

A Practical Approach for Testing the Process Quality

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Abstract—Process capability index C_{pk} is the most widely used index in making managerial decisions since it provides bounds on the process yield for normally distributed processes. However, existent methods for assessing process performance which constructed by statistical inference may unfortunately lead to fine results, because uncertainties exist in most real-world applications. Thus, this study adopts fuzzy inference to deal with testing of C_{pk} . A brief score is obtained for assessing a supplier's process instead of a severe evaluation.

Keywords—Process capability analysis; quality control.

I. INTRODUCTION

PROCESS capability analysis has always been seen as one of the most important engineering decision tools. The well-known index C_{pk} is defined explicitly as:

$$C_{pk} = \text{Min} \left\{ \frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right\} = \frac{d - |\mu - M|}{3\sigma} \quad (1)$$

where μ is the process mean, σ is the process standard deviation, USL and LSL are respectively the upper and lower specification limits, $M = (USL + LSL)/2$ is the mid-point between the lower and the upper specification limits, and $d = (USL - LSL)/2$ is half-length of the specification interval.

C_{pk} is the most prevalent index for making managerial decisions in practice. Statistical testing with alternative hypothesis $H_1: C_{pk} > c$ is often used to evaluate the performance of a supplier's process. By the sampling data, the p -value can be calculated for making the decision of rejecting H_0 or not. If p -value $< \alpha$, then H_0 is rejected, and the process is judged as capable. However, statistical inference for testing process capability may lead to fine or abrupt decision results. Hence, a new trend has been recently inspired of combining randomness and fuzziness in assessing process capability. Chen et al. [1] provided fuzzy inference to conduct the uncertainty in evaluating a supplier's capability for

processes with one-sided specifications. In addition, Chen et al. [2] applied fuzzy inference to select the better supplier based on the index C_{pm} . Unfortunately, literature about the application of fuzzy inference on assessing capability based on the most prevalent index C_{pk} is ignored. Therefore, the main purpose of this study is to provide methods by incorporating the fuzzy inference into capability evaluation based on the index C_{pk} .

II. STATISTICAL TESTING FOR C_{pk}

The formulae of C_{pk} is easy to understand and apply, but the parameters, say μ and σ , are generally unknown in practice. Hence, the C_{pk} value has to be estimated based on the random sample x_1, x_2, \dots, x_n taken from a stable process. The point estimation of C_{pk} , \hat{C}_{pk} , can be obtained by replacing μ and σ with their sample estimators $\bar{x} = \sum_{i=1}^n x_i / n$ and $s = [\sum_{i=1}^n (x_i - \bar{x})^2 / (n - 1)]^{1/2}$.

$$\hat{C}_{pk} = \text{Min} \left\{ \frac{USL - \bar{x}}{3s}, \frac{\bar{x} - LSL}{3s} \right\} = \frac{d - |\bar{x} - M|}{3s} \quad (2)$$

Under the assumption of normality, Lin and Pearn [3] obtained the exact and explicit form of the CDF of \hat{C}_{pk} . Based on the CDF of \hat{C}_{pk} , the p -value for statistical testing is

$$p\text{-value} = \int_0^{3C_p \sqrt{n}} G \left(\frac{(n-1)[3C_p \sqrt{n} - t]^2}{9n(\hat{C}_{pk})^2} \right) \times [\phi(t + 3(C_p - c)\sqrt{n}) - \phi(t - 3(C_p - c)\sqrt{n})] dt \quad (3)$$

III. FUZZY CONTROL SYSTEMS

Zadeh [4] introduced the concept of fuzzy set \tilde{A} of \mathbb{R} by extending the characteristic function. A fuzzy subset \tilde{A} of \mathbb{R} is characterized by its membership function $\eta_{\tilde{A}}: \mathbb{R} \rightarrow [0,1]$, which assigns to each element $x \in \mathbb{R}$ a real number $\eta_{\tilde{A}}(x)$ in the interval $[0, 1]$, where the value of $\eta_{\tilde{A}}(x)$ reflects the membership function of x in \tilde{A} . The set of elements that belong to the fuzzy set \tilde{A} at least to the degree of membership δ is called the δ -cut, denoted by

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$$\tilde{A}[\delta] = \{x | \eta_{\tilde{A}}(x) \geq \delta, x \in \mathbb{R}\}, \text{ where } \delta \in [0,1] \quad (4)$$

The symbol $\tilde{A}[\delta]$ represents a non-empty bounded interval contained in \mathbb{R} , which can be denoted by $\tilde{A}[\delta] = [L_{\tilde{A}}(\delta), R_{\tilde{A}}(\delta)]$, where $L_{\tilde{A}}(\delta)$ and $R_{\tilde{A}}(\delta)$ are the lower and upper bounds of the closed interval, respectively.

An expert system is a computer-based system that emulates the reasoning process of a human expert within a specific domain of knowledge. The most successful application area of fuzzy systems has undoubtedly been the area of fuzzy control. A fuzzy controller consists of four modules: a fuzzy rule base, a fuzzy inference engine, and fuzzification/defuzzification modules. Generally, a fuzzy controller operates by repeating a cycle of the following steps (see Klir and Yuan [5] for more details): Step 1: Define input and output variables; Step 2: Determine fuzzy sets for the variables; Step 3: Formulate fuzzy inference rules; Step 4: Perform fuzzy inference; Step 5: Defuzzification.

IV. FUZZY INFERENCE ON SUPPLIER EVALUATION

As mentioned above, we noted that traditional method for testing process capability may lead to fine or abrupt decision results. Hence, in this section we tempt to provide a method to incorporate the fuzzy inference into the evaluation of a supplier's capability.

Step 1. In the control system, we let p -value be the input variable, and a concise score be the output variable. The range of the p -value is $[0, 1]$, and the range of the score is $[0, 100]$. Moreover, we use seven linguistic states: very small, small, rather small, medium, rather large, large, and very large, to rank the input p -value, and seven linguistic states: very low, low, rather low, medium, rather high, high, and very high, to rank the output score.

Step 2. In this step, seven fuzzy numbers with membership functions, $S_3, S_2, S_1, M_e, B_1, B_2$ and B_3 , are sequentially employed to express the linguistic states of the input variable, where S_3 and B_3 are trapezoid membership functions while S_2, S_1, M_e, B_1 , and B_2 are triangular membership functions. Because the inference results of the output are related to α , a allowance w which can be determined by the producer is taken into account into analysis.

Furthermore, seven fuzzy numbers with membership functions, $L_3, L_2, L_1, m_e, H_1, H_2$ and H_3 , are sequentially employed to express the linguistic states of the output variable.

Step 3. Fuzzy rules are essential factors for successful inference results. A rule base represents the experience and knowledge of experts. Since the larger the p -value, the higher the score, we straightly formulate the rules as;

Rule 1: if (p -value is S_3) then (score is L_3)

Rule 2: if (p -value is S_2) then (score is L_2)

Rule 3: if (p -value is S_1) then (score is L_1)

Rule 4: if (p -value is M_e) then (score is m_e)

Rule 5: if (p -value is B_1) then (score is H_1)

Rule 6: if (p -value is B_2) then (score is H_2)

Rule 7: if (p -value is B_3) then (score is H_3)

Step 4. Suppose that R_i : if $x = A_i$ then $y = B_i$ represents the i th inference rule defined at the former step, the Mamdani inference method [6] can be used to obtain the inference result

Step 5. The defuzzification is a process where an appropriate crisp value $y^* \in Y$ is transferred from the result of fuzzy inference $B'_i(y)$. This study adopts the weight method, which is the most used method, for defuzzification.

The crisp value y^* is the final output score we want to obtain in our control system. The score is range from 0 to 100, and we divide this range into four segments in order to represent the capability grade of the process. Table I lists the defined capability grades, and the suggested corresponding strategies. The practitioners can make decisions according to this table.

V. SUMMARY

Process capability index C_{pk} is the most widely used for quality assurance activities. However, uncertainty often arises in real applications, which causes imprecise measurements of product quality. Hence, this study incorporates the fuzzy inference into process evaluation. Linguistic states are used to rank the p -values, and fuzzy numbers are constructed to express these linguistic states. After fuzzy rules are formulated, the Mamdani method is performed to obtain the inference results, and the weight method is employed for defuzzification. Finally, a concise score is derived from defuzzification, and the practitioners can make flexible decisions by this score, which effectively avoids fine results.

TABLE I
THE STRATEGIES FOR VARIOUS CAPABILITY GRADES

| Item | Fuzzy evaluation | | Strategy |
|------|-----------------------------|---|--|
| 1 | $85 \leq \text{score}$ | The sufficiency of the process capability is rather high. | <ol style="list-style-type: none"> 1. The producer does not need to change his process, but keeps his process capability. 2. The consumer gives the order. |
| 2 | $60 \leq \text{score} < 85$ | The sufficiency of the process capability is acceptable. | <ol style="list-style-type: none"> 1. The producer has to monitor the process, and prevents the capability slipping down. 2. The consumer gives the order. |
| 3 | $45 \leq \text{score} < 60$ | The process capability is insufficient. | <ol style="list-style-type: none"> 1. The producer has to promote the process capability. 2. The consumer rejects to give the order. |
| 4 | $\text{score} < 45$ | The process capability is very insufficient. | <ol style="list-style-type: none"> 1. The producer has to find the flaws in the process, and makes any efforts to improve the process capability. 2. The consumer rejects to give the order. |

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