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Mechanism of Damping in Welded Structures using Finite Element Approach

B. Singh, B. K. Nanda

Abstract**—**The characterization and modeling of the dynamic behavior of many built-up structures under vibration conditions is still a subject of current research. The present study emphasizes the theoretical investigation of slip damping in layered and jointed welded cantilever structures using finite element approach. Application of finite element method in damping analysis is relatively recent, as such, some problems particularly slip damping analysis has not received enough attention. To validate the finite element model developed, experiments have been conducted on a number of mild steel specimens under different initial conditions of vibration. Finite element model developed affirms that the damping capacity of such structures is influenced by a number of vital parameters such as; pressure distribution, kinematic coefficient of friction and micro-slip at the interfaces, amplitude, frequency of vibration, length and thickness of the specimen. Finite element model developed can be utilized effectively in the design of machine tools, automobiles, aerodynamic and space structures, frames and machine members for enhancing their damping capacity.

Keywords**—**Amplitude, finite element method, slip damping, tack welding.

I. INTRODUCTION

THE development of mathematical models for the mechanism of damping along with techniques adopted to improve the damping capacity of layered structures for controlling the adverse effects of vibrations is emphasized in the studies. The characterization and modeling of the dynamic behavior of many built-up structures under vibration conditions is still a subject of current research.

Welded joints are used to fabricate many assembled structures and a little amount of work has been reported till date on the damping capacity of such structures. [1]-[3] have reported on the techniques of improving the damping capacity of welded structures. However, these analytical techniques are only applicable to simple structures such as; beams or plates with classical boundary conditions. In practice, it is often necessary to design damped structures with complicated geometry, hence it is obvious choice to look for the finite element method (FEM) for a solution. However, its application in slip damping analysis is relatively recent and has not received enough attention. Shastry and Rao [4] used the finite element method to obtain critical frequencies and the stability boundaries for a cantilever column under an intermediate periodic concentrated load for various loading positions. Bauchau and Hong [5] studied the non-linear response and stability analysis of beams using finite element technique. Briseghella et al. [6] studied the dynamic stability problems of beams and frames using finite element method. The present study emphasizes the theoretical investigation of slip damping in layered and jointed welded cantilever structures using finite element approach. To validate the finite element model developed, experiments have been conducted on a number of mild steel specimens under different initial conditions of vibration. Finite element model developed affirms that the damping capacity of such structures is influenced by the parameters such as; interface pressure distribution characteristics, kinematic coefficient of friction, micro-slip, amplitude and frequency of vibration, length and thickness of the specimen. Finite element model developed can be utilized effectively in the design of machine tools, automobiles, aerodynamic and aerospace structures, frames and machine members in order maximize their damping capacity.

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II. FORMULATION USING FINITE ELEMENT METHOD

Layered and tack welded cantilever beam model with uniform pressure distribution at the interfaces is shown in Fig. 1.

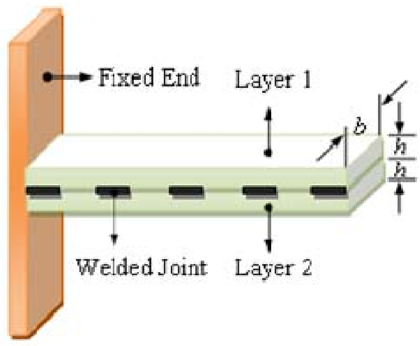

Fig. 1 Two layered tack welded cantilever beam model

A. The displacement description

The finite element model of the damped welded two-layer beam is shown in Fig. 2. It is assumed that every layer has the same transverse displacement as established by [7]. At each node *n*, four displacements $\{\mathbf{q}_n\}$ are introduced, these being the transverse displacement w_n, the rotation θ_n and the axial displacements u_{n1}, u_{n2} of the middle planes of these elastic layers.

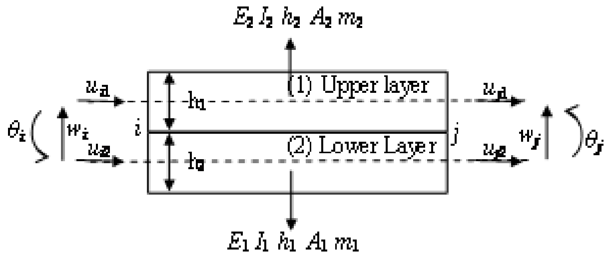

Fig. 2 Finite element model for the damped layered and welded beams

The total set of nodal displacements for the element is given by:

$$\left\{q^e\right\} = \left[w_i \quad \theta_i \quad u_{i1} \quad u_{i2} \quad w_j \quad \theta_j \quad u_{j1} \quad u_{j2}\right]^T \tag{1}$$

With the traditional polynomial shape functions, the displacement field vector {d} may be written as;

$$\begin{Bmatrix} w \\ \theta \\ u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} N \\ N' \\ N_1 \\ N_2 \end{bmatrix} \left\{q^e\right\} \tag{2a}$$

where

$$\left[N_1\right] = \left\{0 \quad 0 \quad 1-\xi \quad 0 \quad 0 \quad 0 \quad \xi \quad 0\right\} \tag{2b}$$

$$\left[N_2\right] = \left\{0 \quad 0 \quad 0 \quad 1-\xi \quad 0 \quad 0 \quad 0 \quad \xi\right\} \tag{2c}$$

$$\left[N\right] = \left\{1-3\xi^2+2\xi^3 \quad \left(\xi-2\xi^2+\xi^3\right)L \quad 0 \quad 0 \quad 3\xi^2-2\xi^3 \quad \left(-\xi^2+\xi^3\right)L \quad 0 \quad 0\right\} \tag{2d}$$

$$\left[N'\right] = \left[\frac{\partial N}{\partial x}\right] = \frac{1}{L}\left[\frac{\partial N}{\partial \xi}\right] \tag{2e}$$

$\xi = x/L$, L=element length

B. Element stiffness matrix

The stiffness matrix for the jointed element is obtained from the bending and extensional strain energies as follows:

$$U_{be} = \frac{1}{2}\int_V \left(\varepsilon_1\sigma_1 + \varepsilon_2\sigma_2\right) dV \tag{3}$$

$$U_{be} = \frac{1}{2}\int_0^L \left(E_1A_1\left(\frac{\partial u_1}{\partial x}\right)^2 + E_2A_2\left(\frac{\partial u_2}{\partial x}\right)^2 + \left(E_1I_1 + E_2I_2\right)\left(\frac{\partial^2 w}{\partial x^2}\right)^2 \right) dx \tag{4}$$

$$U_{be} = \frac{1}{2}\left\{q^e\right\}^T \int_0^1 \begin{pmatrix} \frac{E_1A_1}{L}\left[N_1'\right]^T\left[N_1'\right] + \frac{E_2A_2}{L}\left[N_2'\right]^T\left[N_2'\right] \\ +\left(\frac{E_1I_1+E_2I_2}{L^3}\right)\left[N''\right]^T\left[N''\right] \end{pmatrix} d\xi \left\{q^e\right\} \tag{5}$$

$$U_{be} = \frac{1}{2}\left\{q^e\right\}^T \left[K\right]^e \left\{q^e\right\} \tag{6}$$

Therefore element stiffness matrix is given by;

$$\left[K\right]^e = \int_0^1 \begin{pmatrix} \frac{E_1A_1}{L}\left[N_1'\right]^T\left[N_1'\right] + \frac{E_2A_2}{L}\left[N_2'\right]^T\left[N_2'\right] \\ +\frac{E_1I_1}{L^3}\left[N''\right]^T\left[N''\right] + \frac{E_2I_2}{L^3}\left[N''\right]^T\left[N''\right] \end{pmatrix} d\xi \tag{7}$$

where, E_i, A_i, I_i are the modulus of elasticity, cross-section area and moment of inertia of the ith layer of the beam. Integrating the expression (7), the element stiffness matrix is found to be as;

$$\left[K\right]^e = \begin{bmatrix} \frac{12EI}{L^3} & & & & & & & \\ \frac{6EI}{L^2} & \frac{4EI}{L} & & & & & & \\ 0 & 0 & \frac{E_1A_1}{L} & & & & & \\ 0 & 0 & 0 & \frac{E_2A_2}{L} & & & & \\ \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & 0 & 0 & \frac{12EI}{L^3} & & & \\ \frac{6EI}{L^3} & \frac{2EI}{L} & 0 & 0 & \frac{-6EI}{L^2} & \frac{4EI}{L} & & \\ 0 & 0 & \frac{-E_1A_1}{L} & 0 & 0 & 0 & \frac{E_1A_1}{L} & \\ 0 & 0 & 0 & \frac{-E_2A_2}{L} & 0 & 0 & 0 & \frac{E_2A_2}{L} \end{bmatrix} \tag{8}$$

where $EI = E_1I_1 + E_2I_2$

C. Element mass matrix

Following a similar procedure, the mass matrix for the jointed and welded beam element is obtained from the kinetic energy as follows:

$$T = \frac{1}{2}\int_0^L \left(m_0\dot{w}^2 + m_1\dot{u}_1^2 + m_2\dot{u}_2^2\right) dx \tag{9}$$

where, m_i is the mass per unit length of the ith layer of the beam element and $m_0 = m_1 + m_2$

$$T = \frac{L}{2}\left\{\dot{q}^e\right\}^T \int_0^1 \begin{pmatrix} m_0\left[N\right]^T\left[N\right] + m_1\left[N_1\right]^T\left[N_1\right] \\ +m_2\left[N_2\right]^T\left[N\right] \end{pmatrix} d\xi \left\{\dot{q}^e\right\} \tag{10}$$

$$T = \frac{1}{2}\left\{\dot{q}^e\right\}^T \left[M\right]^e \left\{\dot{q}^e\right\} \tag{11}$$

Therefore, element mass matrix is given by;

$$\left[M\right]^e = \int_0^1 \begin{pmatrix} m_1\left[N\right]^T\left[N\right] + m_2\left[N\right]^T\left[N\right] \\ +m_1\left[N_1\right]^T\left[N_1\right] + m_2\left[N_2\right]^T\left[N_2\right] \end{pmatrix} d\xi \tag{12}$$

Integrating the expression (12), the element mass matrix is found to be;

$$[M]^e = \begin{bmatrix} \frac{13m_0L}{35} & & & & & & & \\ \frac{11m_0L^2}{210} & \frac{m_0L^3}{105} & & & & & & \\ 0 & 0 & \frac{m_1L}{3} & & & & & \\ 0 & 0 & 0 & \frac{m_2L}{3} & & & & \\ \frac{9m_0L}{70} & \frac{13m_0L^2}{140} & 0 & 0 & \frac{13m_0L}{35} & & & \\ \frac{-13m_0L^2}{420} & \frac{-m_0L^3}{140} & 0 & 0 & \frac{-11m_0L^2}{210} & \frac{m_0L^3}{105} & & \\ 0 & 0 & \frac{m_1L}{6} & 0 & 0 & 0 & \frac{m_1L}{3} & \\ 0 & 0 & 0 & \frac{-m_2L}{6} & 0 & 0 & 0 & \frac{m_2L}{3} \end{bmatrix} \quad (13)$$

D. Element damping matrix

Element damping matrix is given by;

$$[C] = \alpha[M] + \beta[K] \quad (14)$$

where, α and β are the constants determined from the experimental results

E. Evaluation of eigenvalues and eigenvectors

The basic computational eigen-solution is determined in terms of mode shape solving the expression as given by;

$$M\ddot{x} + C\dot{x} + Kx = 0 \quad (15)$$

F. Evaluation of loss factor

The eigenvalue problem is solved with a real stiffness matrix to obtain the real modal parameters for the un-damped structure. The loss factor for the damped structure is obtained using the modal strain energy method which is given by;

$$\eta = \frac{\sum_e \{X^e\}^T \{C^e\}\{X^e\}}{\sum_e \{X^e\}^T \{K^e\}\{X^e\}} \quad (16)$$

where $\{X^e\}$, $\{C^e\}$ and $\{K^e\}$ are the mode shape vector, element damping and stiffness matrices respectively.

III. EXPERIMENTAL SET-UP, EXPERIMENTS AND RESULTS

An experimental set-up as shown in Fig. 3 has been fabricated to conduct the experiments. The specimens are prepared from the stock of mild steel flats by tack welding two layers of various thickness and cantilever length. The cantilever specimens are excited at the amplitudes of 0.1, 0.2, 0.3 and 0.4 mm at their free ends with the help of an exciter. The input excitation and output vibration are sensed with vibration pick-ups and the corresponding signal is fed to a digital storage oscilloscope which is connected to the computer with vibration analyzer software i. e., Lab View of National Instruments limited.

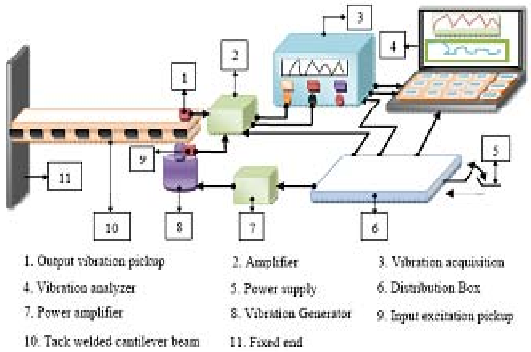

Fig. 3 Experimental set-up

The frequency response for the first four modes of vibration has been generated. Damping matrix is then evaluated using the expression (14) from the corresponding frequency response curves. The frequency response curves at different amplitudes of excitation are shown in Figs. 4 and 5. The loss factor is then evaluated theoretically using the expression (16). Further, the experimental loss factor is evaluated using half power band width method and experimental FRF curves. These numerical results as well as the corresponding experimental ones are plotted as solid and dotted lines respectively for comparison as shown in Figs. 7 and 8. It is observed that both the curves are in good agreement with a maximum variation of 3.8% which shows the authenticity of the theoretical analysis

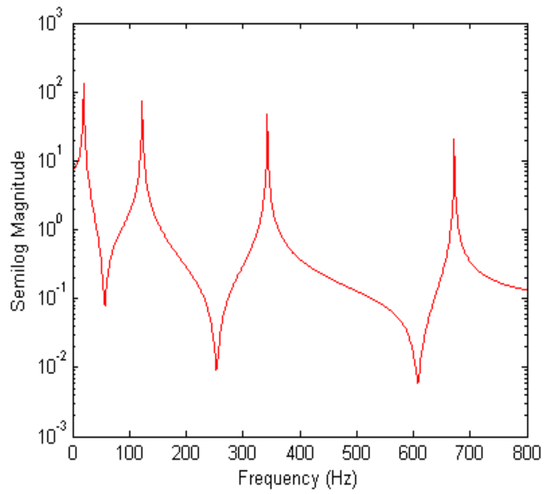

Fig. 4-a FRF plot at 0.1 mm initial amplitude of excitation

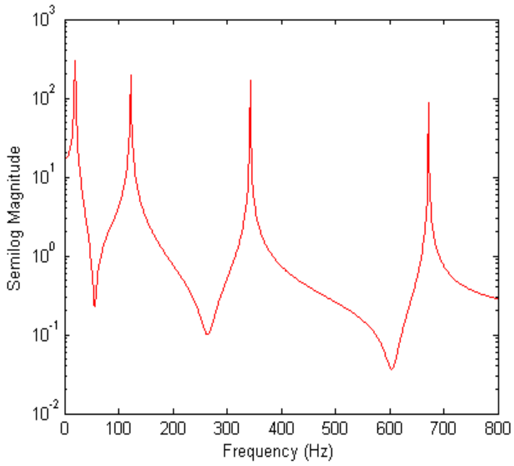

Fig. 4-b FRF plot at 0.2 mm initial amplitude of excitation

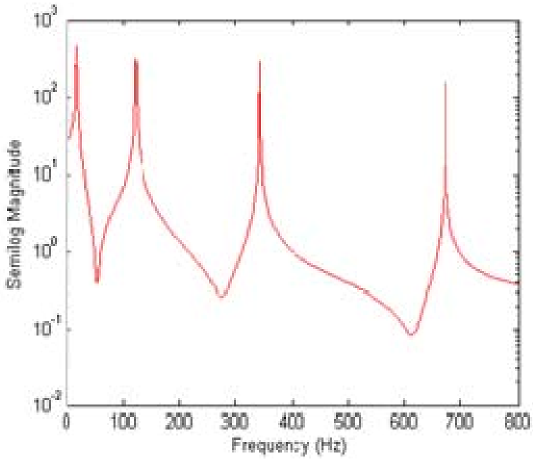

Fig. 5-a FRF plot at 0.3 mm initial amplitude of excitation

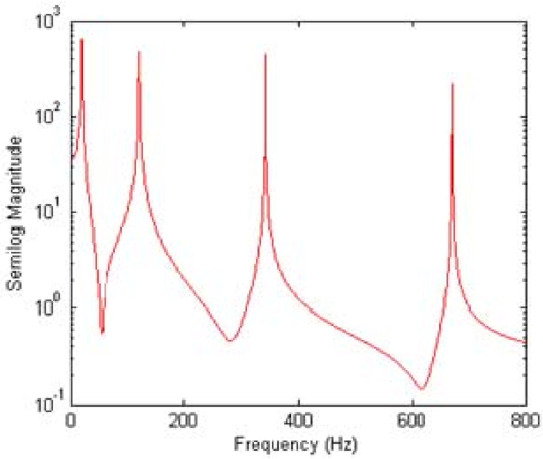

Fig. 5-b FRF plot at 0.4 mm initial amplitude of excitation

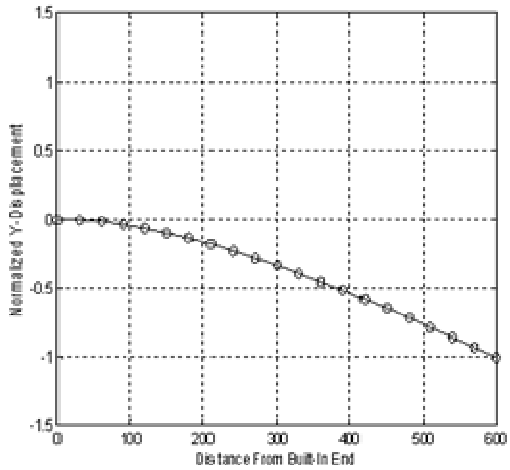

Fig. 6-a Mode shape for first mode

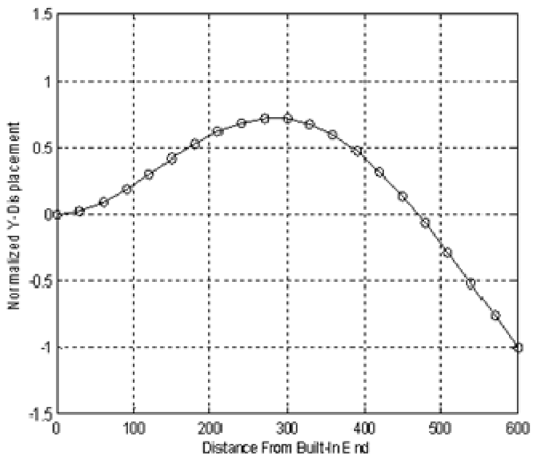

Fig. 6-b Mode shape for second mode

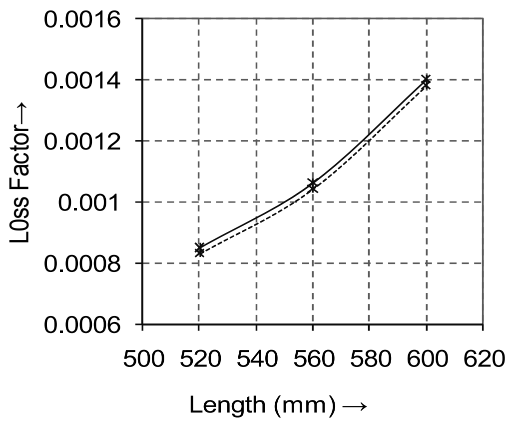

Fig. 7-a Variation of loss factor with length of the cantilever beams

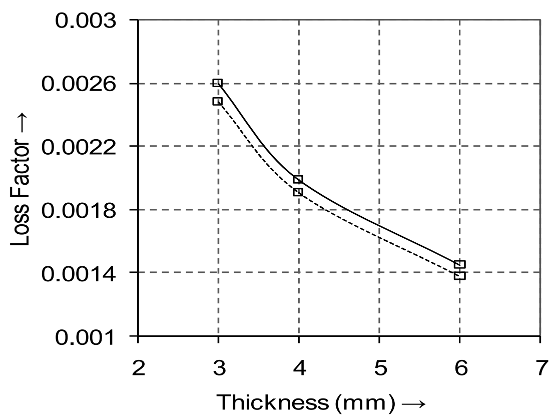

Fig. 7-b Variation of loss factor with thickness of the welded cantilever beams

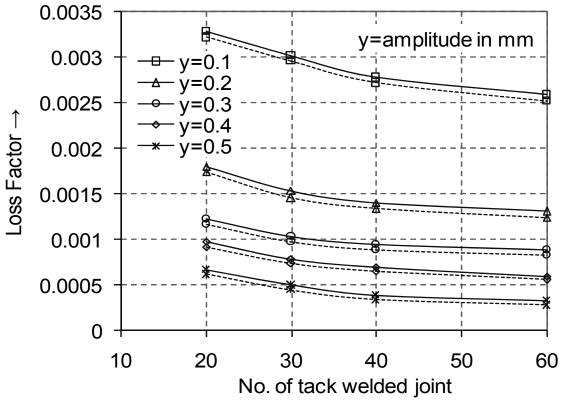

Fig. 8-a Variation of loss factor with amplitude at various number of tack welded joints

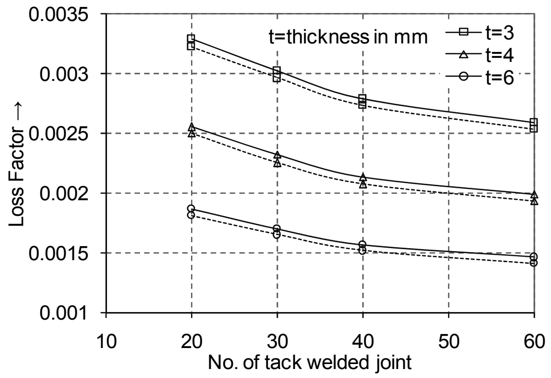

Fig. 8-b Variation of loss factor with amplitude and thickness of the welded cantilever beams at various number of tack welded joints

IV. CONCLUSION

In the present work, a new technique of analyzing layered and jointed beams with slip damping at the interfaces has been proposed considering the finite element method. Unlike finite element method, existing numerical methods use displacement models defined over the whole structure for analysis of slip damping. Finally, experiments are conducted to validate the theory developed. From both the finite element analysis and experimental strategy, it is concluded that the damping capacity of structures jointed with welding is influenced by the parameters such as; intensity of pressure distribution and micro-slip at the interfaces, techniques of welding, relative spacing between tack welds, number of tack joints, amplitude and frequency of vibration, length and thickness of the beam specimens. Finally, it is established that the damping capacity of the layered and jointed welded structures can be improved largely by fabricating the same with tack welds instead of continuous welds. Moreover, more relative spacing between the consecutive tack welds with less no. of tack joints under smaller amplitude of excitation will enhance the damping capacity substantially. The results plotted in the figures and table show the coherence between the experimental and finite element analysis. Therefore, the practicing engineers can use these results to estimate the natural frequencies and mode shapes of damped layered and jointed welded structures accurately. Further, the technique of finite element analysis will strengthen the designers to estimate the dynamic performance of the structures to tailor their dynamic behavior as per the requirements.

REFERENCES

[1] B. M. Belgaumkar, and A. S. R. Murty, “Effect of root fixture conditions on the damping characteristics of cantilever beam,” *Journal of science and Engineering research*, *India*, vol. 12, no.1, pp.147-154, 1968.

[2] M. Masuko, I. Yoshimi, and Y. Keizo, “Theoretical analysis for a damping ration of a jointed cantibeam,” *Bull. JSME*, vol. 16, no.99, pp. 1421-1433, 1973.

[3] N. Nishiwaki, M. Masuko, Y. Ito, and I. Okumura, “A study on damping capacity of a jointed cantilever beam (1st report; experimental results),” *Bulletin of JSME*, vol. 21, no. 153, pp. 524-531, 1978.

[4] B.P. Shastry, and G.V. Rao, “Dynamic stability of a cantilever column with an intermediate concentrated periodic load,” *Journal of Sound and Vibration*, vol. 113, pp. 194 – 197, 1987.

[5] O.A. Bauchau, and C.H. Hong, “Nonlinear response and stability analysis of beams using finite elements in time,” *AIAA Journal*, vol. 26, pp. 1135–1141, 1998.

[6] G. Briseghella, C.E. Majorana, and C. Pellegrino, “Dynamic stability of elastic structures: a finite element approach,” *Computer and structures*, vol. 69, pp. 11-25, 1998.

[7] M. G. Sainsbury, and Q. J. Zhang, “The Galerkin Element Method Applied to the Vibration of Damped Sandwich Beams,” *Computer and structures*, vol. 71**,** no. 3, pp. 239–256, 1999

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