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Positive periodic solutions in a discrete competitive system with the effect of toxic substances

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Abstract—In this paper, a delayed competitive system with the effect of toxic substances is investigated. With the aid of differential equations with piecewise constant arguments, a discrete analogue of continuous non-autonomous delayed competitive system with the effect of toxic substances is proposed. By using Gaines and Mawhin's continuation theorem of coincidence degree theory, a easily verifiable sufficient condition for the existence of positive solutions of difference equations is obtained.

Keywords—competitive system; periodic solution; discrete time delay; topological degree

I. Introduction

It is well known that the dynamical properties of competitive populations has received great attention from both theoretical and mathematical biologists due to its universal prevalence and importance. Many excellent works have been done for a lot of different continuous or impulsive competitive models(see[1,5,8,]). In 2009, Song and Chen[9] proposed a delay two-species competitive system in which two species have toxic inhibitory effects on each other:

$$\begin{cases} \frac{dx}{dt} = x(t)[K_1(t) - \alpha_1(t)x(t) - \beta_1(t)y(t) \\ \qquad - \gamma_1(t)x(t)y(t-\tau_1(t))], \\ \frac{dy}{dt} = y(t)[K_2(t) - \alpha_2(t)y(t) - \beta_2(t)x(t) \\ \qquad - \gamma_2(t)x(t-\tau_2(t))y(t)], \end{cases} \tag{1}$$

where $x(t), y(t)$ stand for the population densities of two competing species, respectively. $K_i(t)(i = 1, 2)$ are the intrinsic growth rates of two competing species; $\alpha_i(t)(i = 1, 2)$ denote the coefficients of interspecific competition; $K_i(t)/\alpha_i(t)(i = 1, 2)$ are the environmental carrying capacities of two competing species; γ_1 and γ_2 stand for, respectively, the rates of toxic inhibition of the species x by the species y and vice versa. More details about the model, one can see [9]. By applying the theory of coincidence degree theory, Song and Chen[9] established the existence of positive periodic solution for system (1).

Considering the impulsive effects and periodic perturbations, Liu et al.[7] investigated the following periodic impulsive delay competitive system with the effect of toxic substances

$$\begin{cases} \frac{dx}{dt} = x(t)[K_1(t) - \alpha_1(t)x(t) - \beta_1(t)y(t) \\ \qquad - \gamma_1(t)x(t)y(t-\tau_1(t))], \quad t \neq t_k, \\ \frac{dy}{dt} = y(t)[K_2(t) - \alpha_2(t)y(t) - \beta_2(t)x(t) \\ \qquad - \gamma_2(t)x(t-\tau_2(t))y(t)], \quad t \neq t_k, \\ x(t_k^+) = x(t_k) + p, t = t_k, \\ y(t_k^+) = (1 + b_k)y(t_k), t = t_k \end{cases} \tag{2}$$

with initial condition $(x(s), y(s) = \phi(s) = (\phi_1(s), \phi_2(s))$, for $-\tau \leq s \leq 0, \phi(0) > 0, \phi \in PC([-\tau, 0], R_+^2)$, where $\tau = \max_{1\leq i\leq 2}\max_{t\in[0,\omega]}\{\tau_i(t)\}.K_i(t), \alpha_i(t), \beta_i(t), \gamma_i(t), \tau_i(t)(i = 1, 2)$ are continuous ω-periodic functions, and $\alpha_i(t), \beta(t), \gamma_i(t)(i = 1, 2)$ are positive and $\tau_i(t)(i = 1, 2)$ are nonnegative. The intrinsic growth rates $K_i(t)(i = 1, 2)$ are not necessarily positive and may be negative. $k \in N$ and N is the set of positive integers. The jump conditions reflect the possibility of impulsive effects on the species x and y. $p > 0$ is the impulsive stocking amount of the species x at $t = t_k$, which implies that the populations are subjected to impulsive stocking at a constant rate p. $b_k y(t_k) < 0$ represent the impulsive harvesting amount of the species y at $t = t_k$, while $b_k y(t_k) > 0$, the perturbations may stand for the impulsive stocking amount of the species y at $t = t_k$. By applying the theory of impulsive differential equation and some analysis techniques, Liu et al.[7] obtained a set of sufficient conditions for the permanence and partial extinction of system (2).

Many authors argue that discrete time models governed by difference equations are more appropriate to describe the dynamics relationship among populations than continuous ones when the populations have non-overlapping generations. Moreover, discrete time models can also provide efficient models of continuous ones for numerical simulations. Therefore, it is reasonable and interesting to study discrete time systems governed by difference equations. Recently, a great deal of research has been devoted to this topics, see[2,3,6,10,12-14]. The principle object of this article is to propose a discrete analogue system (1) and explore its dynamics.

The remainder of the paper is organized as follows: in Section 2, with the help of differential equations with piecewise constant arguments, we first propose a discrete analogue of system (1), modelling the dynamics of time non-autonomous competing system with with the effect of toxic substances where populations have non-overlapping generations. In Section 3, based on the coincidence degree and the related continuation theorem, a easily verifiable sufficient condition for the existence of positive solutions of difference equations is obtained.

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II. Discrete analogue of system (1)

There is several different ways of deriving discrete time version of dynamical systems corresponding to continuous time formulations. One of the way of deriving difference equations modelling the dynamics of populations with non-overlapping generations that we will used in this following is based on appropriate modifications of models with overlapping generations. For more detail about the approach, we refer to [3,11].

In the following, we will discrete the system (1). Assume that the average growth rates in system (1) change at regular intervals of time, then we can obtain the following modified system:

$$\begin{cases} \frac{1}{x(t)}\dot{x}(t) = K_1([t]) - \alpha_1([t])x([t]) - \beta_1([t])y([t]) \\ \qquad - \gamma_1([t])x([t])y([t] - \tau_1([t])), \\ \frac{1}{y(t)}\dot{y}(t) = K_2([t]) - \alpha_2([t])y([t]) - \beta_2([t])x([t]) \\ \qquad - \gamma_2([t])x([t] - \tau_2([t]))y([t]), \end{cases} \tag{3}$$

where $[t]$ denotes the integer part of $t, t \in (0, +\infty)$ and $t \neq 0, 1, 2, \cdots$. Equations of type (3) are known as differential equations with piecewise constant arguments and these equations occupy a position midway between differential equations and difference equations. By a solution of (3), we mean a function $\bar{x} = (x, y)^T$, which is defined for $t \in [0, +\infty)$ and have the following properties:

1. $\bar{x}$ is continuous on $[0, +\infty)$.
2. The derivative $\frac{dx(t)}{dt}, \frac{dy(t)}{dt}$ exist at each point $t \in [0, +\infty)$ with the possible exception of the points $t \in \{0, 1, 2, \cdots\}$,where left-sided derivative exist.
3. The equations in (3) are satisfied on each interval $[k, k+1)$ with $k = 0, 1, 2, \cdots$.

We integrate (3) on any interval of the form $[k, k+1), k = 0, 1, 2, \cdots$, and obtain for $k \leq t < k+1, k = 0, 1, 2, \cdots$.

$$\begin{cases} x(t) = x(k)\exp\{[K_1(k) - \alpha_1(k)x(k) - \beta_1(k)y(k) \\ \qquad -\gamma_1(k)x(k)y(k - \tau_1(k))]\,(t-k)\}, \\ y(t) = y(k)\exp\{[K_2(k) - \alpha_2(k)y(k) - \beta_2(k)x(k) \\ \qquad -\gamma_2(k)x(k - \tau_2(k))y(k)]\,(t-k)\}. \end{cases} \tag{4}$$

Let $t \to k+1$, then (4) takes the following form :

$$\begin{cases} x(k+1) = x(k)\exp\{[K_1(k) - \alpha_1(k)x(k) - \beta_1(k)y(k) \\ \qquad -\gamma_1(k)x(k)y(k - \tau_1(k))]\}, \\ y(k+1) = y(k)\exp\{[K_2(k) - \alpha_2(k)y(k) - \beta_2(k)x(k) \\ \qquad -\gamma_2(k)x(k - \tau_2(k))y(k)]\}, \end{cases} \tag{5}$$

which is a discrete time analogue of system (1), where $k = 0, 1, 2, \cdots$.

III. Existence of positive periodic solutions

For convenience and simplicity in the following discussion ,we always use the notations below throughout the paper:

$$I_\omega := \{0, 1, 2, ..., \omega - 1\},\ \overline{f} := \frac{1}{\omega}\sum_{k=0}^{\omega-1} f(k),\ f^L := \min_{k \in Z}\{f(k)\},$$

$f^M := \max_{k \in Z}\{f(k)\}$, where $f(k)$ is an $\omega-$periodic sequence of real numbers defined for $k \in Z$. We always assume that

(H1) $K_i, \alpha_i, \beta_i, \gamma_i : Z \to R^+ (i = 1, 2)$ are ω periodic.
(H2) $\text{sign}\Big\{\bar{K_1}\bar{\alpha_2} - \bar{K_2}\bar{\alpha_1}\Big\} = \text{sign}\Big\{\bar{K_2}\bar{\alpha_1} - \bar{K_1}\bar{\alpha_2}\Big\} = \text{sign}\Big\{\bar{\alpha_1}\bar{\alpha_2} - \bar{\beta_2}\bar{\beta_2}\Big\} \neq 0.$

In order to explore the existence of positive periodic solutions of (5) and for the reader's convenience, we shall first summarize below a few concepts and results without proof, borrowing from [4].

Let X, Y be normed vector spaces, $L : DomL \subset X \to Y$ is a linear mapping, $N : X \to Y$ is a continuous mapping. The mapping L will be called a fredholm mapping of index zero if $dimKerL = codimImL < +\infty$ and ImL is closed in Y. If L is a fredholm mapping of index zero and there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that $ImP = KerL, ImL = KerQ = Im(I - Q)$, It follows that $L \mid DomL \cap KerP : (I - P)X \to ImL$ is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called $L-$compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I-Q)N : \bar{\Omega} \to X$ is compact. Since ImQ is isomorphic to $KerL$, there exist isomorphisms $J : ImQ \to KerL$.

Lemma 3.1. ([4]Continuation Theorem) *Let L be a Fredholm mapping of index zero and let N be $L-$compact on $\bar{\Omega}$. Suppose*
(a) For each $\lambda \in (0, 1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$;
(b) $QNx \neq 0$ for each $x \in KerL \bigcap \partial\Omega$, and $deg\{JQN, \Omega \bigcap \partial KerL, 0\} \neq 0$;
Then the equation $Lx = Nx$ has at least one solution lying in $DomL \bigcap \bar{\Omega}$.

Lemma 3.2. [3] *Let $g : Z \to R$ be ω periodic, i.e., $g(k+\omega) = g(k)$; then for any fixed $k_1, k_2 \in I_\omega$ and any $k \in Z$, one has*

$$g(k) \leq g(k_1) + \sum_{s=0}^{\omega-1} |g(s+1) - g(s)|,$$

$$g(k) \geq g(k_2) + \sum_{s=0}^{\omega-1} |g(s+1) - g(s)|.$$

Lemma 3.3. *$(\hat{x}(k), \hat{y}(k))$ is an ω periodic solution of (5) with strictly positive components if and only if $(\ln\{\hat{x}(k)\}, \ln\{\hat{y}(k)\})$ is an ω periodic solution of*

$$\begin{cases} x(k+1) - x(k) = u_1(k), \\ y(k+1) - y(k) = u_2(k), \end{cases} \tag{6}$$

where

$$u_1(k) = K_1(k) - \alpha_1(k)\exp(x(k)) - \beta_1(k)\exp(y(k)) - \gamma_1(k)\exp(x(k))\exp(y(k - \tau_1(k))),$$

$$u_2(k) = K_2(k) - \alpha_2(k)\exp(y(k)) - \beta_2(k)\exp(x(k)) - \gamma_2(k)\exp(x(k - \tau_2(k)))\exp(y(k)).$$

Lemma 3.4. *If condition (H2) holds, then the algebraic equations*

$$\begin{cases} \bar{K_1} - \bar{\alpha_1}x - \bar{\beta_1}y = 0, \\ \bar{K_2} - \bar{\alpha_2}y - \bar{\beta_2}x = 0 \end{cases} \tag{7}$$

has a unique positive solution $(x^*, y^*)^T \in R^2$.

The proofs of Lemma 3.3 and Lemma 3.4 are trivial, so we omitted the details here.

Define

$$l_2 = \{z = \{z(k)\} : z(k) \in R^2, k \in Z\}.$$

For $a = (a_1, a_2)^T \in R^2$, define $|a| = \max\{|a_1|, |a_2|\}$. Let $l^\omega \subset l_2$ denote the subspace of all ω periodic sequences equipped with the usual supremum norm $\|.\|$, i.e., $\|z\| = \max_{k\in I} \ |z(k)|$, for any $z = \{z(k) : k \in Z\} \in l^\omega$. It is easy to show that l_ω is a finite-dimensional Banach space.

Let

$$l_0^\omega = \left\{ z = \{z(k)\} \in l^\omega : \sum_{k=0}^{\omega-1} z(k) = 0 \right\}, \tag{8}$$

$$l_c^\omega = \left\{ z = \{z(k)\} \in l^\omega : z(k) = h \in R^2, k \in Z \right\}, \tag{9}$$

then it follows that l_0^ω and l_c^ω are both closed linear subspaces of l^ω and

$$l^\omega = l_0^\omega + l_c^\omega, \qquad diml_c^\omega = 2.$$

In the following, we will ready to establish our result.

Theorem 3.1. *Let B_9 be defined by (36). Suppose that $(H1), (H2)$ and $2\bar{K_1} > \bar{\alpha_1}, 2\bar{K_2} > \bar{\beta_1}, \bar{K_2} > \bar{\gamma_2}\exp(B_9)$ hold, then the system (5) has at least an ω periodic solution.*

Proof. Let $X = Y = l^\omega$,

$$(Lz)(k) = z(k+1) - z(k), \tag{10}$$

$$(Nz)(k) = \begin{pmatrix} u_1(k) \\ u_2(k) \end{pmatrix}, \tag{11}$$

where $z \in X, k \in Z$. Then it is trivial to see that L is a bounded linear operator and

$$KerL = l_c^\omega, \quad ImL = l_0^\omega$$

and

$$dimKerL = 2 = codimImL,$$

then it follows that L is a fredholm mapping of index zero. Define

$$Py = \frac{1}{\omega}\sum_{s=0}^{\omega-1} y(s), \ y \in X, \ Qz = \frac{1}{\omega}\sum_{s=0}^{\omega-1} z(s), \ z \in Y.$$

It is not difficult to show that P and Q are continuous projectors such that

$$ImP = KerL, \ ImL = KerQ = Im(I-Q).$$

Furthermore, the generalized inverse (to L) $k_P : ImL \to KerP \bigcap DomL$ exists and is given by

$$K_P(z) = \sum_{s=0}^{\omega-1} z(s) - \frac{1}{\omega}\sum_{s=0}^{\omega-1}(\omega - s)z(s).$$

Obviously, QN and $K_P(I-Q)N$ are continuous. Since X is a finite-dimensional Banach space, using the Ascoli-Arzela theorem, it is not difficult to show that $\overline{K_P(I-Q)N(\bar{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\bar{\Omega})$ is bounded. Thus, N is $L-$compact on $\bar{\Omega}$ with any open bounded set $\Omega \subset X$.

Now we are at the point to search for an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation $Ly = \lambda Ny, \lambda \in (0, 1)$, we have

$$x(k+1) - x(k) = \lambda u_1(k), \tag{12}$$

$$y(k+1) - y(k) = \lambda u_2(k). \tag{13}$$

Suppose that $z(k) = (x(k), y(k))^T \in X$ is an arbitrary solution of system (12) and (13) for a certain $\lambda \in (0, 1)$, summing both sides of (12) and (13) from 0 to $\omega - 1$ with respect to k respectively, we obtain

$$\sum_{k=0}^{\omega-1} [\alpha_1(k)\exp(x(k)) + \beta_1(k)\exp(y(k)) + \gamma_1(k)\exp(x(k))\exp(y(k-\tau_1(k)))] = \bar{K_1}\omega, \tag{14}$$

$$\sum_{k=0}^{\omega-1} [\alpha_2(k)\exp(y(k)) + \beta_2(k)\exp(x(k)) + \gamma_2(k)\exp(x(k-\tau_2(k)))\exp(y(k))] = \bar{K_2}\omega. \tag{15}$$

It follows from (12), (13), (14) and (15) that

$$\sum_{k=0}^{\omega-1} |x(k+1) - x(k)| \leq 2\bar{K_1}\omega, \tag{16}$$

$$\sum_{k=0}^{\omega-1} |y(k+1) - y(k)| \leq 2\bar{K_2}\omega. \tag{17}$$

In view of the hypothesis that $z = \{z(k)\} \in X$, there exist $\xi_i, \eta_i \in I_\omega$ such that

$$\begin{cases} x(\xi_1) = \min_{k\in I} \ \{x(k)\}, x(\eta_1) = \max_{k\in I} \ \{x(k)\}, \\ y(\xi_2) = \max_{k\in I} \ \{y(k)\}, y(\eta_2) = \max_{k\in I} \ \{y(k)\}. \end{cases} \tag{18}$$

By (14), we have

$$\sum_{k=0}^{\omega-1} a_1(k)\exp(x(k)) < 2\bar{K_1}\omega, \tag{19}$$

$$\sum_{k=0}^{\omega-1} \beta_1(k)\exp(y(k)) < 2\bar{K_1}\omega, \tag{20}$$

then we obtain

$$\exp(x(k)) < \frac{2\bar{K_1}}{\bar{\alpha_1}}, \tag{21}$$

$$\exp(y(k)) < \frac{2\bar{K_1}}{\bar{\beta_1}}. \tag{22}$$

Thus,

$$x(\xi_1) < \ln\left[\frac{2\bar{K_1}}{\bar{\alpha_1}}\right], \tag{23}$$

$$y(\xi_2) < \ln\left[\frac{2\bar{K_1}}{\bar{\beta_1}}\right]. \tag{24}$$

In the sequel, we consider two cases.
(a) If $x(\eta_1) \geq y(\eta_2)$, then it follows from (14) that

$$\overline{(\alpha_1+\beta_1)}\exp(x(\eta_1)) + \bar{\gamma_1}\exp(2x(\eta_1)) \geq \bar{K_1},$$

which leads to

$$x(\eta_1) > \ln\left[\frac{-\overline{(\alpha_1+\beta_1)} + \sqrt{\overline{(\alpha_1+\beta_1)}^2 + 4\bar{\gamma_1}\bar{K_1}}}{2\bar{\gamma_1}}\right]. \quad (25)$$

It follows from (23),(25) and Lemma 3.2 that

$$\begin{aligned} x(k) &\leq x(\xi_1) + \sum_{s=0}^{\omega-1}|x(s+1)-x(s)| \\ &\leq \ln\left[\frac{2\bar{K_1}}{\bar{\alpha_1}}\right] + 2\bar{K_1}\omega := B_1, \quad (26) \\ x(k) &\geq x(\eta_1) - \sum_{s=0}^{\omega-1}|x(s+1)-x(s)| \\ &\geq \ln\left[-\frac{\overline{(\alpha_1+\beta_1)} + \sqrt{\overline{(\alpha_1+\beta_1)}^2 + 4\bar{\gamma_1}\bar{K_1}}}{2\bar{\gamma_1}}\right] \\ &\quad -2\bar{K_1}\omega := B_2. \quad (27) \end{aligned}$$

By (26) and (27), we derive

$$\max_{k\in I}\{x(k)\} < \max\{|B_1|, |B_2|\} := B_3. \quad (28)$$

From (15), we obtain that

$$\bar{\alpha_2}\exp(y(\eta_2)) + \bar{\beta_2}\exp(B_3) + \bar{\gamma_2}\exp(B_3)\exp(y(\eta_2)) \geq \bar{K_2}.$$

Then

$$y(\eta_2) \geq \ln\left[\frac{\bar{K_2} - \bar{\gamma_2}\exp(B_3)}{\bar{\alpha_2} + \bar{\gamma_2}\exp(B_3)}\right]. \quad (29)$$

Thus by (24), (29) and Lemma 3.2, we get

$$\begin{aligned} y(k) &\leq y(\xi_2) + \sum_{s=0}^{\omega-1}|y(s+1)-y(s)| \\ &\leq \ln\left[\frac{2\bar{K_1}}{\bar{\beta_1}}\right] + 2\bar{K_2}\omega := B_4, \quad (30) \\ y(k) &\geq y(\eta_2) - \sum_{s=0}^{\omega-1}|y(s+1)-y(s)| \\ &\geq \ln\left[\frac{\bar{K_2} - \bar{\gamma_2}\exp(B_3)}{\bar{\alpha_2} + \bar{\gamma_2}\exp(B_3)}\right] - 2\bar{K_2}\omega := B_5. \quad (31) \end{aligned}$$

It follows from (30) and (31) that

$$\max_{k\in I}\{y(k)\} < \max\{|B_4|, |B_5|\} := B_6. \quad (32)$$

(b) If $x(\eta_1) < y(\eta_2)$, then it follows from (14) that

$$\overline{(\alpha_1+\beta_1)}\exp(y(\eta_2)) + \bar{\gamma_1}\exp(2y(\eta_2)) \geq \bar{K_1},$$

which leads to

$$y(\eta_2) > \ln\left[\frac{-\overline{(\alpha_1+\beta_1)} + \sqrt{\overline{\alpha_1+\beta_1}^2 + 4\bar{\gamma_1}\bar{K_1}}}{2\bar{\gamma_1}}\right]. \quad (33)$$

By (24),(33) and Lemma 3.2, we have

$$\begin{aligned} y(k) &\leq y(\xi_2) + \sum_{s=0}^{\omega-1}|y(s+1)-y(s)| \\ &\leq \ln\left[\frac{2\bar{K_1}}{\bar{\beta_1}}\right] + 2\bar{K_2}\omega := B_7, \quad (34) \\ y(k) &\geq y(\eta_2) - \sum_{s=0}^{\omega-1}|y(s+1)-y(s)| \\ &\geq \ln\left[\frac{-\overline{(\alpha_1+\beta_1)} + \sqrt{\overline{\alpha_1+\beta_1}^2 + 4\bar{\gamma_1}\bar{K_1}}}{2\bar{\gamma_1}}\right] \\ &\quad -2\bar{K_2}\omega := B_8. \quad (35) \end{aligned}$$

It follows from (34) and (35) that

$$\max_{k\in I}\{y(k)\} < \max\{|B_7|, |B_8|\} := B_9, \quad (36)$$

From (15), we obtain that

$$\bar{\alpha_2}\exp(B_9) + \bar{\beta_2}\exp(x(\eta_1)) + \bar{\gamma_2}\exp(B_9)(x(\eta_1)) \geq \bar{K_2}.$$

Then

$$x(\eta_1) \geq \ln\left[\frac{\bar{K_2} - \bar{\gamma_2}\exp(B_9)}{\bar{\beta_2} + \bar{\gamma_2}\exp(B_9)}\right]. \quad (37)$$

By (23),(37) and Lemma 3.2, we obtain

$$\begin{aligned} x(k) &\leq x(\xi_1) + \sum_{s=0}^{\omega-1}|x(s+1)-x(s)| \\ &\leq \ln\left[\frac{2\bar{K_1}}{\bar{\alpha_1}}\right] + 2\bar{K_2}\omega := B_{10}, \quad (38) \\ x(k) &\geq x(\eta_1) - \sum_{s=0}^{\omega-1}|x(s+1)-x(s)| \\ &\geq \ln\left[\frac{\bar{K_2} - \bar{\gamma_2}\exp(B_9)}{\bar{\beta_2} + \bar{\gamma_2}\exp(B_9)}\right] - 2\bar{K_2}\omega := B_{11}. \quad (39) \end{aligned}$$

It follows from (38) and (39) that

$$\max_{k\in I}\{x(k)\} < \max\{|B_{10}|, |B_{11}|\} := B_{12}. \quad (40)$$

Obviously, $B_i (i = 1, 2, \cdots, 12)$ are independent of $\lambda \in (0, 1)$. Take $M = \max\{B_3, B_6, B_9, B_{12}\} + B_0$, where B_0 is taken sufficiently large such that $\max\{|\ln\{x^*\}|, |\ln\{y^*\}|\} < B_0$, where $(x^*, y^*)^T$ is the unique positive solution of (61). Now we have proved that any solution $z = \{z(k)\} = \{(x(k), y(k))^T\}$ of (12) and (13) in X satisfies $\|z\| < M, k \in Z$.

Let $\Omega := \{z = \{z(k)\} \in X : \|z\| < M\}$, then it is easy to see that Ω is an open, bounded set in X and verifies requirement (a) of Lemma 3.1. When $y \in \partial\Omega \cap KerL, z = \{(x, y)^T\}$ is a constant vector in R^2 with $\|z\| = \max\{|x|, |y|\} = M$. Then

$$QNy = \begin{pmatrix} \bar{K_1} - \bar{\alpha_1}\exp(x) - \bar{\beta_1}\exp(y) - \bar{\gamma_1}\exp(x)\exp(y) \\ \bar{K_2} - \bar{\alpha_2}\exp(y) - \bar{\beta_2}\exp(x) - \bar{\gamma_2}\exp(x)\exp(y) \end{pmatrix} \neq 0.$$

Now let us consider homotopic $\phi(y_1, y_2, \mu) = \mu QNy + (1-\mu)Gy, \mu \in [0, 1]$, where

$$Gy = \begin{pmatrix} \bar{K_1} - \bar{\alpha_1}\exp(x) - \bar{\beta_1}\exp(y) \\ \bar{K_2} - \bar{\alpha_2}\exp(y) - \bar{\beta_2}\exp(x) \end{pmatrix}.$$

Letting J be the identity mapping and by direct calculation, we get

$$\begin{aligned}
&\deg\Big\{JQN(x,y)^T;\partial\Omega\bigcap kerL;0\Big\}\\
&=\deg\Big\{QN(x,y)^T;\partial\Omega\bigcap kerL;0\Big\}\\
&=\deg\Big\{\phi(x,y,1);\partial\Omega\bigcap kerL;0\Big\}\\
&=\deg\Big\{\phi(x,y,0);\partial\Omega\bigcap kerL;0\Big\}\\
&=\text{sign}\left\{\det\left(\begin{array}{cc}-\bar{\alpha_1}\exp(x) & -\bar{\beta}_1\exp(y)\\ -\bar{\beta}_2\exp(x) & -\bar{\alpha_2}\exp(y)\end{array}\right)\right\}\\
&=\text{sign}\{(\bar{\alpha_1}\bar{\alpha_2}-\bar{\beta}_1\bar{\beta}_2)\exp(x^*+y^*)\}\neq 0.
\end{aligned}$$

By now, we have proved that Ω verifies all requirements of Lemma 3.1, then it follows that $Lz = Nz$ has at least one solution in $DomL \cap \overline{\Omega}$, that is to say, (6) has at least one ω periodic solution in $DomL \cap \overline{\Omega}$, say $z^* = \{z^*(k)\} = \{(x^*(k), y^*(k))^T\}$. Let $\bar{x}^*(k) = \exp\{x^*(k)\}, \bar{y}^*(k) = \exp\{y^*(k)\}$ then by Lemma 3.3 we know that $\bar{z}^* = \{\bar{z}^*(k)\} = \{\bar{x}^*(k), \bar{y}^*(k))^T\}$ is an ω periodic solution of system (5) with strictly positive components. The proof is complete.

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