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Numerical investigation of two-dimensional boundary layer flow over a moving surface

Mahmoud Zarrini and R.N. Pralhad

Abstract—In this chapter, we have studied Variation of velocity in incompressible fluid over a moving surface. The boundary layer equations are on a fixed or continuously moving flat plate in the same or opposite direction to the free stream with suction and injection. The boundary layer equations are transferred from partial differential equations to ordinary differential equations. Numerical solutions are obtained by using Runge-Kutta and Shooting methods. We have found numerical solution to velocity and skin friction coefficient.

Keywords—Boundary layer, continuously moving surface, shooting method, skin friction coefficient.

I. Introduction

STUDIES on Boundary layer flows have paved the ways in understanding the wakes or eddies effects on the structure and effective velocity within the zone of boundary layer [1]. Applications of these studies in particular are of high speed flows [fighter air crafts, spaceships and so forth], Pollutants emitting form refineries and its effects on the habitants living around, industrial flows in nuclear reactors cross sectional flows and so forth. In view of its importance boundary layer flow over a moving surface has been taken up in the present chapter with a view that, the present studies has direct relevance over environmental aspect. In the proposed study, we investigate development of the boundary-layer viscous flow on a fixed on continuously moving flat plate in the same or opposite direction to the free stream with suction and injection. The study undertaken is of typical Blasius [2] equation which was proposed by Blasius way been 1908. The present model has an extension to the stream moving with flat plate in the same or opposite direction with stream has suction and injection facility in the flow field. These are various [analytical / numerical] methods ([3], [4] and [5]) have been proposed for finding solution to the problem undertaken. However in the present chapter, we have used shooting method with Runge-Kutta fourth order numerical scheme has been adopted. The advantage of the present model over earlier models has been highlighted.

II. Analysis

Consider a steady and laminar two dimensional flow of viscous fluid through a moving flat plate with constant velocity U_w and free stream is with constant velocity U_∞. The y-axis is normal to the surface and x-axis is parallel to the surface (Fig. 1).

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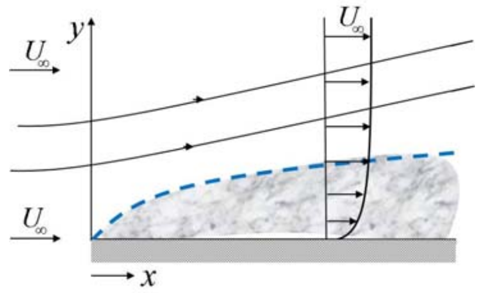

Fig. 1. Flow dyagram

The boundary layer equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho \left(u\,\frac{\partial u}{\partial x} + v\,\frac{\partial v}{\partial y} \right) = \mu\,\frac{\partial^2 u}{\partial y^2} \tag{2}$$

The boundary conditions used are:

$$u = U_w\ ,\ v = V_w \quad at \quad y = 0$$

$$u \to U_\infty \ as \quad y \to \infty \tag{3}$$

Where u , v are velocity components in direction of x , y directions, respectively, V_w is the mass transfer at the surface of the plate, ρ is density, μ is viscosity.

In order to, solve Equations (1) and (2) subjected to Eq. (3), according to Schlichting [1], We consider stream function $\psi(x, y)$, then The mass conservation equation is satisfied :

$$u = \frac{\partial \psi}{\partial y}\ ,\ v = -\frac{\partial \psi}{\partial x}$$

And we are using some transforms that, introduced by Blasius [2]:

$$\eta = \sqrt{\frac{U}{\nu x}}\ y\ ,\ \psi(x, y) = \sqrt{\nu x U} f(\eta) \tag{4}$$

We are taking $U = U_w + U_\infty$, ν is Kinematic viscosity, η is the independent dimensionless variable and $f(\eta)$ dimensionless function. By using Eq. (4), we obtain:

$$u = U\ f^{'}(\eta)\ ,\ v = \frac{1}{2}\sqrt{\frac{\nu U}{x}}(\eta f^{'} - f) \tag{5}$$

and then

$$V_w(x) = \frac{1}{2}\sqrt{\frac{\nu U}{x}} f_w \tag{6}$$

where $f_w = f(0)$ and it is a non-dimensional constant. Equation of (2) can be written as following:

$$f^{'''} + \frac{1}{2} f f^{''} = 0 \quad (7)$$

And boundary conditions (3) will be:

$$f = f_w \ , \ f^{'} = \lambda \ at \ \eta = 0$$
$$f^{'} \rightarrow 1 - \lambda \ as \ \eta \rightarrow \infty \quad (8)$$

Where Equation (7) is ordinary differential equations w.r.t. η, and λ is the velocity ratio parameter defined by

$$\lambda = \frac{U_w}{U}$$

With $\lambda = 0$, $\lambda > 0$ and $\lambda < 0$ indicate that plate is fixed, moving in same direction and opposite directions to the free stream respectively. If $\lambda = 0$ and $\lambda = 1$ then $U_w = 0$ and $U_\infty = 0$ respectively. The skin friction of moving surface is given by [6]

$$C_f = \frac{2\tau_w}{\rho U^2} \quad (9)$$

Where τ_w is shearing stress and it is defined by [6]

$$\tau_w = \mu \frac{\partial u}{\partial y}|_{y=0} \quad (10)$$

$$\tau_w = \mu \frac{\partial^2 \psi}{\partial y^2}|_{y=0} = U \sqrt{\frac{U}{\nu x}} f^{''}(0)$$

where $f^{''}(0)$ is skin-friction coefficient of moving surface.

III. RESULTS AND DISCUSSIONS

The nonlinear ordinary differential Equation (7) with the associated boundary conditions (8) was solved numerically by using the fourth-order Runge - Kutta method and shooting method. For boundary conditions ($\eta \rightarrow \infty$) is replaced with a large value of η that, in this paper is taken $\eta = 14$. The computed results have been shown in Figs. 2-7. Dimensionless velocity ($f^{'}(\eta)$) has been compute for its variation, along η [non-dimensional distance along 'y']. The cases computed is for different suction and injection parameter (f_w) for the plate (λ) moving same (Fig. 2) and opposite direction (Fig. 3). Also $f^{'}(\eta)$ computation with respect to η for various λ's [fluid is moving in same and opposite directions]. [Fig. 4 is for suction ($f_w > 0$) and Fig. 5 is for injection ($f_w < 0$)]. The computation of non-dimensional velocity has also been done for the case of fluid at stationary ($\lambda = 0$) and for the suction and injection (Fig. 6). The results indicate that, the suction values of non-dimensional velocity are higher than injection values. It is of interest to note that, the values of $f^{'}(\eta)$ [for suction] are higher for the case of fluid moving in opposite direction to that of plate in compression to at stationary and at same direction (Fig. 4). These observations are found to be exactly opposite for the case of injection (Fig. 5).

These results are found to be in agreement with physics of flows and deformation since injection must reduce the flow by imparting retarding to the ongoing flow and the stream moving opposite direction to the plate must introduce higher disturbance in the 'y' direction that is in boundary layer region . Results on skin friction coefficient ($f^{''}(0)$) (Fig. 7 and Table I) indicates that, $f^{''}(0)$ decreases with increase in 'λ'[$-0.3 \le \lambda \le 0.3$] and observed that, skin friction values are higher for suction case in compression to injection. This observation on skin friction is also justified since frictional values are higher in case of disturbed medium.

The results of the present model has been compared to that of earlier works of Nazar et al. [5] and Sakiadis [4]. The comparative statements have been shown in Table II. Though they are no significant variations in the values compared for however it could be said here with positive note that, the method of shooting employed in the present model is much simpler and faster when compression to the method adopted by earlier works [Keller box technique [5] and finite difference method [4]].

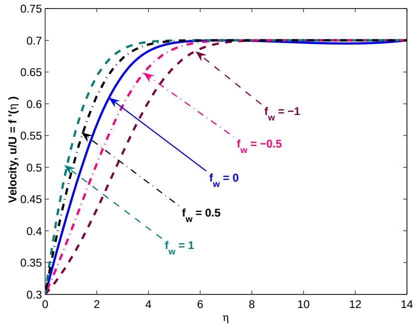

Fig. 2. Variation of velocity with η for various of f_w when $\lambda = 0.3$

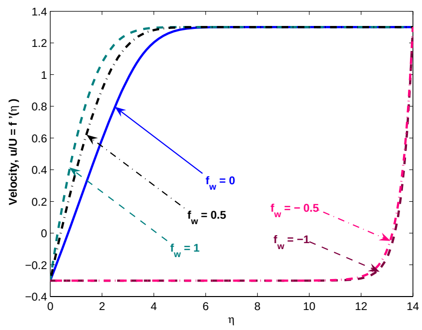

Fig. 3. Variation of velocity with η for various of f_w when $\lambda = -0.3$

TABLE I
VALUES OF THE SKIN FRICTION COEFFICIENTS $f^{''}(0)$ FOR VARIOUS VALUES OF λ AND f_w

f_w	$f^{''}(0)$ with $\lambda = -0.3$	$f^{''}(0)$ with $\lambda = 0$	$f^{''}(0)$ with $\lambda = 0.3$
-1	$1.665244781 \ . \ 10^{-9}$	0.03552589416504	0.04438915252686
-0.5	$5.014562702 \ . \ 10^{-8}$	0.16452102661133	0.08993263244629
0	0.43382263183594	0.33203353881836	0.15016632080078
0.5	0.77709655761719	0.52270889282227	0.22122764587402
1	1.12651557922363	0.72874546051025	0.29999389648438

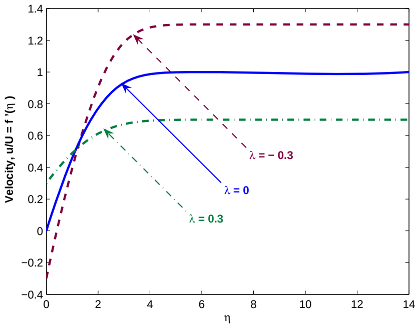

Fig. 4. Variation of velocity with η for various of λ when $f_w = 0.5$

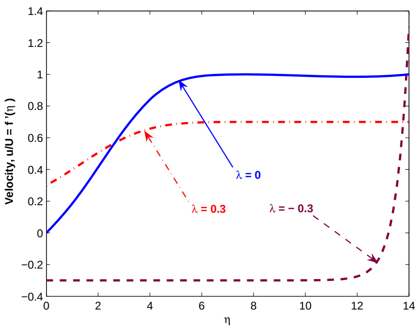

Fig. 5. Variation of velocity with η for various of λ when $f_w = -0.5$

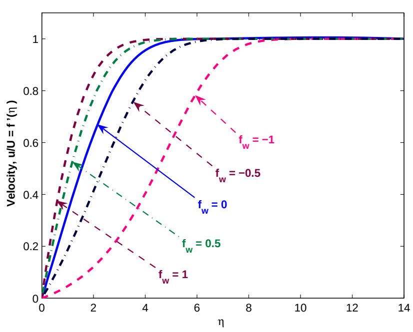

Fig. 6. Variation of velocity with η for various of f_w when $\lambda = 0$

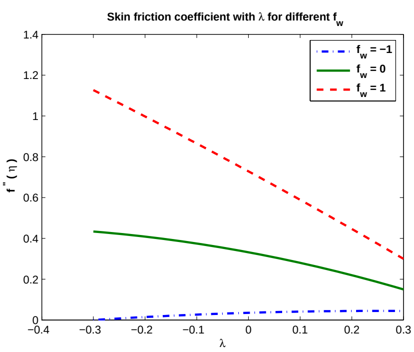

Fig. 7. Variation of skin friction coefficient with λ for several of f_w

TABLE II
VALUES OF THE SKIN FRICTION COEFFICIENTS $f^{''}(0)$ FOR $f_w = 0$

λ	0	0.3	-0.3
Blasius [2]	0.332	-	-
Nazar et al.[5]	0.3321	-	-
Present	0.33203	0.15017	0.43382

IV. Conclusion

Boundary layer equations on a fixed or continuously moving plate in the same or opposite direction to the free stream with suction or injection are studied in the present model. The studies have been taken up in view of its applications in environmental, chemical, engineering and industrial applications. Flow parameters such as non dimensional velocity $f^{'}(\eta)$ and skin friction coefficient $f^{''}(0)$ have been computed for different flow conditions (suction and injection) and for different geometric conditions (plate fixed, moving in same and opposite direction to that of stream). The results have also been compared to that of earlier works and found that the present model agrees well with the physics of flow and with the observations of earlier works. One of the basic thrust on which the model studies have been undertaken is to employ new numerical scheme for the computation of present set of boundary equations and check with the earlier works ([5] and [4]). The present scheme which employs shooting method has been found to be much simpler and effective in comparison to the one used by earlier investigators (Keller box technique [5] and finite difference scheme [4]).

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