

# A Study on Applying 3D Reconstruction to 3D Last Morphing

Shih-Wen Hsiao, Rong-Qi Chen, and Chien-Yu Lin

**Abstract**—When it comes to last, it is regarded as the critical foundation of shoe design and development. A computer aided methodology for various last form designs is proposed in this study. The reverse engineering is mainly applied to the process of scanning for the last form. Then with the minimum energy for revision of surface continuity, the surface reconstruction of last is rebuilt by the feature curves of the scanned last. When the surface reconstruction of last is completed, the weighted arithmetic mean method is applied to the computation on the shape morphing for the control mesh of last, thus 3D last form of different sizes is generated from its original form feature with functions remained. In the end, the result of this study is applied to an application for 3D last reconstruction system. The practicability of the proposed methodology is verified through later case studies.

**Keywords**—Reverse engineering, Surface reconstruction, Surface continuity, Shape morphing.

## I. INTRODUCTION

LAST has played an important role on the shoe design and development, shoe designers can directly design shoe styles on the surface of last. 2D patterns which aids shoe manufacturing is generated through the design process, then applying last to these 2D patterns that will be sewed can complete the production of a pair of shoes. Therefore, during the design stage, the appropriate selection of last types and sizes can realize the design concept effectively and aid shoes manufacturing simultaneously. This will achieve the state of art for selecting appropriate, customer-oriented and function-based last for shoe design and development.

Even though different forms of last will be generated due to the different functional requirements of products, without doubt the measurements of feet size on the last can be obtained through the measurement of human factors or the 3D measuring methodology for customer-oriented product design in accurate demands. For this reason, it will make a great benefit on the shoe design especially the wearing comfort of shoes if the lasts with different sizes can be generated by scaling the 3D model of last. In order to achieve this target, a methodology of applying 3D reconstruction to 3D last morphing will be proposed in the study. In the first stage of research processes, different functional last forms will be scanned by reverse engineering methodology, and the data of scan point will be obtained. Then the data of these scan points will be defined on the segments of surface reconstruction which partitioned by feature curves.

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Furthermore, the grid points for fitting process will be generated which allows B-spline surface be applied to the surface reconstruction in every section. After the revising calculation of surface reconstruction, a 3D last model that consisted of the control mesh of B-spline surface can be constructed. Through systematical methodology which is applied to the scaling for the sizes of 3D last model, the generation and shape morphing of 3D last model in different sizes can be mainly completed by the application of shape blending algorithm based on weighted arithmetic mean method. Thus not only the 3D last model can remain the required features of different functions but also generate a new 3D last model with the requirements of different sizes by shape morphing. In the end, the result of this study will be applied to the constructed application of 3D last model system. Plus, the practicality of methodology will be verified by several cases study. With the frame of the proposed methodology, a 3D last surface with high quality can be generated and various 3D last forms in different sizes can also be constructed by shape morphing.

## II. RELATED WORK

From the previous research of last, constructing 3D last form with 2D profiles is a relative common method [1]. Since various 3D last models with different features can be constructed by blending and adjusting the 2D profiles, a standardized 3D last model which also accurately reconstructs different regional systems of last can be developed through the methodology and fulfill the requirements of public-oriented shoe design. In addition, while measuring the last sizes for human factors, 2D profiles is not the proper way for defining the positions of features, instead the most suitable size of specific last should be gauged by actual implementation of feet measuring [2]. Due to the reason that the relative position for sizes of 3D last model should be calculated and evaluated by reconstructing the new 2D profiles on the 3D last model, this will cause a negative impact on developing the application and system of 3D last model. Although the 3D data of last forms can be obtained by scanning the concrete last form through reverse engineering method, 3D last model will be represented in various ways because of different reconstruction methodologies.

In the process of surface reconstruction from scan data, using parametric surfaces to represent the entire 3D shape is a highly efficient procedure, which can also provide a variety of geometric information to help solving problems. Before surface reconstruction can begin, figuring out how to divide the scan data into appropriate blocks using shape features [3], [4] and re-simplifying the entire scan data using slicing methods [5], [6]

are the most important preparations. When parametric surfaces are needed for the shape reconstruction of already partitioned blocks, using interpolation to obtain the minimum energy is a relatively common method [7]-[11]; it can also be widely used in the reconstruction of various parametric surfaces. However, ensuring both real-time and accuracy are challenges for algorithm design. In addition, gradually obtaining optimal results by adjusting the control points of the parametric surfaces is a solution from a different viewpoint [12], [13], while using a genetic algorithm for the reconstruction of parametric surfaces is an attempt to use a new method [14]. When all parametric surfaces of the entire shape have been obtained, the continuity of surface convergence will be an important challenge as it directly affects the quality of the surfaces, thereby raising different requirements related to different levels of continuity [15]-[19]. To alter the surface control grid by using parameters, and thus obtain different results under conditions in line with the same continuity is a more flexible method [20], but ensuring that such correction will not lead to transition errors is another obstacle to overcome. Finally, to make it easier to alter the form of parametric surfaces, reducing the number of surface control points is a feasible option [21]. However, if shape-blending methods can be applied to systematically change the shapes using parameters [22], [23], the varied requirements in shoe design concerning 3D lasts suitable for different individual sizes can thus be met.

Through the previous research, it is necessary to construct a form reconstruction methodology that standardizes the representation of 3D last form. At the meanwhile, the size scaling of 3D last can be directly defined by the feature curves, and a variety of last form will also be produced with functional features remained.

### III. METHODOLOGY

#### A. Last and Feature Curves

Last has an enormous influence on shoe design and development, as shown in Fig. 1, not only can the last define feet features but also indicate the size for every part of feet by the key feature curves.

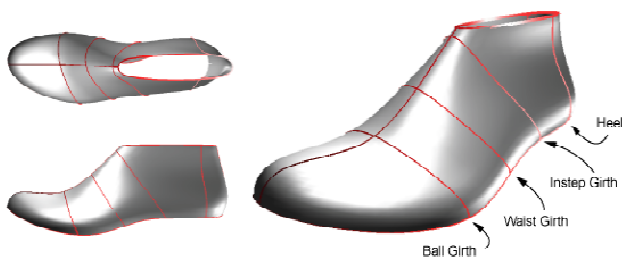


Fig. 1 3D last model and feature curves

The measurement of feet can be represented by basic length and width of feet which includes ball girth, waist girth and instep girth. Furthermore, heel is another key point of features, both the width and height of heel that have direct relation to

form design and wearing comfort of shoes can be defined. Due to the application of feature curves, the 3D last form can be reconstructed by feet features and directly indicate the actual size of feet. The 3D last morphing can be implemented with basis and t operating requirement, more advanced, a variety of last forms can be constructed.

#### B. B-Spline Surface and Surface Fitting

B-spline surface is the extension of B-spline curve in biparametric directions, whose equation is as follows:

$$Q(u, v) = \sum_{i=0}^n \sum_{j=0}^m B_{i,j} N_{i,k}(u) M_{j,l}(v) \quad (1)$$

where  $N_{i,k}$  and  $M_{j,l}$  are the B-spline basis function in the biparametric  $u$  and  $v$  directions, respectively.  $B_{i,j}$  are the vertices of a defining polygon mesh.

In this study, however, to clearly express the contents of the method mentioned, (1) was rewritten as a matrix:

$$[Q] = [B] \cdot [N] \cdot [M] \quad (2)$$

However, unlike forward design, in reverse engineering, a set of point data is often needed to obtain applicable parametric surfaces using the surface fitting computation method, as shown in Fig. 2. First of all, it is necessary to obtain the grid point array subject to fitting from the scan data using the slicing method (Fig. 2 (a)); then, more refined curve points are calculated using the curve fitting method in the  $u$  and  $v$  directions of the point array. Next, the anti-matrix calculation method is used in (3) to obtain the polygon control net of the B-spline surfaces of this grid point (Fig. 2 (b)). Finally, (1) can be used to obtain high-quality B-spline surfaces (Fig. 2 (c)):

$$[B] = [Q] \cdot [M]^T \cdot ([M] \cdot [M]^T)^{-1} \cdot [N]^T \cdot ([N] \cdot [N]^T)^{-1} \quad (3)$$

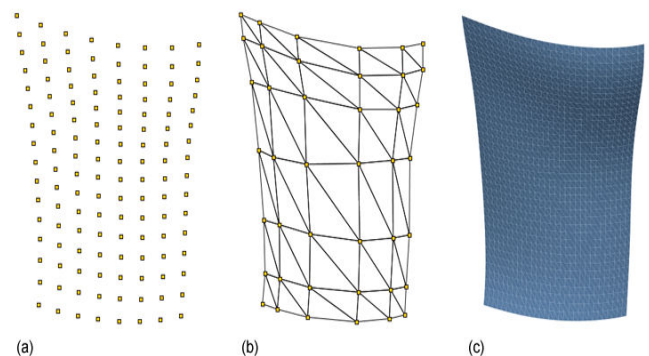


Fig. 2 B-spline surface fitting (a) Points subject to fitting (b) The B-spline polygon mesh (c) B-spline surface

### C. Convergence of Parametric Surfaces

When a shape needs to be formed by multiple surfaces, in order to obtain surfaces with high-quality shapes, the continuity of parametric surface convergence is the primary consideration; it can be divided into position continuity (G0), tangent-plane continuity (G1) and curvature continuity (G2) by its different extents. Therefore, only when low-order continuous conditions are met can the possibility of high-order continuity be explored. Here, make Q(u,v) and S(u,v) two B-spline surfaces to be converged (Fig. 3); this can be expressed as:

$$Q(u, v) = \sum_{i=0}^n \sum_{j=0}^m A_{i,j} N_{i,k}(u) M_{j,\ell}(v) \quad (4)$$

$$S(u, v) = \sum_{i=0}^g \sum_{j=0}^m B_{i,j} L_{i,k}(u) R_{j,\ell}(v)$$

where  $A_{i,j}$  is the sum,  $B_{i,j}$  is the surface control point of, while  $N_{i,k}$ ,  $M_{j,\ell}$  and  $L_{i,k}$ ,  $R_{j,\ell}$  are the basis functions. Therefore, for the discussion of subsequent issues, these two surfaces share the same number of control points and order on a common border.

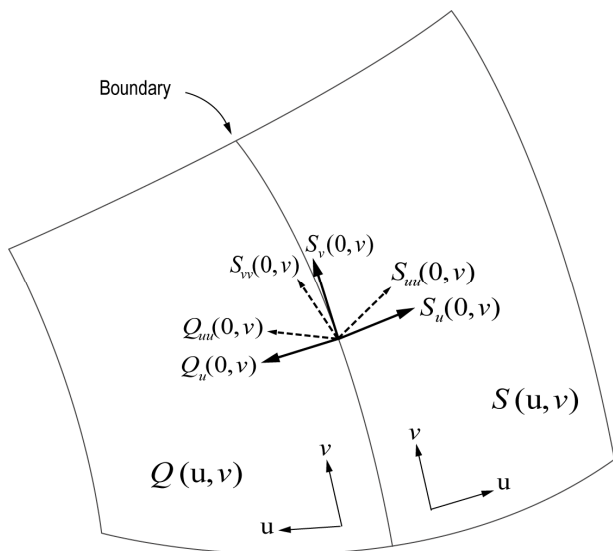


Fig. 3 Two parametric surfaces to be converged

On the other hand, during the process of continuity correction, the first order differential and second order differential vectors in the u and v directions on the common border connecting the two surfaces need to be calculated (Fig. 4). As the surface is an extension of the curve in the biparametric directions, these can be obtained by the differential equation of the B-spline curve. After a set of control point coordinates is given, the first order differential result of each point on the B-spline curve thus generated can be obtained by the computation in (5):

$$P_t(t) = \sum_{i=0}^{n-1} N_{i+1,k-1} \frac{k-1}{x_{i+k} - x_{i+1}} (B_{i+1} - B_i) \quad (5)$$

Similarly, the first order differential result of each point on the curve can be obtained by the computation in (6):

$$P_{tt}(t) = \sum_{i=0}^{n-2} N_{i+2,k-2} \frac{k-2}{x_{i+k} - x_{i+2}} \cdot \left( \frac{k-1}{x_{i+1+k} - x_{i+2}} (B_{i+2} - B_{i+1}) - \frac{k-1}{x_{i+k} - x_{i+1}} (B_{i+1} - B_i) \right) \quad (6)$$

When the two surfaces: Q(u,v) and S(u,v) need to be converged, making the two surfaces' locations continuous on the convergence border is the premise for a discussion of continuity issues which should meet:

$$Q(0, v) = S(0, v) \quad (7)$$

After the aforesaid conditions are met, whether the connection of the two surfaces: Q (u,v) and S (u,v) reaches tangent continuity is the next issue for discussion. It means that on the convergence border, the tangent vector in the u direction should meet the following conditions:

$$\frac{Q_u(0, v)}{|Q_u(0, v)|} = \frac{S_u(0, v)}{|S_u(0, v)|} \quad (8)$$

Before the exploration of curvature continuity of surface convergence (the curvature refers to the Normal Curvature) when the two surfaces connect linearly to achieve G2 continuity, their points on the common border meet (9):

$$K_n = \frac{S_{uu}(0, v) \cdot \bar{n}}{|S_u(0, v)|^2} = \frac{Q_{uu}(0, v) \cdot \bar{n}}{|Q_u(0, v)|^2} \quad (9)$$

where  $\bar{n}$  is the normal vector of the Tangent-Plane.

### D. Shape Morphing Based on Blending

Shape morphing is the process of continual transformation between two shapes or forms that also includes the conversion of size. In order to make the process continuity contained, in this study, the weighted arithmetic mean method in (10) is applied to calculate the degree of scaling between the two forms and then generates the output of shape morphing with the specific form feature remained.

$$S_i = w_a \times S_a + w_b \times S_b$$

$$w_a = \frac{i}{n} \quad (10)$$

$$w_b = 1 - w_a$$

where  $n$  is the step number for the degree of shape morphing process,  $s_a$  and  $s_b$  are the scaling degree of form A and form B respectively,  $w_a$  and  $w_b$  are the weight of form A and form B respectively, and  $s_i$  is the degree of  $i$ -th form of shape morphing.

While applying this shape morphing methodology to the shape morphing of 3D model, the result was shown in Fig. 4. Fig. 4 (a) represents a primary 3D model, and Fig. 4 (b) is the output of shape morphing process which implements profile scaling for the X axis direction based on the profile on Y axis direction. Compare the new model with the primary one, the new 3D model remained the original features.

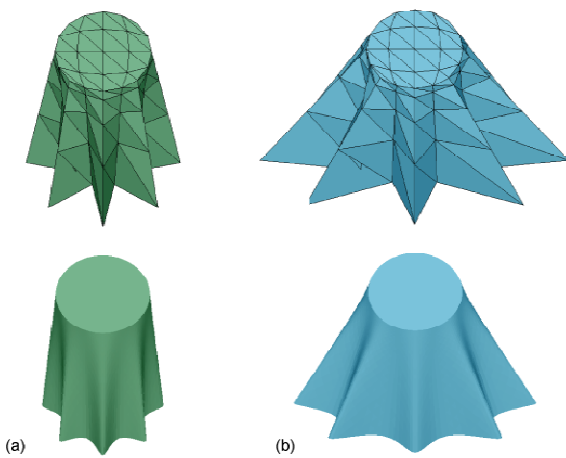


Fig. 4 The application of weighted arithmetic mean method to 3D shape morphing process (a) The primary 3D model (b) The 3D model after shape morphing

#### IV. CASE STUDY

The study constructs an application for 3D last model system based on the frame of proposed methodology mentioned above, as shown in Fig. 5. The application of 3D model system contains two functions: The reconstruction and shape morphing of 3D last model which can generate new 3D last modes in different sizes for shoe design and development, the context is shown as follows.

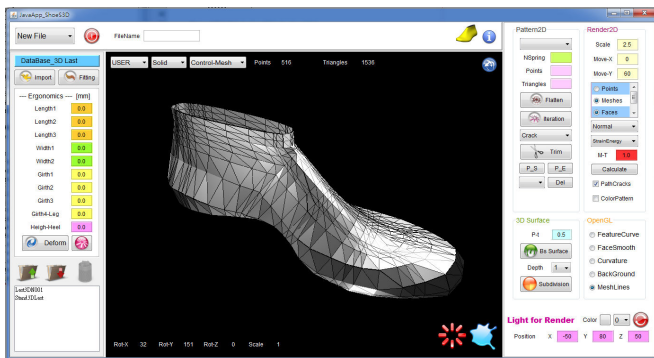


Fig. 5 The application of 3D last model system

#### A. The Reconstruction of 3D Last Model

Even though the 3D last reconstruction is based on the frame of the proposed methodology, it is necessary to implement the preprocedure (as shown in Fig. 6) for the data of 3D last, before the data uploaded to the application of 3D last model system. This stage is to apply reverse engineering method to scan the 3D last form which will be reconstructed later, thus the data of scan points can be obtained as shown in Fig. 6 (a). Next, the data of these scan points will be defined on the segments of surface reconstruction which partitioned by last feature curves. With other feature curves, the whole 3D last model can be partitioned in several sections in different sizes. Then project the grid points which can be utilized on surface fitting algorithm on each section so that a 3D last mesh can be generated, as shown in Fig. 6.

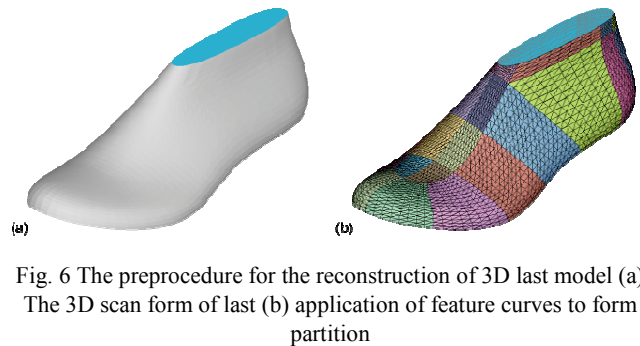


Fig. 6 The preprocedure for the reconstruction of 3D last model (a) The 3D scan form of last (b) application of feature curves to form partition

Then uploading the data of 3D last mesh from the preprocedure to the application for 3D last model system in this study, each section on the 3D last model can be calculated with the B-spline surface fitting. After revising the continuity of the connections among surfaces, the 3D last model consisted of B-spline control mesh can be constructed, as shown in Fig. 7. Meanwhile, input different B-spline parameter  $t$  into the system, the required form and surface of 3D last model can be generated (Fig. 7); the surfaces will be adjusted or transformed due to the specific demands.

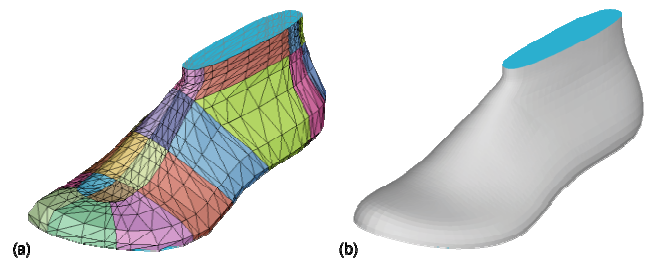


Fig. 7 The surface reconstruction of 3D last model (a) B-spline control mesh (b) B-spline surface

#### B. The Shape Morphing of 3D Last

After the reconstruction of 3D last model, the shape morphing of 3D last is the other target of this study. By inputting different feet measurement, 3D last model with different sizes which adjusted through scaling can be generated with primary functional features remained. As shown in Fig. 8, a 3D last



model in different size will be constructed by shape morphing computation for the reconstruction of 3D last model. The application of this methodology makes a great breakthrough that various 3D last model can be generated in once operation,

so that the developers can be much more concentrate on the research for comfort of shoes wearing caused by different last forms.

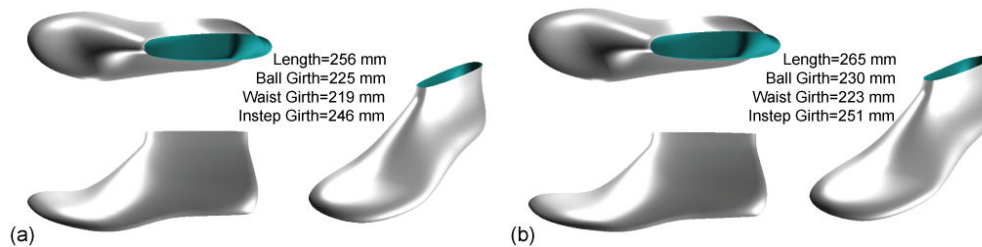


Fig. 8 The application of weighted arithmetic mean method to shape morphing of 3D last model (a) the reconstruction of 3D last model (b) the 3D last model after shape morphing

## V. CONCLUSION

Through the proposed methodology in this study, the data 3D of grid points for actual last can be scanned with features of different functional forms remained. And through the surface reconstruction algorithm, the 3D last model consisted of B-spline surface can be constructed. Due to the reason that the partition of surface was formed by feature curves of last, thus the measurement of feature curves can be converted to the scaling proportion for each section of the 3D model, and a variety of 3D last forms can be generated with the weighted arithmetic mean method. Through the verification for the theoretical application of 3D last model system with cases study, the practicality of proposed methodology was proved, the research target for computer aided design and manufacture will be realized. In the future, the customer-oriented last can be constructed by scaling the size of 3D last model with the auto representation for scan data of each feature curves of lasts.

## ACKNOWLEDGMENT

The authors would like to thank the National Science Council of the Republic of China for supporting this project under contract No. NSC-101-2218-E-006-030.

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