Mathematical Modelling of Single Phase Unity Power Factor Boost Converter
Sanjay L. Kurkute, Pradeep M. Patil, Kakasaheb C. Mohite

Abstract—An optimal control strategy based on simple model, a single phase unity power factor boost converter is presented with an evaluation of first order differential equations. This paper presents an evaluation of single phase boost converter having power factor correction. The simple discrete model of boost converter is formed and optimal control is obtained, digital PI is adopted to adjust control error. The method of instantaneous current control is proposed in this paper for its good tracking performance of dynamic response. The simulation and experimental results verified our design.

Keywords—Single phase, boost converter, Power factor correction (PFC), Pulse Width Modulation (PWM).

I. INTRODUCTION

Single-phase switch mode ac-dc converters are being used as front-end rectifiers for a variety of applications due to the advantages of high efficiency and power density. These classical converters, however, draw non-sinusoidal input ac currents leading to low input power factors and injection of harmonics into the utility lines. Research in improved power quality utility interface has gained importance due to stringent power quality regulation and strict limits on total harmonic distortion (THD) of input current placed by standards such as IEC 61000-3-2 and IEEE 519-1992. This has led to consistent research in the various techniques for power quality improvement. Research into passive and active techniques for input current wave shaping has highlighted their inherent drawbacks. Passive filters have the demerits of fixed compensation, large size and resonance whereas the use of active filters is limited due to added cost and control complexity.

Research into switch mode power factor corrected ac-dc converters has been in two directions namely buck and boost type topologies. The advantage of Buck type topology is that the converter can provide variable output voltage, which is lower than the input voltage. However, when the instantaneous input voltage is below the output voltage the current drops to zero and the results in significant increase in input current THD.

Design of input filters for power factor improvement in buck converters is therefore complex and provides only limited improvement in input current quality. On the other hand the boost type converter generates dc voltage, which is higher than the input ac voltage. However, the input current in these converters flows through the inductor and therefore can easily be actively wave-shaped with appropriate current mode control. Moreover, boost converters provide regulated dc output voltage at unity input power factor and reduced THD of input ac current. These converters have found widespread use in various applications due to the advantages of high efficiency, high power density and inherent power quality improvement at ac input and dc output. The preferred power circuit configuration of single-phase boost converter [1-9] is the most popular and economical PFC converter consisting of diode bridge rectifier with step-up chopper. The related research work is addressed in subsequent work [10-14].

This paper provides a study of single phase boost converter topology. Complete mathematical modeling of PFC converters is carried out. Simulation results are provided for evaluation of converter performance under steady state and dynamic conditions and performance of single-phase boost converter is then experimentally verified.

II. CIRCUIT CONFIGURATION

PFC converter topology considered in this work is described in this section. The single-phase boost converter with unidirectional power flow shown in Figure 1 is realized by cascading single-phase diode bridge rectifier with boost chopper topology.

III. CONTROL SCHEME

The objective of the control scheme of the boost converters is to regulate the power flow ensuring tight output voltage regulation as well as unity input power factor. Cascaded control structure shown in Figure 2 is the most extensively used control scheme for the converter. In this scheme, the output of voltage regulator, limited to a safe value, forms the amplitude of input reference current. This reference amplitude
is then multiplied to a template of input voltage to synchronize the reference with input voltage, as required for unity power factor operation. The inductor current is forced to track its reference current using current controller, which generates appropriate gating signals for the active device(s).

IV. MATHEMATICAL MODELLING OF PFC CONVERTER

The proposed PFC converter system comprises single-phase ac supply, power converter circuit, and control scheme. In this section modeling equations of various components of the converter system are formulated separately to develop a comprehensive model for their performance simulation.

Supply System:
Under normal operating conditions the supply system can be modeled as a sinusoidal voltage source of amplitude $V_m$ and frequency $f_s$. The instantaneous voltage is given as:

$$V_s(t) = V_m \sin \omega t$$

where $\omega = 2\pi f_s$ electrical rad/s and $t =$ instantaneous time.

From sensed supply voltage, a template $u(t)$ is estimated for converter topologies with ac side inductor.

$$u(t) = \frac{V_s(t)}{V_m}$$

For converter topologies with dc side inductor, $u(t)$ is obtained from:

$$u(t) = \frac{V_s(t)}{V_m}$$

DC Voltage Controller:
The objective of dc voltage controller is described earlier. A proportional integral (PI) voltage controller is selected for voltage loop for tight regulation of the output voltage. The dc voltage $V_{dc}$ is sensed and compared with set reference voltage $V_{dc}^*$. The resulting voltage error $V_e(n)$ at nth sampling instant is:

$$V_e(n) = V_{dc}^* - V_{dc}(n)$$

Output of PI voltage regulator $V_o(n)$ at nth sampling instant is:

$$v_o(n) = v_o(n-1) + K_p(v_e(n) - v_e(n-1)) + K_i v_e(n)$$

where $K_p$ and $K_i$ are the proportional and integral gain constants.

$v_e(n-1)$ is the error at the $(n-1)^{th}$ sampling instant.

The output of the controller $v_o(n)$ after limiting to a safe permissible value is taken as amplitude of reference supply current A.

PWM Current Regulator:
Current regulation loop is required for active wave shaping of input current to achieve unity input power factor and reduced harmonics.

Reference Supply Current Generation:
The input voltage template $B$ obtained from sensed supply voltage is multiplied with the amplitude of reference source current $A$ in the multiplier-divider circuit. Moreover, a component of input voltage feed forward $C$ is also added to improve the dynamic response of the converter system to line disturbances [Figure 2]. The resulting signal forms the reference for input current. The instantaneous value of the reference current is given as:

$$i_s = \frac{ABC^*}{2}$$

Active Wave-shaping of Input Current:
The inductor current error is the difference of reference supply current and inductor current ($i_{en} = i_{s}^* - i_s$)

This error signal is amplified and compared to fixed frequency carrier wave to generate gating signals for power devices of the converter. PWM switching algorithm is selected depending on the converter topology.

MODELLING OF SINGLE-PHASE BOOST PFC CONVERTER:
The converters are modeled using first order non-linear differential equations. The number of equations is equal to the number of energy storage components in the system. The boost converter is modelled using two differential equations for inductor current $i_L$ and dc link capacitor voltage $V_{dc}$.

$$pi_L = \frac{(V_d - V_p)}{(L - r(\omega^2))}$$

![Fig. 2 Control scheme of PFC converter](image)
\[ PV_{dc} = \frac{(i_p - \frac{V_p}{C_d})}{C_d} \]  \hspace{1cm} (8)

where \( p \) = differential operator (d/dt),
\( r \) = the resistance of the inductor \( L \),
\( V_d \) = the rectified line voltage at diode rectifier output,
\( R \) = resistance of the load and
\( V_p \) = PWM voltage across the switch

and is defined as
\[ V_p = V_{dc}(1.\delta) \]  \hspace{1cm} (9)

\( i_p \) is the current through the boost diode and is defined as
\[ i_p = i_L(1.\delta) \]  \hspace{1cm} (10)

where \( \delta \) is the switching signal obtained from current regulation loop. Its value is 1 (ON) or 0 (OFF) depending upon the state of the switch.

Performance Characteristics:
Performance simulation of converters modelled and described in the previous section is carried out for different loading conditions at 100-kHz switching frequency. The values of inductor and capacitor are calculated for desired input current ripple and output voltage ripple. A summary of performance evaluation is presented in Table 1.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>EXPERIMENTAL RESULTS OF SINGLE PHASE BOOST CONVERTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>TDH (%)</td>
<td>Power Factor</td>
</tr>
<tr>
<td>Heavy Load</td>
<td>Light Load</td>
</tr>
<tr>
<td>0.3917</td>
<td>1.0572</td>
</tr>
</tbody>
</table>

Steady-State and Dynamic Performance: Topology considered in this work, provide smooth dc voltage at a power factor close to unity and show excellent steady state (Figures 2). Input current THD is well below the limits stipulated by IEC61000-3-2 and other standards. These converters exhibit satisfactory voltage regulation at load variations from 325 W to 1625 W of nominal. This makes these converters suitable for applications with significant load variation. Half-bridge converter and voltage source converter exhibit best characteristics in terms of dc voltage regulation and input current THD. Input current distortion at zero crossovers is also non-existent in these converters.

Efficiency:
Voltage drops across semi-conductor devices can have significant effect on the overall efficiency of the converters. Number of voltage drops across single-phase boost converter is three.

Application Potential of PFC Converter:
Boost Converter is most economical and optimal converter in terms of performance and efficiency and provides unidirectional power flow. This converter is used as power factor pre-regulators for power supplies; electronic ballast and low power drive applications where bi-directional power flow is not required.

IV. EXPERIMENTAL TESTING OF SINGLE PHASE BOOST CONVERTER

Performance of single-phase boost converter is experimentally tested to identify the numerous non-topological factors that can impact the quality of current drawn by these converter as well as to develop greater confidence in the simulation studies carried out in the paper.

A Texas Instrument IC-UC3854A power factor controller based implementation is carried out for a 500 W, universal line (80 V-270 V) single-phase boost converter. The power circuit is fabricated with IRFP460 MOSFET and MUR460 fast recovery diode with \( L_s = 1.1 \) mH and \( C_d = 560 \) mF. The converter is fed from ac lines via an auto transformer followed by an isolation transformer to provide variable input voltage and protection, respectively. Figure 3 shows the distortion in the input voltage caused by distorted current when the system is operated as an uncontrolled diode bridge rectifier. Harmonic current interacting with the source impedance, primarily due to utility lines and transformers, causes this distortion. This distortion is corrected by triggering the power factor controller and wave shaping the input current into desired sinusoid as shown in Fig. 4.
Rectified line voltage and current is presented in Photo 1. The input current and voltage waveforms shown in Photo 2 matches with the simulation results shown in Figure 4. This validates the mathematical model as well as simulation tools used in the comparative evaluation.

The input current THD of the boost converter is slightly lower under simulated conditions as several factors like nonlinearity in multiplier, source impedance, deviation of input voltage from desired sinusoidal shape, addition of EMI filter, tolerances in passive components and limited bandwidth and non-linearity in sensor circuits were neglected in developing the mathematical model of the converter for performance simulation. These factors have a very local influence and can vary significantly in different systems as well as utility connections. Moreover, incorporating these factors in mathematical model can complicate analysis and simulation.

VI. CONCLUSION

Modelling and simulation of PFC converter is carried out and is experimentally tested to verify the simulation results. From the study it can be inferred that single-phase boost topology is optimal in terms of performance, efficiency, cost and power density and is therefore suitable for most applications.

REFERENCES