Effect of thermal radiation on temperature variation in 2-D stagnation-point flow

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Abstract—Non-isothermal stagnation-point flow with consideration of thermal radiation is studied numerically. A set of partial differential equations that governing the fluid flow and energy is converted into a set of ordinary differential equations which is solved by Runge-Kutta method with shooting algorithm. Dimensionless wall temperature gradient and temperature boundary layer thickness for different combination of values of Prandtl number Pr and radiation parameter Nr are presented graphically. Analyses of results show that the presence of thermal radiation in the stagnation-point flow is to increase the temperature boundary layer thickness and decrease the dimensionless wall temperature gradient.

Keywords—Stagnation-point flow, Similarity solution, Thermal radiation.

I. INTRODUCTION

STAGNATION-POINT flow is a flow in which the component of velocity normal to a wall is toward the wall everywhere in the region concerned, so that the vorticity created at the wall will be convected toward the wall, in opposition to viscous diffusion away from it. Stagnation-point flow has been found in numerous applications in engineering and technology[1]. Hiemenz[2] discovered that stagnation-point flow can be analyzed by the Navier-Stokes (NS) equations through similarity solution in which the number of variables can be reduced by on or more by a coordinate transformation. Chiam[3] studied heat transfer in stagnation-point flow with variable conductivity. Wang[4] found a similarity solution of stagnation slip flow and convective heat transfer on a moving plate.

Stagnation-point flow with consideration of thermal radiation is rarely and this is important for flow with high temperature environment[5][6]. Recently Bataller[7] consider the radiation effects in the Blasius flow over a flat plate with variable conductivity. The present paper extends the results of [7] by considering the effect of radiation effect in non-isothermal stagnation-point flow. The Navier-Stokes and energy equations are transferred to two uncoupled ordinary different equations which are solved numerically by the shooting method given in [8]. The outline of the paper is as follows. In Section 2, we introduce governing equations for non-isothermal stagnation-point flow with thermal radiation and the process of finding similarity solution. Numerical results of the similarity solution are given and discussed in Section 3. Finally conclusions are drawn in Section 4.

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Where $\alpha = \frac{k}{\rho c_p}$ is the molecular thermal diffusivity. Eq. (8) indicates that the presence of thermal radiation can be interpreted as an increase in the molecular thermal diffusivity. Let $N_R = \frac{k_{\text{eff}}}{\alpha}T_{\infty}$ be the radiation parameter, Eq. (8) can be rewritten as

$$\frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \alpha_{\text{eff}} \frac{\partial^2 T}{\partial y^2},$$  

(9)

where $\alpha_{\text{eff}} = \frac{\alpha}{k_0}$ and $k_0 = \frac{3N_R}{3N_R + 1}$. It can be seen that the effect of thermal radiation is to replace the original molecular diffusivity, $\alpha$, by an equivalent molecular diffusivity, $\alpha_{\text{eff}}$, in the energy equation at which no thermal radiation is considered. It should be noted that $k_0 \leq 1$ and $\alpha_{\text{eff}} \geq \alpha$. The condition that $k_0 = 1$ or $N_R \rightarrow \infty$ represents the stagnation-point flow without thermal radiation.

Following [9] by introducing a similarity variable $\eta$, a dimensionless stream function $F(\eta)$, a dimensionless temperature $\theta$, and the Prandtl number $Pr$, such that

$$\eta = \frac{y}{\sqrt{u' B}}, \quad u = Bx F' (\eta), \quad v = -\sqrt{u' B} F(\eta),$$

(10)

and

$$\theta = \frac{T - T_w}{T_{\infty} - T_w}, \quad Pr = \frac{\nu}{\alpha}.$$  

(11)

where $B$ is a constant. Eq. (1) is satisfied automatically. Eqs. (2), (3), and (9) can be reduced to two uncoupled ordinary differential equations

$$F'''' + FF' + (1 - F'^2) = 0$$

(12)

$$\theta'' + Prk_0 F\theta' = 0.$$  

(13)

The boundary conditions are

$$F(0) = F'(0) = F'(\infty) = 1 = 0$$

(14)

and

$$\theta(0) = \theta(\infty) = 1 = 0.$$  

(15)

The two-point boundary value problem of Eqs. (12)-(15) is solved numerically with Runge-Kutta method of fourth-order[8].

### III. RESULTS AND DISCUSSION

In present study of non-isothermal stagnation-point flow, the fluid flow problem governing by Eq. (12) is uncoupled with the thermal problem of Eq. (13). Hence change of the values of $Pr$ and $N_R$ will have no effect on the fluid flow velocity. Eqs. (12)-(15) are solved with $N_R = 0.005$. Three quantities of interest in this study are wall heat flux which is related to dimensionless wall temperature gradient $\theta'(0)$, dimensionless temperature boundary layer thickness $\eta_\delta$, and dimensionless parameter $\eta_{\text{in}}$. The dimensionless temperature boundary layer thickness $\eta_\delta$ is defined as the distance ($\eta$) away from the wall such that $\theta(\eta_\delta) = 0.90$. The dimensionless parameter $\eta_{\text{in}}$ is defined as the value of $\eta$ at which $\theta'(\eta_{\text{in}}) = 10^{-4}$.

Numerical results of variation of $\theta'(0)$, $\eta_\delta$, and $\eta_{\text{in}}$ with $Pr$ without thermal radiation (i.e., $k_0 = 1$) are given in Table 1. It is noted that the dimensionless wall temperature gradient $\theta'(0)$ increases with increase of Prandtl number, but both the temperature boundary layer thickness $\eta_\delta$ and the dimensionless parameter $\eta_{\text{in}}$ decrease with increase of Prandtl number $Pr$. It can be explained by the definition of Prandtl number that Prandtl number is inversely proportional to the molecular thermal diffusivity $\alpha$. Table 2 gives variation of results with $N_R$ for $0.5 \leq N_R \leq 1000$ at $Pr = 5$. It is observed that values of the dimensionless wall temperature gradient $\theta'(0)$ increases with increasing $N_R$, but both the temperature boundary layer thickness $\eta_\delta$ and dimensionless parameter $\eta_{\text{in}}$ decrease with increasing $N_R$. It should be noted that all values of $\theta'(0)$ obtained in Table 2 with the presence of thermal radiation at $Pr = 5$ are less than 1.0434, which is the value of $\theta'(0)$ calculated in Table 1 without thermal radiation at $Pr = 5$. Effect of variation of $Pr$ for $0.5 \leq Pr \leq 1000$ at $N_R = 5$ on the numerical results are shown in Table 3. Again increase of $\theta'(0)$, as well as decrease of both $\eta_\delta$ and $\eta_{\text{in}}$ with increasing $Pr$ are found.

Temperature profiles for three different values of radiation parameter ($N_R = 0.5, 1, 10$) and three different values of Prandtl number ($Pr = 0.5, 5, 10$) are given in Figs. 1-3. Results obtained for the case without thermal radiation (i.e., $k_0 = 1$) are also shown in these figures for comparison.
Radiation, the difference in temperature distribution for the cases with and without thermal radiation is quite large for all three values of \( Pr \). When the value of \( N_R \) is large, or the effect of thermal radiation is small, as shown in Fig. 3, it is found that the temperature distribution with or without thermal radiation are close each other for all values of \( Pr \). This is physically consistent because of the fact that \( k_0 \) equals to one, or \( N_R \) tends to infinity, represents the case without thermal radiation. On the other hand the effect of variation of \( Pr \) on the thermal boundary layer is more pronounced for small value of \( N_R \), especially in the case where \( Pr \) is less than one. This is consistent with the founding given in Table 3. It is attributed to the fact that decreasing either the \( Pr \) or \( N_R \) will increase \( \alpha \) or \( \alpha_{eff} \), which in turn, will increase the temperature boundary layer thickness.

It is also seen from Figs. 1-3 that the effect of thermal radiation is to increase the temperature boundary layer thickness for a fixed value of Prandtl number. This effect is diminishing when \( N_R \) is increasing as expected because a large value of \( N_R \) represent a decreasing of thermal radiation effect. All these findings in Figs. 1-3 are consistent with the results shown in Table 2.

IV. CONCLUDING REMARK

Numerical calculation of non-isothermal stagnation-point flow with thermal radiation has been investigated by similarity solution of solving the ordinary differential equations. This two-point boundary value problem is solved by the Runge-Kutta method and results are presented as both tables and figures. Analyses of the results find that the effect of thermal radiation is to increase the temperature boundary layer thickness and to decrease the wall temperature gradient. But this effect become negligible when radiation parameter \( N_R \) is large. On the other hand, increasing the Prandtl number tends to reduce the temperature boundary layer thickness and enlarge the wall temperature gradient for both of the cases with and without thermal radiation.

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