An Augmented Automatic Choosing Control Designed by Extremizing a Combination of Hamiltonian and Lyapunov Functions for Nonlinear Systems with Constrained Input

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Abstract—In this paper we consider a nonlinear feedback control called augmented automatic choosing control (AACC) for nonlinear systems with constrained input. Constant term is included in it. This constant term is treated as a coefficient of a stable zero dynamics. The given nonlinear system approximately makes up a set of augmented linear systems, to which the optimal linear control theory is applied to get the linear quadratic (LQ) controls[2]. These LQ controls are smoothly united by sigmoid type automatic choosing functions to synthesize a single nonlinear feedback controller, which is limited to be satisfied with the constrained condition.

This controller is of a structure-specified type which has some parameters, such as the number of division of the domain, regions of the subdomains, points of Taylor expansion, and gradients of the automatic choosing function. These parameters must be selected optimally so as to be just the controller’s fit. Since they lead to a nonlinear optimization problem, we are able to solve it by using the genetic algorithm (GA)[9] suboptimally. In this paper the suboptimal values of these parameters are obtained by acquiring both minimization of Hamiltonian and maximization of a stable region in the sense of Lyapunov.

This approach is applied to a field excitation control problem of power system, which is Ozeki-Power-Plant of Kyushu Electric Power Company in Japan, to demonstrate the splendidness of the AACC. Simulation results show that the new controller using the GA is able to improve performance remarkably well.

Keywords—Augmented Automatic Choosing Control, Nonlinear Control, Genetic Algorithm, Hamiltonian, Lyapunov function
Considering the nonlinearity of the system (1), introduce a vector-valued function \( C : \mathbb{D} \rightarrow \mathbb{R}^L \) which defines the separative variables \( \{C_i(x)\} \), where \( C = [C_1 \ldots C_j \ldots C_L]^T \) is continuously differentiable. Let \( \mathbb{D} \) be a domain of \( C^{-1} \). For example, if \( x[2] \) is the element which has the highest nonlinearity of (1), then
\[
C(x) = x[2] \in \mathbb{D} \subset \mathbb{R} \quad (L = 1)
\]
(see Section 4). The domain \( \mathbb{D} \) is divided into some subdomains: \( \mathbb{D} = \bigcup_{i=0}^{M} \mathbb{D}_i \), where \( \mathbb{D}_i = \mathbb{D} - \bigcup_{j=i+1}^{M} \mathbb{D}_j \) and \( C^{-1}(\mathbb{D}_i) \ni 0 \). \( \mathbb{D}_i (0 \leq i \leq M) \) endowed with a lexicographic order is the Cartesian product \( \mathbb{D}_i = \prod_{j=1}^{L} [a_{ij}, b_{ij}] \), where \( a_{ij} < b_{ij} \).

Introduce a stable zero dynamics:
\[
\dot{x}[n+1] = -\sigma x[n+1] + g(x)[u]
\]
(3)
where the value of \( \sigma \) shall be selected so that \( -\dot{x}[n+1]/x[n+1] \leq -\dot{x}[k]/x[k] \) holds for all \( k = 1 \ldots n \). This tries to keep \( x[n+1] \approx 1 \) for a good while when the system (1) is not on \( C^{-1}(\mathbb{D}_0) \) (see Appendix).

Combine (1) with (3) to form an augmented system
\[
\dot{X} = f(X) + g(X)u
\]
(4)
where
\[
X = \begin{bmatrix} x \\ x[n+1] \end{bmatrix} \in D \times R
\]
\[
\begin{bmatrix} f(x) \\ -\sigma x[n+1] \end{bmatrix}, \quad g(X) = \begin{bmatrix} g(x) \\ 0 \end{bmatrix}
\]

Let a cost function be
\[
J = \frac{1}{2} \int_{0}^{\infty} (X^T Q X + u^T R u) dt
\]
(5)
where
\[
Q = \begin{bmatrix} Q & 0 \\ 0 & q \end{bmatrix}, \quad R \ni q > 0,
\]
\[
Q = Q^T > 0 \quad \text{and} \quad R = R^T > 0
\]
which denote positive symmetric matrices. Values of \( Q \) and \( R \) are properly determined based on engineering experience.

On each \( \mathbb{D}_i \), the nonlinear system is linearized by the Taylor expansion truncated at the first order about a point \( \hat{X}_i \in C^{-1}(\mathbb{D}_i) \) and \( \hat{X}_0 = 0 \) (see Fig. 1):
\[
f(x) + g(x)u \simeq A_i x + w_i + B_i u
\]
(6)
where
\[
A_i = \left. \frac{\partial f(x)/\partial x^T} \right|_{x = \hat{X}_i}, \quad B_i = g(\hat{X}_i),
\]
\[
w_0 = 0, \quad u_i = f(\hat{X}_i) - A_i \hat{X}_i.
\]
That is, an approximation of (4) is
\[
\dot{X} = \hat{A_i} X + \hat{B}_i u \quad \text{on} \quad C^{-1}(\mathbb{D}_i) \times R
\]
(7)
where
\[
\hat{A}_i = \begin{bmatrix} A_i \\ 0 \end{bmatrix}, \quad \hat{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}
\]

Introduce a stable zero dynamics:
\[
\dot{X} = \hat{A}_i X + \hat{B}_i u \quad \text{on} \quad C^{-1}(\mathbb{D}_i) \times R
\]
(7)
where
\[
\hat{A}_i = \begin{bmatrix} A_i \\ 0 \end{bmatrix}, \quad \hat{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}
\]

An application of the linear optimal control theory[2] to (5) and (7) yields
\[
u_i(X) = F_i X
\]
(8)
\[
F_i = -R^{-1} \hat{B}_i^T P_i
\]
(9)
where the \((n+1) \times (n+1)\) matrix \( P_i \) satisfies the Riccati equation:
\[
P_i \hat{A}_i + \hat{A}_i^T P_i + Q - P_i \hat{B}_i R^{-1} \hat{B}_i^T P_i = 0
\]
(10)
Introduce an automatic choosing function of sigmoid type:
\[
I_i(x) = \prod_{j=1}^{L} \left( 1 - \frac{1}{1 + \exp(2N_1(C_j(x) - a_{ij}))} \right)
\]
(11)
where \( N_1 \) is positive real value, \(-\infty \leq a_{ij} \leq b_{ij} \leq \infty\). \( I_i(x) \) is analytic and almost unity on \( C^{-1}(\mathbb{D}_i) \), otherwise almost zero (see Fig. 2).

Fig. 1 Sectionwise linearization

Fig. 2 Automatic Choosing Function \( (N_i = 3.0, 6.0) \)

Uniting \( \{u_i(X)\} \) of (8) with \( \{I_i(x)\} \) of (11) yields
\[
\hat{u}(X) = [\hat{u}(X)[1], \ldots, \hat{u}(X)[j], \ldots, \hat{u}(X)[r]]^T = \sum_{i=0}^{M} u_i(X) I_i(x).
\]

We finally have an augmented automatic choosing control which is satisfied with the constraint condition (2) by
\[
u(X) = [u(X)[1], \ldots, u(X)[j], \ldots, u(X)[r]]^T
\]
(12)
In order to design the optimal control by Hamiltonian and expand the stable region in the sense of Lyapunov as wide as possible, we define a performance

\[
P_I = \omega_1 \int_\Omega |H(X, u, \lambda)|/X^T X dX + \omega_2 \int_\Omega \left\| \lambda + \partial H(X, u, \lambda)/\partial X \right\|/X^T X dX - \omega_3 \cdot \gamma,
\]

where \(\omega_i (i = 1, 2, 3)\) are weights.

A set of parameters included in the control (12):

\[
\bar{\Omega} = \{ M, N_1, N_2, a_{ij}, b_{ij}, \bar{x}_i, \eta_i \}
\]

is suboptimally selected by minimizing \(P_I\) with the aid of GA[9] as follows.

<ALGORITHM>
step1: Apriori: Set values \(\bar{\Omega}_{apriori} \) appropriately.
step2: Parameter: Choose a subset \(\Omega \subset \bar{\Omega} \) to be improved and rewrite it by \(\Omega = \{ M, N_1, \ldots \} = \{ \alpha_k : k = 1, \ldots, K \} \).
step3: Coding: Represent each \(\alpha_k\) with a binary bit string of \(L\) bits and then arrange them into one string of \(L \cdot K\) bits.
step4: Initialization: Randomly generate an initial population of \(\bar{q}\) strings \(\{ \Omega_p : p = 1, \ldots, \bar{q} \} \).
step5: Decoding: Decode each element \(\alpha_k\) of \(\Omega_p\) by \(\alpha_k = (\alpha_{k, max} - \alpha_{k, min}) A_k / (2^L - 1) + \alpha_{k, min}\), where \(\alpha_{k, max}\): maximum, \(\alpha_{k, min}\): minimum, and \(A_k\): decimal values of \(\alpha_k\).
step6: Adjacent: Make \(\lambda = \lambda(X)\), \((p = 1, \ldots, \bar{q})\) for \(\Omega_p\) by using Eq.(14).
step7: Fitness value calculation: Calculate \(P_I\) by Eqs.(15) and (19), or fitness \(F_p = -P_I\). Integration of \(P_I\) is approximated by a finite sum.
step8: Reproduction: Reproduce each of individual strings with the probability of \(P_{F_p} / \sum_{j=1}^{\bar{q}} F_j\).
step9: Crossover: Pick up two strings and exchange them at a crossing position by a crossover probability \(P_c\).
step10: Mutation: Alter a bit of string (0 or 1) by a mutation probability \(P_m\).
step11: Repetition: Repeat step5~step10 until prespecified \(G\)-th generation. If unsatisfactory, go to step2.

As a result, we have a suboptimal control \(u(X)\) for the string with the best performance over all the past generations.

IV. NUMERICAL EXAMPLE

Consider a field excitation control problem of power system. Fig. 3 is a diagram of Ozeki-Power-Plant of
Kyushu Electric Power Company in Japan. This system is assumed to be described by

\[ M \ddot{\delta} + \tilde{D}(\delta) \dot{\delta} + P_r(\delta) = P_m \]

\[ P_r(\delta) = E_2^2 Y_{11} \cos \theta_{11} + E_1 \tilde{V} Y_{12} \cos(\theta_{12} - \delta) \]

\[ E_1 + T_{d0} \dot{\delta}^2 = E_{fd} \]

\[ E_1 = E' + (X_d - X_q') I_d(\delta) \]

\[ I_d(\delta) = -E_1 Y_{11} \sin \theta_{11} - \tilde{V} Y_{12} \sin(\theta_{12} - \delta) \]

\[ \dot{D}(\delta) = \tilde{V}^2 \left[ \frac{T''_{d0} (X_d' - X_q'')}{(X_d + X_q)^2} \sin^2 \delta + \frac{T''_{q0} (X_q - X_q'')}{(X_q + X_d)^2} \cos^2 \delta \right] \]

where \( \delta \): phase angle, \( \dot{\delta} \): rotor speed, \( \tilde{M} \): inertia coefficient, \( \tilde{D}(\delta) \): damping coefficient, \( P_m \): mechanical input power, \( P_r(\delta) \): generator output power, \( \tilde{V} \): reference bus voltage, \( E_1 \): open circuit voltage, \( E_{fd} \): field excitation voltage, \( X_d \): direct axis synchronous reactance, \( X_q \): direct axis transient reactance, \( Y_{11} \): external impedence, \( Y_{12} \): self-admittance of the network, \( Y_{12} \dot{\delta} \): mutual admittance of the network, and \( I_d(\delta) \): direct axis current of the machine. Put \( x = [x_1, x_2, x_3]^T = [E_1 - \dot{E}_1, \dot{\delta}, \delta] \) and \( u = E_{fd} - \dot{E}_{fd} \), so that

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
f_1(x) \\
f_2(x) \\
f_3(x)
\end{bmatrix} +
\begin{bmatrix}
g_1(x) \\
0 \\
0
\end{bmatrix} u \quad (20)
\]

where

\[
f_1(x) = -\frac{1}{k T_{d0}^2} \left( x_1 + \dot{E}_1 \right) + \frac{(X_d - X_d') \tilde{V} Y_{12}}{k} x_3 \cos \left( \theta_{12} - x_2 - \dot{\delta} \right)
\]

\[
f_2(x) = \frac{x_3}{k}
\]

\[
f_3(x) = -\frac{\tilde{V} Y_{12}}{M} \left( x_1 + \dot{E}_1 \right) \cos \left( \theta_{12} - x_2 - \dot{\delta} \right) - \frac{Y_{11} \cos \theta_{11}}{M} \left( x_1 + \dot{E}_1 \right)^2 \frac{\tilde{D}(x)}{M x_3} + \frac{P_m}{M}
\]

\[
\dot{D}(x) = \tilde{V}^2 \left\{ \frac{T''_{d0} (X_d' - X_q'')}{(X_d + X_q)^2} \sin^2 x_2 + \frac{T''_{q0} (X_q - X_q'')}{(X_q + X_d)^2} \cos^2 x_2 \right\} + \frac{P_m}{M}
\]

\[
g_1(x) = \frac{1}{k T_{d0}^2}, \quad k = 1 + (X_d - X_d') Y_{11} \sin \theta_{11}.
\]

Assume that the constrained input is subject to

\[ u_{\min} + \dot{E}_{fd} \leq E_{fd} \leq u_{\max} + \dot{E}_{fd} \]

Parameters are

\[
\begin{align*}
M &= 0.016095 [pu] \quad T_{d0} = 5.09907 [sec] \\
\tilde{V} &= 1.0 [pu] \quad P_{in} = 1.2 [pu] \\
X_d &= 0.875 [pu] \quad X_d' = 0.422 [pu] \\
Y_{11} &= 1.04276 [pu] \quad Y_{12} = 1.03084 [pu] \\
\theta_{11} &= -1.56495 [pu] \quad \theta_{12} = 1.56189 [pu] \\
X_c &= 1.15 [pu] \quad X_q' = 0.238 [pu] \\
X_q &= 0.6 [pu] \quad X_q'' = 0.3 [pu] \\
T_{d0}' &= 0.0299 [pu] \quad T_{q0}' = 0.02616 [pu].
\end{align*}
\]

Steady state values are

\[ \dot{E}_1 = 1.52243 [pu] \quad \dot{\delta}_0 = 48.57^\circ \]

\[ \dot{\delta} = 0.0 [deg/sec] \quad E_{fd} = 1.52243 [pu]. \]

Fig. 3 Diagram of Ozeki-Power-Plant

Set \( X = [x^T, x]^T = [x_1, x_2, x_3, x_4]^T, \ n = 3, \ X_0 = 0, \ X_0 = 0 = 48.57^\circ, \ C(x) = x_2, \ L = 1, \ Q = \text{diag}(1, 1, 1, 1), \ R = 1, \ \omega_1 = \omega_2 = 1, \ P = 1, \) and \( x_4(0) = 1, \) where \( I \) is \((n+1) \times (n+1)\) unit matrix. Experiments are carried out for the new control(AACC), the automatic choosing control(ACC)[6], and the ordinary linear optimal control(LOC)[1][2].

1) AACC(\( \omega_3 = 10 \))

We experiment a case of \( \omega_3 = 10 \) and the unknown parameter subset \( \Omega = \{M, N_1, N_2, a_{ij}, b_{ij}, X_c, \eta_i\} \). To reduce the overwork of computer, we select the Taylor expansion points \( \{X_k\} \) from among candidates \( \{X_k : k = 1, \cdots, 26\} \) which are prepared from \( 55^\circ \) to \( 180^\circ \) at intervals of \( 5^\circ \). Put \( X_0 = 48.57^\circ, \ a_0 = -\infty \) and \( b_M = \infty \). Set \( \sigma_0 = \sigma_1 = \cdots = \sigma_{26} = 0.3262 \) at \( 3 \) (because \( \min[\sigma_{pm} : 0 \leq i \leq 26] = 0.3262 \)) using Appendix. The parameters are suboptimally selected along the algorithm of section 3, where \( G = 100, \ \tilde{g} = 100, \ L = 8, \ P_{in} = 0.8, \ P_m = 0.03, \ D = [-0.5, 0.5] \times [-0.2, 0.2] \times [0, 1.0] \). The constrained input value is \( u_{\max} = -u_{\min} = 0.5 \). As a result, we have that \( M = 3, N_1 = 4.91, N_2 = 1.21, a_1 = b_0 = 54.1^\circ, a_2 = b_1 = 113.0^\circ, a_3 = b_2 = 171.7^\circ, \ X_1 = 55^\circ, \ X_2 = 145^\circ, \ X_3 = 180^\circ \) and \( \eta_1 = \eta_2 = 1.11 \).

2) AACC(\( \omega_3 = 100 \))

The parameters are suboptimally selected by using a similar way of the AACC(\( \omega_3 = 10 \)) under \( \omega_3 = 100 \). As a result, we have that \( M = 2, N_1 = 4.76, N_2 = 2.94, a_1 = b_0 = 53.3^\circ, a_2 = b_1 = 90.3^\circ, \ X_1 = 70^\circ, \ X_2 = 115^\circ \) and \( \eta_1 = \eta_2 = 1.44 \).

3) AACC(\( \omega_2 = 0 \))
The parameters are suboptimally selected by using a similar way of the AACC(ω3 = 10) under ω2 = 0, which does not include the differential coefficient of the adjoint vector in Eq.(14). As a result, we have that M = 3, N1 = 7.18, N2 = 1.56, a1 = b0 = 53.5°, a2 = b1 = 147.5°, a3 = b2 = 177.8°, Ẋ1 = 60°, Ẋ2 = 175°, Ẋ3 = 180° and η1 = η2 = 8.22.

4) ACC:

The parameters are suboptimally selected by using a similar way of the AACC(ω3 = 0) under the same condition as it when Ω={M, N, aij, bji, X1}. As a result, we have that M = 1, N = 7.0, a1 = b0 = 64.8° and Ẋ1 = 75°.

Table 1 shows performances by the AACC, the ACC and the LOC. The cost function of Table 1 is

\[ J = \frac{1}{2} \int_0^{t_0} (X^T Q X + u^T R u) \, dt. \]

Figs. 4 and 5 show the responses in case of \( x^T(0) = [0, 1, 0, 0] \). Figs. 6 and 7 show the responses in case of \( x^T(0) = [0, 1, 1, 4, 6, 0] \). These results indicate that the AACC(ω3 = 10, 100) with constraint input is better than the AACC(ω2 = 0), ACC and LOC.

V. CONCLUSIONS

We have studied an augmented automatic choosing control using zero dynamics for nonlinear systems with constrained input. This approach has been applied to a field excitation control problem of power system. Simulation results have shown that the new controller is able to improve performance remarkably well. The followings are left for the future works: problem of optimum selection of \( \sigma_i \) and \( \omega_i \), application to more complicated systems such as multi-machines power systems[7].

REFERENCES


APPENDIX

In the AACC, we make a linear approximation system in (6): \( f(x) \approx A_i x + w_i x [n+1] \) by using an approximation of Taylor expansion: \( f(x) \approx A_i x + w_i \).

Thus we would like to keep \( x[n+1] \approx 1 \), namely \( x[n+1] \in R \ (x[n+1] \approx 1) \) changes slower than the state vector \( x \in R^n \) on \( C^{-1}(D_i) \subset D(0 \neq 0) \). Whenever \( x \in R^n \) enters into \( C^{-1}(D_0) \) which has the steady state point, this AACC almost becomes the ordinary LQ control. That is, stay \( x[n+1] \approx 1 \) for a good while except on \( C^{-1}(D_0) \). We shall show how to do it.

Substituting (8) into (7) yields

\[ \dot{X} = (A_i - B_i R^{-1} B_i^T P_i) X. \] (21)

Assume the controllability of the linear feedback system described by (21). We define \( \lambda_{ik} (0 \leq i \leq M, 1 \leq k \leq n+1) \) being eigenvalues of \( (A_i - B_i R^{-1} B_i^T P_i) \):

\[
\begin{vmatrix}
A_i - B_i R^{-1} B_i^T P_i - \lambda_{ik} I
\end{vmatrix} = \begin{vmatrix} A_i - B_i R^{-1} B_i^T P_i - \lambda_{ik} I \quad w_i \\
0 & -\sigma_i - \lambda_{ik} \end{vmatrix} = (-\sigma_i - \lambda_{ik}) \left| (A_i - B_i R^{-1} B_i^T P_i) - \lambda_{ik} I \right| = 0
\text{ (22)}
\]

where \( | \cdot | \) denotes determinant, and \( P_i = P_i^T > 0 \) is a solution of the Riccati equation: \( P_i A_i + A_i^T P_i + Q - P_i B_i R^{-1} B_i^T P_i = 0 \).

The asymptotically stable condition of (21) is

\[ Re(\lambda_{ik}) < 0 \text{ or } Re(-\lambda_{ik}) > 0 \text{ for all } \lambda_{ik} \]

where \( Re(\cdot) \) denotes the real part.

Define \( \sigma_{im} \) as the minimal value of \( \{Re(-\lambda_{ik})\} \) for \((A_i - B_i R^{-1} B_i^T P_i)\) by \( \sigma_{im} = \min Re(-\lambda_{ik} : 1 \leq k \leq n+1) \).

That is, these \( \lambda_{ik} \) are the solution of \((A_i - B_i R^{-1} B_i^T P_i) - \lambda_{ik} I = 0 \) from (22).

We should be able to select \( \sigma_i \) of (3) from \( \sigma_i \in (0, \sigma_{im}) \subset R \) properly.
### Table I

<table>
<thead>
<tr>
<th>Method</th>
<th>$x^T(0)$</th>
<th>[0, 0.6, 0]</th>
<th>[0, 1.0, 0]</th>
<th>[0, 1.4, 0]</th>
<th>[0, 1.446, 0]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOC</td>
<td>2.587</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>ACC</td>
<td>2.096</td>
<td>2.388</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>AAC($\omega_2 = 0$)</td>
<td>1.991</td>
<td>2.172</td>
<td>2.705</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>AAC($\omega_2 = 10$)</td>
<td>1.995</td>
<td>2.178</td>
<td>2.665</td>
<td>3.096</td>
<td>x</td>
</tr>
<tr>
<td>AAC($\omega_3 = 100$)</td>
<td>1.997</td>
<td>2.182</td>
<td>2.705</td>
<td>3.050</td>
<td>4.988</td>
</tr>
</tbody>
</table>

$x$ : very large value

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**Fig. 4** Responses of LOC, ACC, AAC($\omega_2 = 0$) ($x^T(0) = [0, 1.4, 0]$)

**Fig. 5** Responses of AAC($\omega_3 = 10, 100$) ($x^T(0) = [0, 1.4, 0]$)

**Fig. 6** Response of AAC($\omega_3 = 10$) ($x^T(0) = [0, 1.446, 0]$)

**Fig. 7** Response of AAC($\omega_3 = 100$) ($x^T(0) = [0, 1.446, 0]$)