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Particle Swarm Optimization for Design of Water Distribution Systems

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***Abstract*—**Particle Swarm Optimization (PSO) technique is applied to design the water distribution pipeline network. A simulation-optimization model is formulated with the objective of minimizing cost and is applied to a benchmark water distribution system optimization problem. The benchmark problem taken for the application of PSO technique to optimize the pipe size of the water distribution network is New York City water supply system problem. The results from the analysis infer that PSO is a potential alternative optimization technique when compared to other heuristic techniques for optimal sizing of water distribution systems.

***Keywords*—**New York water supply system, Optimization, Particle swarm optimization, Swarm intelligence, Water distribution systems.

I. INTRODUCTION

WATER distribution networks are considered as the most important and expensive entity in the urban infrastructure systems. Design optimization of water distribution networks has been thoroughly researched over the past few decades due to its computational and engineering complexity. The nonlinearity between flow and head loss along with the presence of discrete variables (pipe diameter) in design optimization makes this highly challenging. In the last decade, researchers have attempted to explore several new non-traditional optimization techniques to such NP-hard combinatorial optimization problems [1], [2], [3].

A significant literature exists for optimizing water distribution networks using linear programming, nonlinear programming, enumeration techniques, heuristic methods and evolutionary techniques focusing on the objective of cost minimization. Reference [4] developed the linear programming gradient method that has been improved by many researchers [5], [6], [7], [8], [9]. Due to the limitations of linear programming, researchers applied the nonlinear programming (NLP) optimization approach to pipe network problems [10], [11], [12], [13], [14]. NLP approaches also had their limitations as they rely on the initial solution and so the researchers applying the heuristic approaches to the optimal design of water distribution networks. Applications to water distribution networks design include genetic algorithms, simulated annealing, harmony search optimization, shuffled frog leaping algorithm, ant colony optimization, memetic algorithms and differential evolution. Genetic algorithms have been used for solving network design problem by [15], [16], [17], [18], [19], [20], [21] and [22]. Simulated annealing has been applied by [23] and [24]. Reference [25] developed the harmony search optimization approach to solve network design problems while reference [26] developed the shuffled frog leaping algorithm. Reference [27] applied ant colony optimization approach and outperformed genetic algorithms both in terms of computational efficiency and their ability to find near global optimal solutions. Reference [2] analyzed the performance of memetic algorithms for optimal design of looped water distribution systems and demonstrated that they work better when the size of the problem increases while [3] successfully applied differential evolution optimization technique.

The escalating complexity of the real world applications similar to the one stated above has demanded researchers to find the possible ways of easing the solution of such problems. This has motivated the researchers to grasp ideas from the nature and implant it in the engineering sciences. This way of thinking led to emergence of many biologically inspired algorithms that have proven to be efficient in handling the computationally complex problems with competence such as Evolutionary Algorithms and Swarm Intelligence (SI) techniques [28]. Particle swarm optimization (PSO) is a swarm intelligence technique developed by [29], inspired by social behavior of bird flocking or fish schooling. PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA). However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. PSO has been successfully applied in many research and application areas. It is demonstrated that PSO gets better results in a faster, cheaper way compared with other methods. Another reason that PSO is attractive is that there are few parameters to work with. In this paper, Particle swarm optimization has been used to the pipe-sizing problem, in comparison with widely used algorithms in the literature for the case study of the New York water supply system. The remainder of the paper is structured as follows: First, the mathematical formulation of the design problem (minimization of network cost) is presented, and then working of PSO technique is discussed. The development of the simulation-optimization model for the New York water supply system is then presented followed by the results and discussion. The paper closes with concluding remarks.

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II. Water Distribution Optimization Model Formulation

The mathematical formulation of the optimal design of water distribution networks is often viewed as a least-cost optimization problem in which the decision variables are the pipe diameters for each pipe in the network. The objective is to find the combination of different sizes of pipe that give the least-cost subject to constraints. The problem is constrained by the physical laws of mass and energy conservation. Minimum head constraints at pipe junctions (nodes) and pipe size restrictions are imposed. The following is the objective to minimize the network cost for the New York water supply system to find the optimal pipe diameter for each pipe in the network [30]:

$$Minimize \quad C = \sum_{i=1}^{np} (1.1 D_i^{1.24} L_i) \tag{1}$$

where $C =$ cost [\$]; D_i = diameter [inches]; L_i = length [feet]; np = number of pipes in the system.

The constraints for the above formulated model are as follows:

a. *Continuity Constraint*: For each junction node (other than the source), a continuity constraint should be satisfied.

$$\sum Q_{in} - \sum Q_{out} = Q_e \tag{2}$$

where Q_{in} = flow into the junction; Q_{out} = flow out of the junction; and Q_e = external inflow or demand at the junction node.

b. *Energy Conservation Constraint*: The sum of head loss around a pipe must be equal to zero.

$$\sum_{i \in Loopl} \Delta H_i = 0, \qquad \forall l \in NL \tag{3}$$

where ΔH_i = head loss in the pipe i and NL = total number of loops in the system.

c. *Minimum Head Constraint*:

$$H_j \geq H_j^{\min}, \qquad j = 1, \ldots, nn \tag{4}$$

III. Particle Swarm Optimization

Swarm intelligence is a new area of research inspired by the social behavior of bird flocking and shares many similarities with evolutionary algorithms such as Genetic Algorithms (GA), Differential Evolution (DE) etc. Particle Swarm Optimization (PSO) is a population based stochastic optimization algorithm in swarm intelligence [28]. This algorithm is becoming popular due to its simplicity of implementation and ability to quickly converge to a reasonably good solution. The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the search space by following the current optimum particles. Each particle keeps track of its coordinates in the problem space which are associated with the best solution (fitness) it has achieved so far. The fitness value is also stored is called *pbest*. Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the neighbors of the particle. This location is called *lbest*. When a particle takes all the population as its topological neighbors, the best value is a global best and is called *gbest*. The particle swarm optimization concept consists of, at each time step, changing the velocity of (accelerating) each particle toward its p*best* and *lbest* locations. Acceleration is weighted by a random term, with separate random numbers being generated for acceleration toward *pbest* and *lbest* locations. The velocity (v) and the position (x) of the i_{th} swarm are manipulated according to the following two equations:

$$v_{ij}^{k+1} = \chi \left[\omega v_{ij}^k + C_1 R_1 \left(p_{ij}^k - x_{ij}^k \right) + C_2 R_2 \left(p_{gj}^k - x_{ij}^k \right) \right] \tag{5}$$

$$x_{ij}^{k+1} = x_{ij}^k + v_{ij}^{k+1} \tag{6}$$

where i denotes the number of particles; j denotes number of decision variables; k denotes the iteration counter; χ is the constriction factor which controls and constricts the magnitude of the velocity; g denotes the *gbest* of a particle; p denotes the *pbest* of a particle; ω denotes the intertia weight which is often used as a parameter to control exploration and exploitation in the search space; R_1 and R_2 are random variables uniformly distributed within [0, 1]; and C_1, C_2 are acceleration coefficients, also called the *cognitive* and *social* parameters respectively. C_1 and C_2 are popularly chosen to vary within {0, 2} [31]. The search is terminated if the one of the following criteria is satisfied: (i) the number of iterations reaches the maximum allowable number or (ii) the accuracy between the best solution of two successive generations reached a pre-specified number.

In past several years, PSO has been successfully applied in many research and application areas. It is demonstrated that PSO gets better results in a faster, cheaper way compared with other methods. Another reason that PSO is attractive is that there are few parameters to adjust. One version, with slight variations, works well in a wide variety of applications. Particle swarm optimization has been used for approaches that can be used across a wide range of applications, as well as for specific applications focused on a specific requirement.

IV. Simulation-Optimization Model

A simulation-optimization model is used in this study, which involves the application of PSO linked to the hydraulic simulation software EPANET version 2.0 [32]. In PSO, the generated decimal pipe diameter values are encoded to the nearest discrete pipe diameter before they are passed to the hydraulic simulation software. Although the continuity constraint and energy conservation constraint are satisfied externally via EPANET, the other constraints must be satisfied within PSO. A simple additive penalty function approach is used in order to convert the constrained problem into unconstrained problem. In the present study, the simulation-optimization model has been applied to a case study of the New York water supply system for which the layout is shown in Fig. 1. Numerous researchers have examined the chosen case study [30], [5], [33], [34], [19], [21], [27], [26], [3]. The same input data existing pipe data, discrete set of available diameters, minimum head and demand at each node are used in this study and presented in Tables I and II. The Hazen-Williams coefficient value for new pipes is taken as 100. The imperial system of units is used to enable comparisons with previous studies.

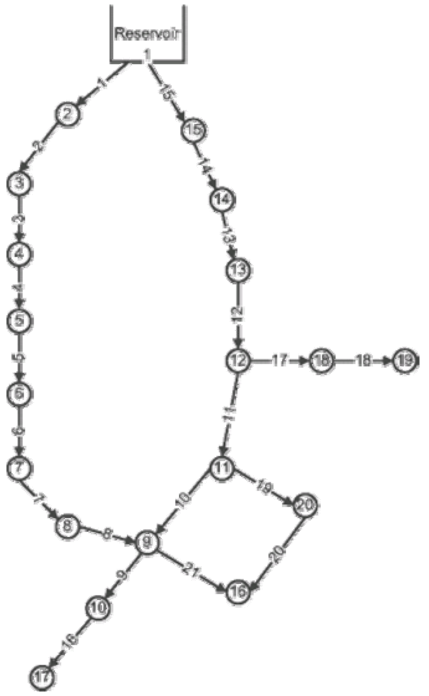

Fig. 1 Layout of New York Water Supply System

Because of pipe aging, the existing gravity flow tunnels are inadequate to meet the pressure requirements (at nodes 16, 17, 18, 19 and 20) for the projected demands. Therefore, the network is planned to be expanded by adding new pipes parallel to the existing pipes to meet the minimum pressure head requirements. The available market pipe diameters (in inches) options are 6, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168, 180, 192 and 204. There are 16 possible decisions for each pipe as there are 15 market pipe diameter sizes available for each pipe in the network and the "do nothing" option makes the sixteenth. Considering all the 21 pipes for possible duplication, the search space for the optimal solution equals to 16^{21} possible network designs.

V. Results and Discussion

The developed simulation-optimization model is applied to the expansion of New York water supply system. The population size is varied from 50 to 300 to determine the optimal solution. The model is run for 20 different trails to determine the optimal network cost. It has been observed from the results that the maximum number of function evaluations was 6825 which yields an optimal network cost of $38.64 million. The best discrete solutions found in the previous studies as compared with that obtained using this model is summarized in Table II where the values are the new pipe diameters to be added in parallel to the respective existing pipes in the network. Table III lists the nodal head values determined for [19], [21], [26] and the optimal pipe diameters simulated using PSO for critical nodes 16, 17 and 19.

TABLE I
NETWORK DATA FOR NEW YORK CITY TUNNEL SYSTEM

Node Data			Pipe Data		
Node	Demand (ft^3/s)	Minimum Head (ft)	Pipe	Length (ft)	Diameter (in)
1	-2017.5	300.0	1	11600	180
2	92.4	255.0	2	19800	180
3	92.4	255.0	3	7300	180
4	88.2	255.0	4	8300	180
5	88.2	255.0	5	8600	180
6	88.2	255.0	6	19100	180
7	88.2	255.0	7	9600	132
8	88.2	255.0	8	12500	132
9	170.0	255.0	9	9600	180
10	1.0	255.0	10	11200	204
11	170.0	255.0	11	14500	204
12	117.1	255.0	12	12200	204
13	117.1	255.0	13	24100	204
14	92.4	255.0	14	21100	204
15	92.4	255.0	15	15500	204
16	170.0	260.0	16	26400	72
17	57.5	272.8	17	31200	72
18	117.1	255.0	18	24000	60
19	117.1	255.0	19	14400	60
20	170.0	255.0	20	38400	60
			21	26400	72

TABLE II
SOLUTIONS FOR NEW YORK CITY WATER SUPPLY SYSTEM OBTAINED BY USING DISCRETE DIAMETER METHODS FOR MINIMIZATION OF NETWORK COST

Pipe	Pipe Diameter D (in.)							
	[34]	[19][1]	[19][2]	[21][3]	[27]	[26][3]	[3]	PSO
1	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-
4	-	-	-	-	-	-	-	-
5	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-
7	-	108	-	132	144	132	144	144
8	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-
11	-	-	-	-	-	-	-	-
12	-	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-
14	-	-	-	-	-	-	-	-
15	120	-	144	-	-	-	-	-
16	84	96	84	96	96	96	96	96
17	96	96	96	96	96	96	96	96
18	84	84	84	84	84	84	84	84
19	72	72	72	72	72	72	72	72
20	-	-	-	-	-	-	-	-
21	72	72	72	72	72	72	72	72
Total Cost ($ million)	38.80	37.13	40.42	38.13	38.64	38.13	38.64	38.64

[1]Solution obtained by using genetic algorithm and w=10.5088. This solution results in infeasible pressures at nodes 16, 17 and 19 when the network is simulated using EPANET hydraulic solver.
[2]Solution obtained by using genetic algorithm and w=10.9031.
[3]This solution results in infeasible pressures at nodes 17 and 19 when the network is simulated using EPANET hydraulic solver.

Table II shows that the optimal network cost evolved using PSO ($38.64 million) is slightly higher than three previous studies with optimal costs of $37.13 million, $38.13 million and $38.13 million found by [19], [21] and [26] respectively. However, when the evolved optimal pipe diameters using these studies is simulated using EPANET version 2.0 hydraulic solver to determine the nodal pressure heads, it is observed that the pressure head at critical nodes 16, 17 and 19 violate the minimum nodal pressure requirement as evident from Table II. The minimum nodal pressure requirement for all nodes except 16 and 17 is 255 ft and for nodes 16 and 17, it is 260 ft and 272.8 ft respectively.

TABLE III
PRESSURE HEADS FOR CRITICAL NODES USING EPANET FOR MINIMIZATION OF NETWORK COST

Node	Pressure (ft)			
	[19][1]	[21]	[26]	PSO
16	259.79* (260.16)	260.00	260.00	260.08
17	272.58* (272.86)	272.79*	272.79*	272.87
19	254.80* (255.21)	254.98*	254.98*	255.05

[1]Solution obtained for w=10.5088 is shown in parentheses
*Infeasible nodal pressure

In case of [19], the numeric conversion constant *w* in the Hazen-Williams formula for head-loss is lowered to 10.5088 to achieve the network cost of $37.13 million. The studies by [21] and [26] obtained a network cost of $38.13 million allowing a violation of 0.05 ft from the minimum nodal pressure requirements. Therefore, although these studies evolve a lower network cost than the previous studies, their optimal solutions violate the minimum head constraint, thus making their solutions infeasible. The results of PSO replicated the feasible lowest cost solution obtained by [27], [3] and proved to be a potential alternative optimization technique for solving water distribution network problems. In addition, the number of function evaluations required to converge to the optimal solution is encouraging. Minor changes in the model could make it suitable for similar water distribution network design optimization as well.

VI. Conclusions

Optimal water distribution network design is a computationally complex problem. This paper describes the development of simulation-optimization model (particle swarm optimization algorithm linked with EPANET simulation hydraulic solver). The efficiency of the model is tested with the New York water supply system for minimization of network cost. PSO matched the feasible lowest cost solution of $38.64 million when compared with the earlier studies in the literature. The robustness of the technique is also examined carrying out sensitivity on its governing optimization parameters. It is concluded that PSO is clearly a potential alternative optimization technique for solving water distribution network design problems.

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