Mathematical Modeling of Elastically Creeping State of Arbitrarily Orientated Cavities in the Transversally Isotropic Massif

N. Azhikhanov, T. Turimbetov, Zh. Masanov, and N. Zhunisov

Abstract**—**It can be determined in preference between representative mechanical and mathematical model of elastic-creeping deformation of transversally isotropic array with doubly periodic system of tilted slots, and offer of the finite elements calculation scheme, and inspection of the states of two diagonal arbitrary profile cavities of deep inception, and in setting up the tense and dislocation fields distribution nature in computing processes.

Keywords**—**Mathematical model, tunnel, transversally isotropic, finite elements.

PACS**—**46.25.Cc, 47.11.Fg.

I. INTRODUCTION

THE elastically creeping and stressed-deformed state of two superficially laying cavities are investigated in this work in heavy (transversally isotropic) transtropic massif depending on discontinuity of adhesion shallow sloping layers.

Depending on directions of their longitude axis to trend line of inclined layers the horizontal subterranean air holes were subdivided in three groups: drift-cavities that curves on the line of the course of layers, crosscut manufacture, filled transversally to the course of layers and diagonal air holes manufacture, interstitial position between drifts and crosscuts.

II. GOVERNING EQUATIONS

Except the interlaying spacing, the depth of the location, the shape of the cross section the stressedly-deformed mode of these horizontal air holes depends on elastic and creeping properties of the surrounding rock massif as well.

The investigating of law of the distribution of elastically creeping strain and of the displacements near air holes of arbitrary depth of location and shapes of cross section also subjected to nonuniformly cracked structure considered interesting not only theoretically but in direct practical using.

Let's take into consideration arbitrarily orientated underground cavities of superficial laying in heavy transtropic massif that depend on the degree of the discontinuity adhesion of shallow layers inclined through φ angle, considering the longitude axis of air holes and the trend line of the plane of isotropy are forming the angle ψ, the latter plane agrees with fissure plane. H is a depth of the location of key seat with center distances L.

In regard to the rectangular coordinate system Oxyz (Fig. 1) the equation of the generalized Hooke law of anisotropic massif that contains cavities in generalized plane deformation represented by [1]:

$$\{\sigma\} = [\overline{D}]\{\varepsilon\}; \tag{1}$$

where $\{\sigma\} = (\sigma_x, \sigma_z, \tau_{yz}, \tau_{xz}, \tau_{xy})^T$,
$\{\varepsilon\} = (\varepsilon_x, \varepsilon_z, \gamma_{yz}, \gamma_{xz}, \gamma_{xy})^T$, $[D^0] = [d_{ij}]$, $(i, j = 1, 2, ..., 6)$

deformation coefficients d_{ij} equal to:

$$
\begin{aligned}
& d_{11} = a_{11}\cos^4\psi + (2a_{12} + a_{66})\sin^2\psi\cos^2\psi + a_{22}\sin^4\psi, \\
& d_{22} = a_{33}, \\
& d_{13} = a_{25}\sin^3\psi + (a_{15} - a_{46})\sin\psi\cos^2\psi, \\
& d_{14} = a_{15}\cos^3\psi + a_{25}\cos\psi\sin^2\psi, \\
& d_{15} = 2(a_{11} - a_{12})\cos^3\psi\sin\psi + 2(a_{12} - a_{22})\sin^3\psi\cos\psi - 0.5a_{66}\cos 2\psi\sin 2\psi, \\
& d_{23} = a_{35}\sin\psi, \quad d_{24} = a_{35}\cos\psi, \\
& d_{25} = 2(a_{13} - a_{23})\cos\psi\sin\psi, \quad d_{34} = (a_{44} - a_{55})\cos\psi\sin\psi, \\
& d_{35} = a_{46}\cos^3\psi + (2a_{15} - 2a_{25} - a_{46})\sin^2\psi\cos\psi, \\
& d_{45} = a_{46}\sin^3\psi + (2a_{15} - 2a_{25} - a_{46})\cos^2\psi\sin\psi. \\
& d_{33} = a_{44}\cos^2\psi + a_{55}\sin^2\psi, \quad d_{44} = a_{44}\sin^2\psi + a_{55}\cos^2\psi, \\
& d_{55} = (a_{11} + a_{22} - 2a_{12} - a_{66})4\sin^2\psi\cos^2\psi + a_{66}, \\
& d_{12} = a_{13}\cos^2\psi + a_{23}\sin^2\psi, \\
& a_{11} = \frac{1}{E_1}\cos^4\varphi + \frac{1}{4}\left(\frac{1}{G_2} - \frac{2\nu_1}{E_1}\right)\sin^2 2\varphi + \frac{1}{E_2}\sin^4\varphi, \\
& a_{22} = \frac{1}{E_1}, \\
& a_{33} = \frac{1}{E_1}\sin^4\varphi + \frac{1}{4}\left(\frac{1}{G_2} - \frac{2\nu_1}{E_1}\right)\sin^2 2\varphi + \frac{1}{E_2}\cos^4\varphi, \\
& a_{44} = \frac{2(1+\nu_1)}{E_1}\sin^2\varphi + \frac{1}{G_2}\cos^2\varphi, \\
& a_{55} = \frac{1}{G_2} + \left(\frac{1+2\nu_2}{E_1} + \frac{1}{E_2} - \frac{1}{G_2}\right)\sin^2 2\varphi, \\
& a_{66} = \frac{1+2\nu_1}{E_1}\cos^2\varphi + \frac{1}{G_2}\sin^2\varphi,
\end{aligned}
\tag{2}
$$

N. Azhikhanov, T. Turimbetov, Zh. Masanov, and N. Zhunisov are with Caspian State University of Technologies and Engineering named after Sh. Yessenov, Kazakhstan (e-mail: ajihanov1@mail.ru).

$$a_{12} = -\frac{\nu_1}{E_1}\cos^2\varphi - \frac{\nu_2}{E_1}\sin^2\varphi,$$

$$a_{13} = \frac{\nu_2}{E_1} + \frac{1}{4}\left(\frac{1+2\nu_2}{E_1} + \frac{1}{E_2} - \frac{1}{G_2}\right)\sin^2 2\varphi,$$

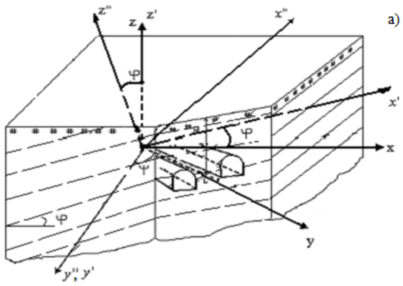
$$a_{15} = \left(\frac{1+\nu_2}{E_1}\cos^2\varphi - \left(\frac{1}{E_2} + \frac{\nu_2}{E_1}\right)\sin^2\varphi - \frac{1}{2G_2}\cos 2\varphi\right)\sin 2\varphi,$$

$$a_{23} = -\frac{\nu_1}{E_1}\sin^2\varphi - \frac{\nu_2}{E_1}\cos^2\varphi,$$

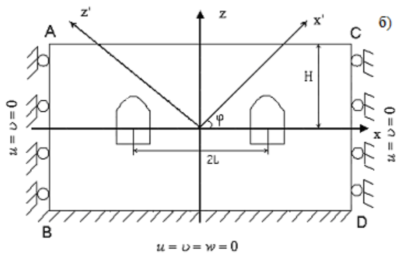
$$a_{25} = -\frac{\nu_1 - \nu_2}{E_1}\sin 2\varphi,$$

$$a_{35} = \left(\frac{1+\nu_2}{E_1}\sin^2\varphi - \left(\frac{1}{E_2} + \frac{\nu_2}{E_1}\right)\cos^2\varphi + \frac{1}{2G_2}\cos 2\varphi\right)\sin 2\varphi,$$

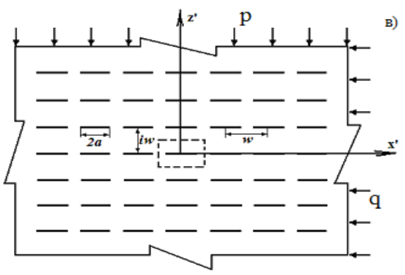
$$a_{46} = -\frac{1}{2}\left(\frac{1}{G_2} - \frac{2(1+\nu_1)}{E_1}\right)\sin 2\varphi$$



(a)



(b)



(c)

Fig. 1 Calculation diagram in investigating of strained state of the anisotropic massif (a) spatial view (b) generalized planar view (c) plane with periodical system of fissures

III. Numerical Studies

In these formulas $E_k^{Э}, \nu_k^{Э}, G_2^{Э} (k=1,2)$ - effective elastic constants of transtropic massifs that equivalent on rigidity to anisotropic massif with fissures which depends on the elastic constants $E_k, \nu_k, G_2 (k=1,2)$ for the second and the geometry of fissures $a, \omega, i\omega$ [2]-[4].

The law of the creeping stress distribution and of the displacements near diagonal pair air holes in the ponderable transrope massif was investigated thanks to the final element methods for the instance of generalized biaxial deformation and to the theory of creep of rocks suggested by Zh. S. Yerzhanov.

Time-dependent processes (at t>0) near underground air holes are caused by effects of creeping properties of the surrounding rocks. Within investigating of them, considering the general principles of creeping rocks theory of Zh. S. Yerzhanov, (2) the elastic constants $E_1^{Э}, E_2^{Э}, G_2^{Э}$ and Poisson ratio $\nu_1^{Э}, \nu_2^{Э}$ were replaced by time-dependent operators

$$\tilde{E}_1^{Э}, \tilde{E}_2^{Э}, \tilde{G}_2^{Э}, \tilde{\nu}_1^{Э}, \tilde{\nu}_2^{Э} .$$

$$\tilde{E}_n^{Э} = E_n^{Э}\left(1 - E_n^*\right), \left(\tilde{E}_3^{Э} = \tilde{G}_2^{Э}\right),\ \nu_k = \nu_k^{Э}\left(1+\nu_k^*\right), (k=1,2;\ n=1,2,3) \qquad (3)$$

$$E_n^* f = \int_0^t M_n(t-\tau) f(\tau) d\tau, \qquad \nu_k^* f = \int_0^t L_k(t-\tau) f(\tau) d\tau;$$

$M_n(t-\tau)$, $L_k(t-\tau)$ - heredity kernel.

The negligibility of variation in creeping parameters of the anisotropic rocks with different directions was demonstrated in laboratory researches [5].

$\tilde{E}_1^{Э}, \tilde{E}_2^{Э}, \tilde{G}_2^{Э}, \tilde{\nu}_1^{Э}, \tilde{\nu}_2^{Э}$ Therefore the time-dependent operators here given by

$$\tilde{E}_n^{Э} = E_n^{Э}\left[1 - \aleph\,Э_\alpha^*(-\beta)\right],$$

$$\tilde{\nu}_k^{Э} = \nu_k^{Э},\ \tilde{E}_i^{Э} / \tilde{E}_j^{Э} = E_i^{Э} / E_j^{Э} = const,\ (n,i,j = 1,2,3; k=1,2) \cdot$$

The creeping parameters of rocks on Abelian kernel of creeping can be defined by these formulas

$$E_{k,t}^{Э} = E_k^{Э}(1+\Phi_t)^{-1},\ \nu_{k,t}^{Э} = 0.5 - (0.5 - \nu_k)(1+\Phi_t)^{-1},$$

$$\Phi_{k,t} = \delta(1-\alpha)^{-1} t^{1-\alpha};$$

α, δ – The creeping parameters of rocks, t – time.

The values of time modules for t=120h и t=600h were used in calculation of stressed mode of the diagonal cavities in condition if there is an appearance of isotropy of creep properties of rocks [2].

In heavy virgin massif the general stress distribution is similar with the side pressures λ_x, λ_y, which are the functions of elastic constants of the continuum and angles φ, ψ.

In research the numerical methods of FEM analysis with isoparametric calculated elements at the generalized plane deformation is used because of the complexity of the rigorous solution of a problem of the strained *mode of the* diagonal air holes in the ponderable massif under rock creep conditions.

The measurement environment of cavities of the investigation is automatically divided into the isoparametric elements within soft FEM3D in object orientated environment DELPHI. Each junction is under the vertical project of weight.

In research the numerical methods of FEM analysis with isoparametric calculated elements at the generalized plane deformation is used because of the complexity of the rigorous solution of a problem of the strained mode of the diagonal air holes in the ponderable massif under rock creep conditions.

The measurement environment of cavities of the investigation is automatically divided into the isoparametric elements within soft FEM3D in object orientated environment DELPHI. Each junction is under the vertical project of weight.

Within splitting the field of study into elements relatively to the displacement components from N junctions the general combined equations on the 3N-th order can be resolved by the Seidel-Gauss iterative technique with over relaxation coefficient $\beta (1 \le \beta \le 2)$. Bundled software was pretested on solutions of known test problems.

Multivariate simulation results on calculation of components of stress near cavities of arched cross section at the different parameters were obtained: location depths, angles φ, ψ and layer cohesion degrees which determined by fissure periods $\omega, i\omega$.

Much attention was paid to the distribution regularity of the vertical stress on trimmer block between cavities and on the components of displacements and five stress components at the points of contours of the vicinity of drifts (ψ=0) .

Table I contains the values of the pressure σ_z and vertical displacement w at cavity contour points (Fig. 2). Step-by-step numbering of cavity contour points is shown in this figure.

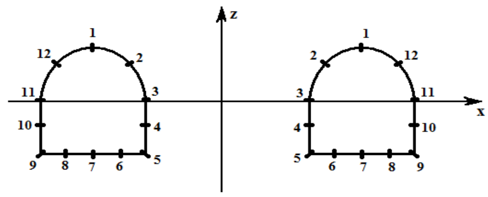


Fig. 2 Scheme of an Arrangement of Numbering of Contours of Cavities

TABLE I
ELASTICALLY CREEPING DISPLACEMENTS AND PRESSURE VALUES AT A POINT ON THE ROCK CONTOURS OF AIR HOLES IN MASSIF WITH DISCONTINUITY FLAW COHESION OF LAYERS AT T=120H. AND T=600H. ω/a =6

Left cavity	**t=120h.**				**t=600h.**			
w/a=6.0	$10^6 u$	$10^6 w$	$\sigma_x / \gamma H$	$\sigma_z / \gamma H$	$10^6 u$	$10^6 w$	$\sigma_x / \gamma H$	$\sigma_z / \gamma H$
1	0,045	-0,610	0,001	0,000	0,071	-0,943	0,001	0,000
2	0,053	-0,588	0,001	0,001	0,082	-0,908	0,001	0,000
3	0,043	-0,514	0,000	0,001	0,067	-0,795	0,000	0,001
4	0,012	-0,420	0,000	0,001	0,018	-0,649	0,000	0,001
5	-0,013	-0,348	0,000	0,000	-0,020	-0,538	0,000	0,000
6	0,003	-0,251	0,001	0,001	0,005	-0,388	0,001	0,001
7	0,000	-0,113	0,001	0,000	0,001	-0,174	0,001	0,000
8	0,002	-0,248	0,001	0,001	0,002	-0,384	0,001	0,001
9	0,037	-0,348	0,000	0,000	0,058	-0,538	0,000	0,000
10	0,037	-0,409	-0,001	0,001	0,058	-0,632	-0,001	0,001
11	0,031	-0,499	0,000	0,001	0,048	-0,772	0,000	0,001
12	0,036	-0,575	0,001	0,000	0,055	-0,890	0,001	0,000

IV. CONCLUSIONS

The calculation data analysis shows the growth of vertical shifts during reduction of the parameter ω/a. The relationship of vertical pressure and ω/a is essential and the angle affect of the inclination of fissure planes to the pressure quantity and displacement is significant; their distribution on contours is asymmetric; there is the growth of displacements on rock contours (Table I) from hanging or pendulous sides than from interlaying sides and such tendency in value of displacements grows with reduction in center-to-center distance of air holes.

REFERENCES

[1] Erzhanov Zh. S., Aitaliev Sh. M., Masanov Zh. K. *Stability of horizontal developments in slantwise-flaky massif,* Alma-Ata, "Science" Kaz SSR, 1971.- 160p.

[2] Erzhanov Zh. S., Aitaliev Sh. M., Massanov Zh. K. *Seismological-stress condition of underground structures in anisotropic layered massi.* Alma-Ata, "Science" КазССР, 1980.- 212p.

[3] Massanov Zh. K., Omarov A. D., Mahмetova N. M. *Static and seismological-stress of transport underground structures in anisotropic vectorially nonlinear massif,* – Alma-Ata: Bastau, 2002.-244p.

[4] Aitaliev Sh. M., Massanov Zh. K. Account of mountain pressure in directed every which way horizontal developments. *Mountain pressure in capital and preparatory developments,* Novosibirsk: 1973. – pp. 21-26.

[5] Sarseмbaev A. A., Siniyaev A. Y., Matveeva V. P., Kudashov E. F. About definition of elastic and temporary deformations on layered samples // In кн.: " Researches on the mechanics of mountain breeds ". Alma-Ata, "Science".-1965.-pp.45-50.