefit and cost criteria can be either crisp or fuzzy. Values under objective criteria may have different units and thus must be normalized into a comparable scale for calculation rationale. Herein, the normalization is completed by a suggested approach from Chu and Lin [8], which preserves the property where the ranges of normalized triangular fuzzy numbers belong to [0,1]. Suppose $x_{ij} = (o_{ij}, p_{ij}, q_{ij})$ is the performance of alternative i versus objective criterion j, the normalized value can be denoted as:

$$x_{ij} = \left(\frac{o_{ij} - o_j^*}{d_j^*}, \frac{p_{ij} - o_j^*}{d_j^*}, \frac{q_{ij} - o_j^*}{d_j^*} \right), j \in Benefit, j = (g+1) \sim h;$$

$$x_{ij} = \left(\frac{q_j^* - q_{ij}}{d_j^*}, \frac{q_j^* - p_{ij}}{d_j^*}, \frac{q_j^* - o_{ij}}{d_j^*} \right), j \in Cost, j = (h+1) \sim n. \quad (6)$$

where $o_j^* = \min_i o_{ij}$, $q_j^* = \max_i q_{ij}$, $d_j^* = q_j^* - o_j^*$, $i = 1,...,m$, $j = g+1,...,n.$

E. Develop Membership Functions

The membership function of the final fuzzy evaluation value, $T_i, i = 1,...,m$ of each alternative can be developed as follows:

$$T_i^\alpha = \frac{1}{g \times k} \sum_{j=1}^{g} \sum_{t=1}^{k} \left(w_{jt}^\alpha \times x_{ijt}^\alpha \right) + \frac{1}{[h-(g+1)] \times k} \sum_{j=g+1}^{h} \sum_{t=1}^{k} \left(w_{jt}^\alpha \times x_{ij}^\alpha \right) - \frac{1}{[n-(h+1)] \times k} \sum_{j=h+1}^{n} \sum_{t=1}^{k} \left(w_{jt}^\alpha \times x_{ij}^\alpha \right) \quad (7)$$

The membership functions are developed as:

$$w_{jt}^\alpha = \left[\left(b_{jt} - a_{jt} \right) \alpha + a_{jt,} \left(b_{jt} - c_{jt} \right) \alpha + c_{jt} \right] \quad (8)$$

$$x_{ijt}^\alpha = \left[\left(p_{ijt} - o_{ijt} \right) \alpha + o_{ijt,} \left(p_{ijt} - q_{ijt} \right) \alpha + q_{ijt} \right] \quad (9)$$

To simplify equations, we assume:

$$A_{i1} = \frac{1}{g \times k} \sum_{j=1}^{g} \sum_{t=1}^{k} \left(b_{jt} - a_{jt} \right) \left(p_{ijt} - o_{ijt} \right);$$

$$A_{i2} = \frac{1}{[h-(g+1)] \times k} \sum_{j=g+1}^{h} \sum_{t=1}^{k} \left(b_{jt} - a_{jt} \right) \left(p_{ijt} - o_{ij} \right);$$

$$A_{i3} = \frac{1}{[n-(h+1)] \times k} \sum_{j=h+1}^{n} \sum_{t=1}^{k} \left(b_{jt} - a_{jt} \right) \left(p_{ij} - o_{ij} \right);$$

$$B_{i1} = \frac{1}{g \times k} \sum_{j=1}^{g} \sum_{t=1}^{k} \left(o_{ijt} \left(b_{jt} - a_{jt} \right) + a_{jt} \left(p_{ijt} - o_{ijt} \right) \right);$$

$$B_{i2} = \frac{1}{[h-(g+1)] \times k} \sum_{j=g+1}^{h} \sum_{t=1}^{k} \left(o_{ij} \left(b_{jt} - a_{jt} \right) + a_{jt} \left(p_{ij} - o_{ij} \right) \right);$$

$$B_{i3} = \frac{1}{[n-(h+1)] \times k} \sum_{j=h+1}^{n} \sum_{t=1}^{k} \left(o_{ij} \left(b_{jt} - a_{jt} \right) + a_{jt} \left(p_{ij} - o_{ij} \right) \right);$$

$$C_{i1} = \frac{1}{g \times k} \sum_{j=1}^{g} \sum_{t=1}^{k} \left(b_{jt} - c_{jt} \right) \left(p_{ijt} - q_{ijt} \right);$$

$$C_{i2} = \frac{1}{[h-(g+1)] \times k} \sum_{j=g+1}^{h} \sum_{t=1}^{k} \left(b_{jt} - c_{jt} \right) \left(p_{ij} - q_{ij} \right);$$

$$C_{i3} = \frac{1}{[n-(h+1)] \times k} \sum_{j=h+1}^{n} \sum_{t=1}^{k} \left(b_{jt} - c_{jt} \right) \left(p_{ij} - q_{ij} \right);$$

$$D_{i1} = \frac{1}{g \times k} \sum_{j=1}^{g} \sum_{t=1}^{k} \left(q_{ijt} \left(b_{jt} - c_{jt} \right) + c_{jt} \left(p_{ijt} - q_{ijt} \right) \right);$$

$$D_{i2} = \frac{1}{[h-(g+1)] \times k} \sum_{j=g+1}^{h} \sum_{t=1}^{k} \left(q_{ij} \left(b_{jt} - c_{jt} \right) + c_{jt} \left(p_{ij} - q_{ij} \right) \right);$$

$$D_{i3} = \frac{1}{[n-(h+1)] \times k} \sum_{j=h+1}^{n} \sum_{t=1}^{k} \left(q_{ij} \left(b_{jt} - c_{jt} \right) + c_{jt} \left(p_{ij} - q_{ij} \right) \right);$$

$$O_{i1} = \frac{1}{g \times k} \sum_{j=1}^{g} \sum_{t=1}^{k} \left(a_{jt} \right) \left(o_{ijt} \right);$$

$$O_{i2} = \frac{1}{[h-(g+1)] \times k} \sum_{j=g+1}^{h} \sum_{t=1}^{k} \left(a_{jt} \right) \left(o_{ij} \right);$$

$$O_{i3} = \frac{1}{[n-(h+1)] \times k} \sum_{j=h+1}^{n} \sum_{t=1}^{k} \left(a_{jt} \right) \left(o_{ij} \right);$$

$$p_{i1} = \frac{1}{g \times k} \sum_{j=1}^{g} \sum_{t=1}^{k} \left(b_{jt} \right) \left(p_{ijt} \right);$$

$$p_{i2} = \frac{1}{[h-(g+1)] \times k} \sum_{j=g+1}^{h} \sum_{t=1}^{k} \left(b_{jt} \right) \left(p_{ij} \right);$$

$$p_{i3} = \frac{1}{[n-(h+1)] \times k} \sum_{j=h+1}^{n} \sum_{t=1}^{k} \left(b_{jt} \right) \left(p_{ij} \right);$$

$$Q_{i1} = \frac{1}{g \times k} \sum_{j=1}^{g} \sum_{t=1}^{k} \left(c_{jt} \right) \left(q_{ijt} \right);$$

$$Q_{i2} = \frac{1}{[h-(g+1)] \times k} \sum_{j=g+1}^{h} \sum_{t=1}^{k} \left(c_{jt} \right) \left(q_{ij} \right);$$

$$Q_{i3} = \frac{1}{[n-(h+1)] \times k} \sum_{j=h+1}^{n} \sum_{t=1}^{k} \left(c_{jt} \right) \left(q_{ij} \right).$$

By the above assumption and (8)-(9), (7) can be arranged as:

$$T_i^\alpha = [\left(A_{i1} + A_{i2} - C_{i3} \right) \alpha^2 + \left(B_{i1} + B_{i2} - D_{i3} \right) \alpha + \left(O_{i1} + O_{i2} - Q_{i3} \right), \left(C_{i1} + C_{i2} - A_{i3} \right) \alpha^2 + \left(D_{i1} + D_{i2} - B_{i3} \right) \alpha + \left(Q_{i1} + Q_{i2} - O_{i3} \right)] \quad (10)$$

To get simplified equations as:

$$\left(A_{i1} + A_{i2} - C_{i3} \right) \alpha^2 + \left(B_{i1} + B_{i2} - D_{i3} \right) \alpha$$

$$+\left(O_{i1}+O_{i2}\quad Q_{i3}\right)\quad x-0\quad -\quad = \tag{11}$$

$$\left(C_{i1}+C_{i2}-A_{i3}\right)\ ^2\quad \left(D_{i1}\alpha\quad D_{i2}+B_{i3}\right)\quad +\quad -\quad \alpha$$
$$+\left(Q_{i1}+Q_{i2}\quad O_{i3}\right)\quad x-0\quad -\quad = \tag{12}$$

Assume $A_i=A_{i1}+A_{i2}\quad C_{i3}$; $B_i=B_{i1}+B_{i2}\quad D_{i3}$; $C_i=C_{i1}+C_{i2}\quad A_{i3}$; $D_i=D_{i1}+D_{i2}\quad B_{i3}$; $O_i=O_{i1}+O_{i2}\quad Q_{i3}$; $P_i=P_{i1}+P_{i2}\quad P_{i3}$; $Q_i=Q_{i1}+Q_{i2}\quad O_{i3}$.

By the above assumption, (11)-(12) can be arranged as:

$$A_i\alpha^2+B_i\quad \alpha O_i+x\quad 0-\quad = \tag{13}$$

$$C_i\alpha^2+D_i\quad \alpha Q_i+x\quad 0-\quad = \tag{14}$$

The right and left membership functions of T_i can be obtained as follows:

$$f^L_{T_i}(x)=\frac{-B_i+\sqrt{B_i^2\quad 4A_i\left(x\quad O_i\right)}}{2A_i}, O_i\le x\quad P_i; \tag{15}$$

$$f^R_{T_i}(x)=\frac{-D_i-\sqrt{D_i^2\quad 4C_i\left(x\quad Q_i\right)}}{2C_i}, P_i\le x\quad Q_i \tag{16}$$

$$g^L_{T_i}\left(y\right)=A_iy^2+B_iy\quad O_i, 0\quad y\quad 1 \tag{17}$$

$$g^R_{T_i}\left(y\right)=C_iy^2+D_iy\quad Q_i, 0\quad y\quad 1 \tag{18}$$

F. Rank Fuzzy Numbers

Herein the distance between the original point and the relative centroid is applied to defuzzify the final fuzzy evaluation values to rank alternatives. T_i is the final fuzzy evaluation value of alternative A_i. First we find out the two centroids of a fuzzy number on horizontal axis and vertical axis respectively as follows [20]:

$$x(T_i)=\frac{\int_{-\infty}^{+\infty}x f_{T_i}(x)dx}{\int_{-\infty}^{+\infty}f_{T_i}(x)dx}=\frac{\int_{O_i}^{P_i}x f^L_{T_i}(x)dx+\int_{P_i}^{Q_i}x f^R_{T_i}\ x\ (dx)}{\int_{O_i}^{P_i}f^L_{T_i}(x)dx+\int_{P_i}^{Q_i}f^R_{T_i}\ x\ (dx)} \tag{19}$$

$$y\left(T_i\right)=\frac{\int_0^1 y\left[g^R_{T_i}\left(y\right)-g^L_{T_i}\quad y\right]dy}{\int_0^1\left[g^R_{T_i}\left(y\right)-g^L_{T_i}\quad y\right]dy} \tag{20}$$

Applying (15)-(16) to (19) to produce (21):

$$x(T_i)=\frac{\int_{O_i}^{P_i}x\left[\frac{-B_i+\sqrt{B_i^2\quad 4A_i(x\quad O_i)}}{2A_i}\right]dx+\int_{P_i}^{Q_i}x\left[\frac{D_i\quad\sqrt{D_i^2\quad 4C_i(x\quad Q_i)}}{2C_i}\right]dx}{\int_{O_i}^{P_i}\left[\frac{-B_i+\sqrt{B_i^2\quad 4A_i(x\quad O_i)}}{2A_i}\right]dx+\int_{P_i}^{Q_i}\frac{D_i\quad\sqrt{D_i^2\quad 4C_i(x\quad Q_i)}}{2C_i}dx} \tag{21}$$

Let $\left[B_i^2+4A_i\left(x-O_i\right)\right]^{1/2}=t$

to get $\left(4A_ix+B_i^2-4A_iO_i\right)^{1/2}=t,\quad x=\frac{t^2+4A_iO_i-B_i^2}{4A_i}$ and

$dx=\frac{tdt}{2A_i}$

$x=O_i \Rightarrow t=\left(4O_iA_i+B_i^2-4A_iO_i\right)^{1/2}=B_i$ and

$x=P_i=\quad t\quad \left(4P_iA_i\quad B_i^2\quad 4A_iO_i\right)^{1/2}$

Let $\left(4P_iA_i+B_i^2-4A_iO_i\right)^{1/2}=Y_i'$

$$\frac{1}{2A_i}\int_{O_i}^{P_i}\left[B_i^2+4A_i\left(x-O_i\right)\right]^{1/2}xdx$$
$$=\frac{1}{16A_i^3}\left[\frac{Y_i'^5-B_i^5}{5}+\left(4A_iO_i\quad B_i^2\right)\frac{Y_i'^3-B_i^3}{3}\right]$$

By applying the above procedure, (21) can be solved as the following equation:

$$x\left(T_i\right)=\frac{\frac{B_i\left(O_i^2-P_i^2\right)}{4A_i}+\frac{1}{16A_i^3}\left[\frac{Y_i'^5-B_i^5}{5}+\left(4A_iO_i\quad B_i^2\right)\frac{Y_i'^3-B_i^3}{3}\right]}{\frac{B_i\left(O_i-P_i\right)}{2A_i}+\frac{Y_i'^3-B_i^3}{12A_i^2}}$$
$$\frac{+\frac{D_i\left(P_i^2-Q_i^2\right)}{4C_i}-\frac{1}{16C_i^3}\left[\frac{D_i^5-Y_i''^5}{5}\quad\left(4C_iO_i\quad D_i^2\right)\frac{D_i^3-Y_i''^3}{3}\right]}{+\frac{D_i\left(P_i-Q_i\right)}{2C_i}-\frac{D_i^3-Y_i''^3}{12C_i^2}} \tag{22}$$

where $\left(4C_iP+D_i^2-4C_iQ_i\right)^{1/2}=Y_i''$

In addition, $y\left(T_i\right)=\frac{\int_0^1 y\left[g^R_{T_i}\left(y\right)-g^L_{T_i}\quad y\right]dy}{\int_0^1\left[g^R_{T_i}\left(y\right)-g^L_{T_i}\quad y\right]dy}$

$$y(T_i)=\frac{\int_0^1 y\left[\left(C_i y^2+D_i y+Q_i\right)-\left(A_i y^2+B_i y+O_i\right)\right]dy}{\int_0^1\left[\left(C_i y^2+D_i y+Q_i\right)-\left(A_i y^2+B_i y+O_i\right)\right]dy}=\frac{\frac{C_i-A_i}{4}+\frac{D_i-B_i}{3}+\frac{Q_i-O_i}{2}}{\frac{C_i-A_i}{3}+\frac{D_i-B_i}{2}+Q_i-O_i} \quad (23)$$

By (22)-(23), distance from the relative centroid to the original point of each final fuzzy evaluation value can be obtained as Eq. (24). The larger value of $d(T_i)$, the higher ranking order of the alternative.

$$d(T_i)=\sqrt{x^2(T_i)+y^2(T_i)} \quad (24)$$

IV. Numerical Example

Suppose a company has five new product projects, A_1, A_2, A_3, A_4 and A_5, under development, and these projects must be ranked for priority management. Suppose the evaluation committee is formed by three professional persons D_1, D_2 and D_3 who have knowledge background in new product development projects. Further suppose six qualitative criteria, such as technology capacity(C_1), market factor (C_2), personnel factor (C_3),R&D capacity(C_4), contribution to social development(C_5),and resource availability (C_6), one benefit quantitative criterion, return on investment (C_7), and three cost quantitative criteria, time factor (C_8), production cost (C_9) and development cost (C_{10}), must be considered.

Ratings assigned by decision makers to alternatives versus qualitative criteria are evaluated in linguistic values represented by triangular fuzzy numbers. These linguistic values and their fuzzy numbers are shown in Table I. The importance weights are given by decision makers to each criterion and these weights are evaluated in linguistic values represented triangular fuzzy numbers in section II.E (Linguistic Values).

Further suppose ratings of new product development projects candidates versus qualitative criteria are given by decision makers as shown in Table II. The suitability values of candidates versus quantitative criteria can be shown in Table III. By (6), the normalized values of candidates versus quantitative criteria can be obtained as shown in Table IV.

In addition, assume that the importance weights of criteria in linguistic values and fuzzy numbers are shown in Table V. By (7)-(16), the $f_{T_i}^L(x)$ and $f_{T_i}^R(x)$ of the value$x(T_i)$ can be obtained as shown in Table VI. By Eqs. (17)-(18), the $g_{T_i}^L(y)$ and $g_{T_i}^R(y)$ membership functions of the final fuzzy evaluation value $y(T_i)$ can be obtained as shown in Table VII.

By using (19)-(24), centroids on x-axis and y-axis and distance values can be obtained as shown in Table VIII. According to Table VIII, the ranking order of the five candidate new product development projects is A_2>A_3 >A_4>A_1>A_5. A_2 has the largest value 0.541 ; therefore A_2 is the most suitable new product development project for decision makers under the evaluation procedure of the proposed model.

TABLE I
LINGUISTIC VALUES AND FUZZY NUMBERS FOR RATINGS

Linguistic values	Triangular fuzzy numbers
Very Unsatisfactory (VU)	(0.0,0.1,0.3)
Unsatisfactory (U)	(0.1,0.3,0.5)
Ordinary (O)	(0.3,0.5,0.7)
Satisfactory (S)	(0.5,0.7,0.9)
Very Satisfactory (VS)	(0.7,0.9,1.0)

TABLE II
RATINGS OF PROJECTS VERSUS QUALITATIVE CRITERIA

Candidate	Criteria	Decision Makers		
		D_1	D_2	D_3
A_1	C_1	S	O	S
	C_2	S	S	S
	C_3	O	S	S
	C_4	VS	S	O
	C_5	O	S	O
	C_6	O	S	O
A_2	C_1	S	S	VS
	C_2	S	O	VS
	C_3	O	S	VS
	C_4	VS	S	S
	C_5	S	S	O
	C_6	O	S	S
A_3	C_1	VS	S	VS
	C_2	S	S	VS
	C_3	VS	S	VS
	C_4	VS	S	S
	C_5	S	VS	O
	C_6	S	S	S
A_4	C_1	S	VS	VS
	C_2	VS	O	VS
	C_3	VS	S	S
	C_4	VS	S	S
	C_5	O	U	S
	C_6	VS	VS	S
A_5	C_1	S	VS	O
	C_2	S	VS	VS
	C_3	S	VS	VS
	C_4	S	VS	VS
	C_5	O	VS	VS
	C_6	VS	O	VS

TABLE III
VALUES OF PROJECTS VERSUS QUANTITATIVE CRITERIA

Criteria	Candidates				
	A_1	A_2	A_3	A_4	A_5
C_7	(26,27,31)	(27,29,33)	(19,20,22)	(22,25,27)	(24,25,27)
C_8	(23,24,25)	(19,20,23)	(26,28,29)	(23,25,27)	(22,23,24)
C_9	(21,22,25)	(25,26,27)	(24,25,26)	(24,25,26)	(20,21,22)
C_{10}	(19,20,24)	(21,22,23)	(27,28,29)	(26,28,30)	(27,18,20)

TABLE VII
LEFT AND RIGHT MEMBERSHIP FUNCTIONS OF $y(Ti)$

	Left and Right Membership functions
$g_{T_1}^{L}(y)$	$0.033y^2+0.5557y+0.045, 0\le y\le 1$
$g_{T_1}^{R}(y)$	$0.0162y^2-0.8396y+1.457, 0\le y\le 1$
$g_{T_2}^{L}(y)$	$0.047y^2+0.6069y+0.167, 0\le y\le 1$
$g_{T_2}^{R}(y)$	$0.0346y^2-0.8436y+1.63, 0\le y\le 1$
$g_{T_3}^{L}(y)$	$0.0263y^2+0.4183y+0.079, 0\le y\le 1$
$g_{T_3}^{R}(y)$	$0.0261y^2-0.5067y+1.004, 0\le y\le 1$
$g_{T_4}^{L}(y)$	$0.049y^2+0.5868y+0.103, 0\le y\le 1$
$g_{T_4}^{R}(y)$	$0.041y^2-0.8075y+1.5061, 0\le y\le 1$
$g_{T_5}^{L}(y)$	$0.033y^2+0.5847y+0.02, 0\le y\le 1$
$g_{T_5}^{R}(y)$	$0.0159y^2-0.6617y+1.20, 0\le y\le 1$

TABLE IV
NORMALIZATION OF QUANTITATIVE CRITERIA

Criteria	New product development projects	Normalization of Quantitative Criteria		
C_7	A_1	0.5	0.571	0.857
	A_2	0.571	0.714	1
	A_3	0	0.071	0.214
	A_4	0.214	0.428	0.785
	A_5	0.357	0.428	0.571
C_8	A_1	0.4	0.5	0.6
	A_2	0.6	0.9	1
	A_3	0	0.1	0.3
	A_4	0.2	0.4	0.6
	A_5	0.5	0.6	0.7
C_9	A_1	0.285	0.714	0.857
	A_2	0	0.142	0.285
	A_3	0.142	0.285	0.428
	A_4	0.142	0.285	0.428
	A_5	0.714	0.857	1
C_{10}	A_1	0.461	0.769	0.846
	A_2	0.538	0.615	0.632
	A_3	0.1	0.153	0.23
	A_4	0	0.153	0.307
	A_5	0.769	0.923	1

TABLEV
IMPORTANCE WEIGHTS OF CRITERIA

Criteria	Decision Makers		
	D_1	D_2	D_3
C_1	VI	VI	AI
C_2	VI	AI	I
C_3	AI	AI	VI
C_4	AI	VI	I
C_5	VI	VI	I
C_6	I	I	VI
C_7	AI	VI	AI
C_8	VI	I	I
C_9	I	OI	I
C_{10}	I	OI	OI

TABLE VIII
CENTROIDS AND DISTANCES

$x(T_1)$	$x(T_2)$	$x(T_3)$	$x(T_4)$	$x(T_5)$
0.138	0.426	0.375	0.275	0.1214
$y(T_1)$	$y(T_2)$	$y(T_3)$	$y(T_4)$	$y(T_5)$
0.334	0.338	0.333	0.3337	0.309
$d(T_1)$	$d(T_2)$	$d(T_3)$	$d(T_4)$	$d(T_5)$
0.3616	0.5417	0.5023	0.4329	0.3322

TABLE VI
LEFT AND RIGHT MEMBERSHIP FUNCTIONS OF $x(T_i)$

A_i		Left and Right Membership functions
A_1	$f_{T_1}^{L}(x)$	$\dfrac{-0.5557+[(0.5557)^2+4(0.033)(x-0.0459)]^{\frac{1}{2}}}{2(0.033)}$,$0.0459\le x\le 0.6346$
	$f_{T_1}^{R}(x)$	$\dfrac{-(-0.8396)-[(-0.8396)^2+4(0.0162)(x-1.4579)]^{\frac{1}{2}}}{2(0.0162)}$,$0.6346\le x\le 1.4579$
A_2	$f_{T_2}^{L}(x)$	$\dfrac{-0.6069+[(0.6069)^2+4(0.0473)(x-0.1675)]^{\frac{1}{2}}}{2(0.0473)}$,$0.1675\le x\le 0.8216$
	$f_{T_2}^{R}(x)$	$\dfrac{-(-0.8436)-[(-0.8436)^2+4(0.0346)(x-1.6306)]^{\frac{1}{2}}}{2(0.0346)}$,$0.8216\le x\le 1.6306$
A_3	$f_{T_3}^{L}(x)$	$\dfrac{-0.4183+[(0.4183)^2+4(0.0263)(x-0.0793)]^{\frac{1}{2}}}{2(0.0263)}$,$0.0793\le x\le 0.5239$
	$f_{T_3}^{R}(x)$	$\dfrac{-(-0.5067)-[(-0.5067)^2+4(0.0261)(x-1.0045)]^{\frac{1}{2}}}{2(0.0261)}$,$0.52639\le x\le 1.004$
A_4	$f_{T_4}^{L}(x)$	$\dfrac{-0.5868+[(0.5868)^2+4(0.0497)(x-0.1037)]^{\frac{1}{2}}}{2(0.0497)}$,$0.1037\le x\le 0.7404$
	$f_{T_4}^{R}(x)$	$\dfrac{-(-0.8075)-[(-0.8075)^2+4(0.0471)(x-1.5061)]^{\frac{1}{2}}}{2(0.0471)}$,$0.7404\le x\le 1.5061$
A_5	$f_{T_5}^{L}(x)$	$\dfrac{-0.5847+[(0.5847)^2+4(0.033)(x-0.021)]^{\frac{1}{2}}}{2(0.033)}$,$0.021\le x\le 0.555$
	$f_{T_5}^{R}(x)$	$\dfrac{-(-0.6617)-[(-0.6617)^2+4(0.0159)(x-1.2008)]^{\frac{1}{2}}}{2(0.0159)}$,$0.555\le x\le 1.2008$

V. Conclusions

A fuzzy multiple criteria decision making model is proposed, where qualitative and quantitative criteria as well as the different importance of all criteria are considered. Membership function of the final fuzzy evaluation value of each alternative can be developed. The ranking approach of the distance between the relative centriod and the original point is suggested to defuzzify all the final fuzzy evaluation values for the ranking of alternatives. Ranking procedure can be formulated. In this work, ratings of alternatives and importance weights of criteria are subjectively assigned by decision makers. Some other ranking approaches other than the suggested one may be used for the proposed fuzzy MCDM model. However, the ranking results may be different. Fuzzy numbers other than triangular can also be used for the proposed model, a comparison may be needed. Finally, the linguistic values and their corresponding fuzzy numbers used in this research can be adjusted to fit different applications. A model may be needed to objectively produce these values.

Acknowledgments

This work was supported in part by National Science Council, Taiwan, R.O.C., under Grant NSC 101-2410-H-218-004-MY2.

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