

# The System Identification and PID Lead-lag Control for Two Poles Unstable SOPDT Process by Improved Relay Method

V. K. Singh and P. K. Padhy

**Abstract**—This paper describes identification of the two poles unstable SOPDT process, especially with large time delay. A new modified relay feedback identification method for two poles unstable SOPDT process is proposed. Furthermore, for the two poles unstable SOPDT process, an additional Derivative controller is incorporated parallel with relay to relax the constraint on the ratio of delay to the unstable time constant, so that the exact model parameters of unstable processes can be identified. To cope with measurement noise in practice, a low pass filter is suggested to get denoised output signal to improve the exactness of model parameter of unstable process. PID Lead-lag tuning formulas are derived for two poles unstable (SOPDT) processes based on IMC principle. Simulation example illustrates the effectiveness and the simplicity of the proposed identification and control method.

**Keywords**—IMC structure, PID Lead-lag controller, Relay feedback, SOPDT

## I. INTRODUCTION

TIME delayed unstable process commonly encountered in process industries and difficult to control due to right-half plane (RHP) poles. A time delay is introduced into the transfer function description of such system due to the measurement delay or an actuator delay or by the approximation of higher dynamics of the system by that of a lower order plus delay systems. Stabilization and control of unstable process with time-delay are one of the most challenging tasks in the industry. Many chemical processes, such as heating boilers and continuous-stirred-tank reactors (CSTRs), have integrating or unstable dynamics by nature. Model-based control strategies have been developed for such processes with effective set-point tracking and disturbance rejection. For the purpose of modeling of system, relay feedback identification has attracted increasing attention because it is such a closed loop exercise that the process response does not drift far away from the set-point during the test. A theory of relay control systems based on the concepts of transfer functions and frequency characteristics is presented by Tsytkin [1] that helps to develop computational methods to analyze general properties of relay control systems. A more practical approach could combine one of the model-based methods with a robust and reliable identification method, The well-known relay feedback

method, proposed by Astrom and Hagglund [2] in which ultimate gain and frequency are obtained using a closed loop relay test, strictly applies only to open-loop stable SISO processes having high-frequency phase lags greater than  $-180^\circ$ . Kaya and Atherton [3] proposed identification methods for integrating unstable processes in terms of the so called A-locus analysis. Ananth and Chidambaram [4] and Marchetti, Scali, and Lewin [5] have been also proposed relay based identification methods for unstable processes.

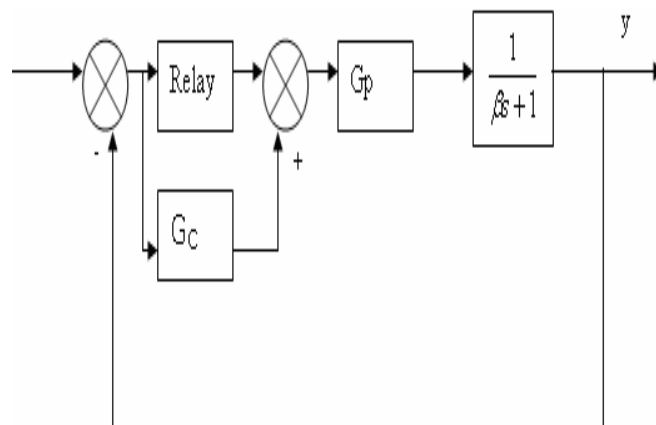


Fig. 1 Block diagram for improved relay feedback system.

In this paper improved relay identification method for open-loop two poles unstable process is proposed. The improved relay identification method is extended up to open-loop unstable SOPDT process and the exact expressions in term of the periods and amplitudes of limit cycles under relay feedback test are derived for two poles unstable SOPDT model. Three parameters of unstable SOPDT model can be identified by means of a single relay test and without any prior information about the dead time or the static gain. An additional  $G_c$  controller in form of derivative ( $sK_d$ ) controller is incorporated parallel with the relay feedback system in fig. 1. to relax the constraint on the ratio of delay to the unstable time constant. The proposed method enables the relay to give a sustained oscillation of the process output for  $\theta/T < .693/K_p$ . Low pass filter is suggested to deal with measurement noise.

## II. IMPROVED RELAY FEEDBACK TEST

The merits of the relay feedback identification method proposed by Astrom and Hagglund [2] is that the auto tuning process is carried out under a closed loop and the whole

V. K. Singh is with the Venkateshwar Institute of Technology, Indore, M.P. 453331 India (e-mail: vinaysi@iiitdmj.ac.in).

P. K. Padhy is with the PDPM Indian Institute of Information Technology, Design & Manufacturing, Jabalpur, M. P. 482005 India (e-mail: prabin16@iiitdmj.ac.in).

system is still in normal operation near operating points, which can not only keep the system work in a good order but also overcome the influence on parameter tuning due to the nonlinearity of the real process. Here some modification is shown in Fig.1 done to relay method in this paper so that it can be more suitable for long time delay process identification. Relay feedbacks are introduced into the control system, in which one relay feedback is followed by a derivative element. Then the limit cycle information of the controlled process located at  $\omega_{90}$  and  $\omega_{180}$  in Fig.2 can be obtained, respectively. The presence of large time delay leads to the decrease of the process corner frequency and the oscillation frequency will decrease consequently. The identification accuracy of the relay feedback to long time delay process can be improved by incorporating a derivative element after the relay feedback system.

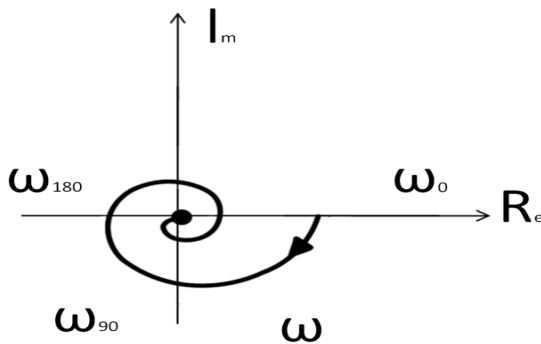


Fig. 2 The Nyquist curve of the controlled process

### III. IDENTIFICATION OF PROCESS MODEL

The identification of stable process can be performed easily using the conventional relay feedback technique. Unlike the usual situation for a simple stable process transfer function under relay control, the unstable two poles SOPDT process does not experience a limit cycle. In this section, a new identification method is proposed. figure 1. is rearranged as shown in figure 3. according to block diagram reduction technique. That depicts how a process is subjected to the D-derivative controller ( $G_C$ ) in the inner feedback path during the relay test. The inner loop helps in stabilizing the open-loop unstable two poles SOPDT processes particularly.

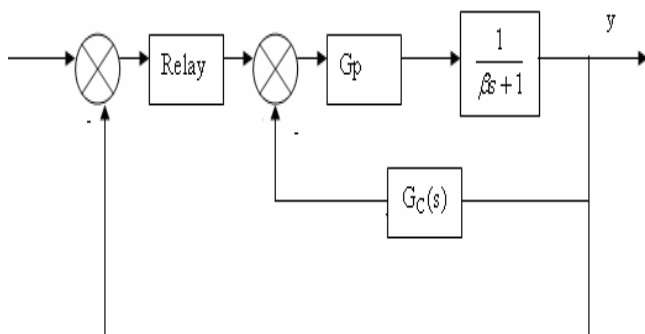


Fig. 3 Equivalent structure of figure 1

Let  $G_C$  controller be

$$G_C(s) = sK_d \quad (1)$$

and transfer function model of SOPDT process be

$$G_P(s) = \frac{K_p e^{-\theta s}}{(T_1 s - 1)(T_2 s - 1)} \quad (2)$$

The describing function of the ideal relay is

$$N = \frac{4h}{\pi A} \quad (3)$$

where h is the amplitude of the relay function and A is the peak amplitude of the output oscillation. The describing function analysis indicates that the closed-loop relay system will oscillate at the frequency  $\omega_c$  and  $\omega_b$  where two non-zero frequency points

$$NG(\omega_c) = -1 \quad NG(\omega_b) = -jK_{90} \quad (4)$$

and a low pass filter combined with process become:

$$G_{PF}(s) = \frac{K_p e^{-\theta s}}{(T_1 s - 1)(T_2 s - 1)(\beta s + 1)} \quad (5)$$

For the parameter identification of model  $G_P(s)$ , suppose the frequency characteristic of  $G_P(s)$  at the point of  $\omega_{90}$  and  $\omega_{180}$  in Fig.2 is equal that of:

$$G_{PF}(\omega_c) [N + G_C(\omega_c)] = -1 \quad (6)$$

$$G_{PF}(\omega_b) [N + G_C(\omega_b)] = -jK_{90} \quad (7)$$

Then eqn.6 and 7 become:

$$\frac{K_p e^{-\theta \omega c}}{(T_1 j \omega c - 1)(T_2 j \omega c - 1)(\beta j \omega c + 1)} \left[ \frac{4h}{\pi A} + j \omega_c K_d \right] = -1$$

$$\frac{K_p e^{-\theta \omega c}}{(T_1 j \omega c - 1)(T_2 j \omega c - 1)(\beta j \omega c + 1)} [x + jy] = -1 \quad (8)$$

$$\text{where } x = \frac{4h}{\pi A} \quad y = \omega_c K_d$$

$$\frac{K_p e^{-\theta \omega b}}{(T_1 j \omega b - 1)(T_2 j \omega b - 1)(\beta j \omega b + 1)} \left[ \frac{4h}{\pi A} + j \omega_b K_d \right] = -jK_{90}$$

$$\frac{K_p e^{-\theta \omega b}}{(T_1 j \omega b - 1)(T_2 j \omega b - 1)(\beta j \omega b + 1)} [x + jz] = -jK_{90} \quad (9)$$

$$\text{where } x = \frac{4h}{\pi A}, \quad z = \omega_b K_d$$

Simplify equation (8) and (9) we get this kind of equation

$$-T_1 T_2 \omega c^2 - j \omega c (T_1 + T_2) + 1 =$$

$$\frac{-(1 + j\beta\omega c)}{(x + jy)(\cos(\theta\omega c) - j\sin(\theta\omega c)Kp)} \quad (10)$$

$$\begin{aligned} & -T_1T_2\omega b^2 - j\omega b(T_1+T_2) + I = \\ & \frac{-j(1 + j\beta\omega b)K_{90}}{(x + jz)(\cos(\theta\omega b) - j\sin(\theta\omega b)Kp)} \quad (11) \end{aligned}$$

If we assume that

$$A = \frac{-(1 + j\beta\omega c)}{(x + jy)(\cos(\theta\omega c) - j\sin(\theta\omega c)Kp)}$$

$$B = \frac{-j(1 + j\beta\omega b)K_{90}}{(x + jz)(\cos(\theta\omega b) - j\sin(\theta\omega b)Kp)}$$

Then equation (10) and (11) becomes:

$$-T_1T_2\omega c^2 - j\omega c(T_1+T_2) = A - I \quad (12)$$

$$-T_1T_2\omega b^2 - j\omega b(T_1+T_2) = B - I \quad (13)$$

For solving the value of  $T_1$  and  $T_2$ , we again assume that

$V = T_1T_2$   $U = (T_1 + T_2)$  and now solve the eqn. for  $V$  and  $U$  then the values of

$$V = \frac{(\omega b - \omega c) + (B\omega c - A\omega b)}{\omega c\omega b(\omega c - \omega b)} = T_1T_2 \quad (14)$$

$$U = \frac{(\omega b^2 - \omega c^2) + (B\omega c^2 - A\omega b^2)}{j\omega c\omega b(\omega b - \omega c)} = T_1 + T_2 \quad (15)$$

$$\text{Then } T_1 - T_2 = \pm \sqrt{U^2 - 4V}$$

Now we can solve for value  $T_1$  and  $T_2$  and we get the value of:

$$T_1 = 1/2 \left( \frac{(\omega b^2 - \omega c^2) + (B\omega c^2 - A\omega b^2)}{j\omega c\omega b(\omega b - \omega c)} + \sqrt{U^2 - 4V} \right)$$

$$T_2 = 1/2 \left( \frac{(\omega b^2 - \omega c^2) + (B\omega c^2 - A\omega b^2)}{j\omega c\omega b(\omega b - \omega c)} - \sqrt{U^2 - 4V} \right)$$

and value of delay time  $\theta$  is calculated with the help of

$$f(\theta) = 2\sqrt{U^2 - 4V}$$

#### IV. RELAY FEEDBACK WITH INNER DERIVATIVE CONTROLLER

It is often desirable to be able to carry out a relay feedback test for the unstable FOPDT process with  $\theta/T \leq 0.693$  in this paper for two poles unstable process D-derivative feedback is proposed for a relay auto tuning test to give new constraint.

Let the dynamic model of a unstable FOPDT plant be represented by

$$G_1(s) = \frac{Kpe^{-\theta s}}{(Ts - 1)} \quad (16)$$

Under the limit cycle condition for relay feedback identification for unstable FOPDT Process.

$$N G_1(j\omega c) = -I \quad (17)$$

$$N \frac{Kpe^{-j\omega\theta}}{(Tj\omega - 1)} = -I \quad (18)$$

Compared with equation (8) becomes

$$\begin{aligned} N \frac{Kpe^{-j\omega\theta}}{(Tj\omega - 1)} = \\ \frac{Kpe^{-j\omega\theta}}{(T_1j\omega c - 1)(T_2j\omega c - 1)(\beta j\omega c + 1)} [N + j\omega c K_d] \end{aligned}$$

Comparison of magnitude part of both side of equation then we get.

$$T = \sqrt{T_1^2 + T_2^2 + \beta^2 - \frac{Kd^2}{N^2}} \quad (19)$$

This paper proposed new constraint for relay feedback identification for two poles unstable SOPDT process

$$\frac{\theta}{\sqrt{T_1^2 + T_2^2 + \beta^2 - \frac{Kd^2}{N^2}}} \leq \frac{0.693}{Kp} \quad (20)$$

where

$$Kd \sqrt{N^2(T_1^2 + T_2^2 + \beta^2)} \quad (21)$$

#### V. AUTOTUNING STRUCTURE

A complete design procedure for auto tuning structure includes two parts in figure 4. Model identification and controller structure. Identification part of model parameter has been already done in above and controller structure has been proposed in previous paper [6]. That controller used as PID controller with series Lead-lag filter for autotuning of process.

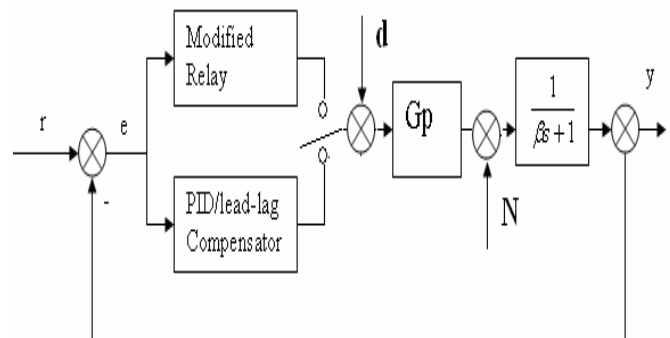


Fig. 4 Auto tuning structure for two poles unstable SOPDT process

PID Lead-lag parameters [6] values are following.

$$C_{PID}(s) = K_C (1 + T_d s + I/T_I * s) \frac{(1 + as)}{(1 + 0.1as)}$$

Where

$$K_C = \frac{\alpha_1}{K_p(\theta + 4\lambda - \alpha_1)} \quad (22)$$

$$T_I = \alpha_1 \quad (23)$$

$$T_d = \frac{\alpha_2}{\alpha_1} \quad (24)$$

$$a = (\beta + \theta/2) \quad (25)$$

### VI. SIMULATION RESULT

The application of the identification approach is demonstrated next in example. The example considers a typical two poles unstable SOPDT process in this section for the simulation studies show that robustness and effectiveness of the new modified relay with ( $G_c=3s$ ) derivative controller for identification method, a low-pass filter ( $\beta = .0483$ ) has been suggested [7] to recover the corrupted limit cycle data from measurement noise shown in figure 5 to figure 6.

$$G_p = \frac{2e^{-0.3s}}{(3s-1)(s-1)}$$

This example also illustrate PID Lead-lag controller auto-tuning with the proposed method under a load disturbance shown in figure 7. at  $t=20s$ , a step load disturbance with an amplitude of 1 enters at the input into the process same as previous paper [6]. Note that the limit cycle oscillation with the basic method becomes asymmetrical while the new method regains the symmetry automatically after an initial transient following the occurrence of the disturbance signal.

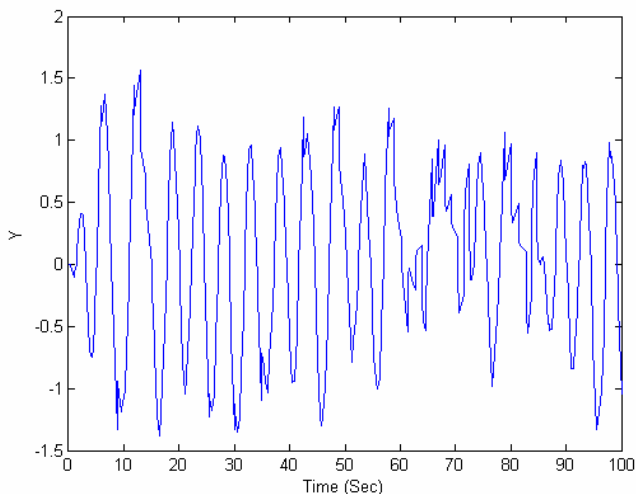


Fig. 5 Limit cycle output for two pole unstable SOPDT process without low pass filter

A set point filters used [8] in figure 4. To getting robust and smooth response compared with previous proposed method

The proposed method shows excellent control performances in terms of the overshoot, speed of response and settling time in figure. 7. The main advantage of this control method is that the inner feedback Dderivative controller manipulates the stabilization problem.

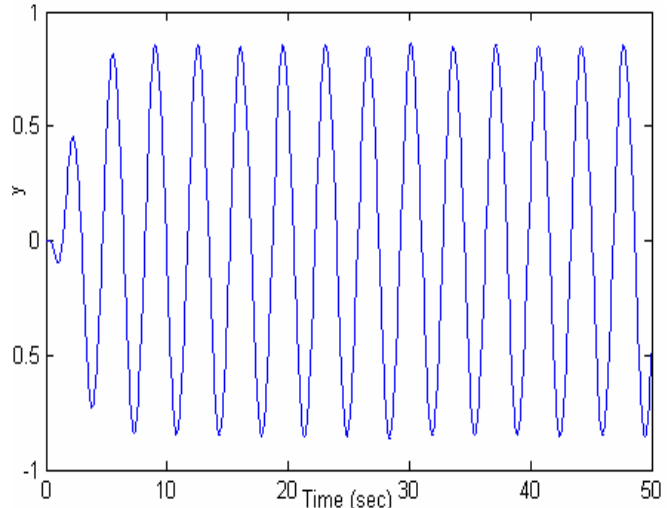


Fig. 6 Limit cycle output for two pole unstable SOPDT process

### VII. CONCLUSION

The new modified relay identification methods for open loop unstable process are presented, extensive simulation have shown that the proposed methods are accurate and feasible with no prior knowledge. Furthermore, the proposed relay feedback method with inner D-derivative controller can identify the two poles unstable processes with time delay. The advantages of this method are single relay feedback test and parameters identification of two poles unstable process. This not only establishes the basis for the PID controller design but also provides a convenient approach for its tuning.

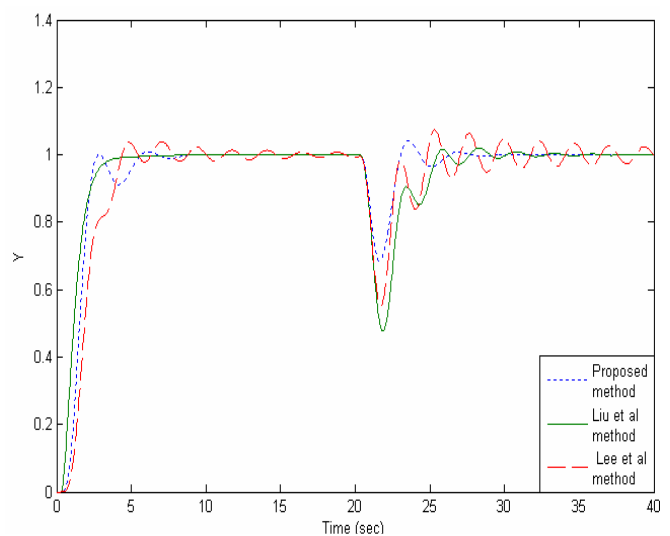


Fig. 7 Unit step output responses with PID Lead-lag controller

#### REFERENCES

- [1] Y. Z. Tsyppkin, "Relay control systems," Cambridge University Press, 1984.
- [2] K. J. Astrom and T. Hagglund, "Automatic tuning of simple regulators with specifications on phase and amplitude margins," *Automatica*, vol. 20, pp. 645-651, 1984.
- [3] I.Kaya, "Parameter estimation for integrating processes using relay feedback control under static load disturbances," *Industrial and Engineering Chemistry Research*, vol. 45, pp. 4726-4731, 2001.
- [4] M. Chidambaram and I. Ananth, "Closed loop identification of transfer function model for unstable process," *Journal of the Franklin Institute*, 336, 1055, 1999.
- [5] G. Marchetti, C. Scali and D. R. Lewin, "Identification and control of open loop unstable processes by relay methods," *Automatica*, 37, 2049, 2001
- [6] V. K. Singh and P. K. Padhy, "IMC Based PID Controller Tuning for Unstable SOPDT Processes," *Proc. PEIE*, vol. ccis 102, pp. 110-114, 2010.
- [7] P. K. Padhy and S. Majhi, "IMC based PID controller for FOPDT stable and unstable processes," *Proc. of 30th National System Conference*, Dona Paula, Goa 2006
- [8] Y. Lee, J. Lee and S Park, "PID controller tuning for integrating and unstable processes with time delay," *Chemical Engineering Science*, vol. 55, pp. 3481-3493, 2000.