M-band Wavelet and Cosine Transform Based Watermark Algorithm Using Randomization and Principal Component Analysis

Tong Liu, Xuan Xu, Xiaodi Wang

Abstract**—**Computational techniques derived from digital image processing are playing a significant role in the security and digital copyrights of multimedia and visual arts. This research presents discrete M-band wavelet transform (DMWT) and cosine transform (DCT) based watermarking algorithm by incorporating the randomization and principal component analysis (PCA). The proposed algorithm is expected to achieve higher perceptual transparency. Specifically, the developed watermarking scheme can successfully resist common signal processing, such as geometric distortions, and Gaussian noise. In addition, the proposed algorithm can be parameterized, thus resulting in more security. To meet these requirements, the image is transformed by a combination of DMWT & DCT. In order to improve the security further, we randomize the watermark image to create three code books. During the watermark embedding, PCA is applied to the coefficients in approximation sub-band. Finally, first few component bands represent an excellent domain for inserting the watermark.

Keywords**—**Discrete cosine transform, discrete M-band wavelet transform, principal component analysis, randomization.

I. INTRODUCTION

PCA is a valuable technique that can achieve more robust watermark embedding [6],[12]. And up to now, most of the PCA based watermarking methods are done in projection space; they often connected with some transform approaches such as DMWT or DCT as in [4],[8],[9],[10],[11]. However, watermark algorithms just using MWT-PCA or DCT-PCA are not secured enough to defeat attackers.

Different from most traditional algorithms, we propose a new algorithm which combines MWT and DCT, applies RNG to the watermark image, and then performs PCA to determine the best location to embed the watermark image in order to make it a lot harder to be destroyed. Comparing our method with others, our method should over perform other algorithms by improving the security and robustness of the authentication of images. In fact, using only one of MWT and DCT can only commit encryption once, so only one crucial step is needed to remove the watermark. But here we commit encryption more times; many steps are needed to remove the watermark. So it can be widely used to protect copyrights of multimedia data and digital visual arts.

Tong Liu is with Affiliated High School to Jilin University, Changchun, Jilin Province, 130021, China (email: 1808772401@qq.com).
Xuan Xu is with Affiliated High School to Jilin University, Changchun, Jilin Province, 130021, China (email: laoyingtwo@163.com).
Xiaodi Wang is with the Math Department, Western Connecticut State University, CT 06810 USA (phone: 203-837-9355; e-mail: wangx@wcsu.edu).

Fig. 1 (a) Original image (b) Watermark image

II. MWT-DCT-RNG-PCA

A. Discrete M-band Wavelet Transform (DMWT)

Discrete M-Band wavelet transform uses a set of M filter banks (M≥2) to break a signal into M different frequency levels. Daubechies wavelets are classical 2-Band wavelets. A 3-Band wavelet transform decomposes an image into one approximation (low frequency) component and 8 detail (high frequency) components. The 2D discrete *M*-Band wavelet transform of a 2D image F is done by multiplying a wavelet transform matrix to the left side of input image, and then by the transpose of the wavelet transform matrix to the right side of input image, WFW^T, where W is the wavelet transform matrix which is orthonormal, and hence $W^T=W^{-1}$.

In order to apply DMWT to a color image, we decompose the color image into three matrices：

$$\text{Red matrix:} \quad F_1=\begin{bmatrix} F_1(1,1) & \cdots & F_1(1,n) \\ \vdots & \ddots & \\ F_1(n,1) & \cdots & F_1(n,n) \end{bmatrix} \quad \vdots \tag{1}$$

$$\text{Green matrix:} \quad F_2=\begin{bmatrix} F_2(1,1) & \cdots & F_2(1,n) \\ \vdots & \ddots & \\ F_2(n,1) & \cdots & F_2(n,n) \end{bmatrix} \quad \vdots \tag{2}$$

$$\text{Blue matrix:} \quad F_3=\begin{bmatrix} F_3(1,1) & \cdots & F_3(1,n) \\ \vdots & \ddots & \\ F_3(n,1) & \cdots & F_3(n,n) \end{bmatrix} \quad \vdots \tag{3}$$

We then apply DMWT to each one of them to obtain

$$G_1= WF_1 W^T,\ G_2= WF_2W^T,\ G_3= WF_3W^T \tag{4}$$

respectively.

An example of 3-band wavelet transform matrix W is given below:

$$W=\begin{bmatrix}
\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 \\
\alpha_4 & \alpha_5 & \alpha_6 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_1 & \alpha_2 & \alpha_3 \\
\beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 & \beta_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 & \beta_6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 & \beta_6 \\
\beta_4 & \beta_5 & \beta_6 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_1 & \beta_2 & \beta_3 \\
\gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & \gamma_5 & \gamma_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & \gamma_5 & \gamma_6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & \gamma_5 & \gamma_6 \\
\gamma_4 & \gamma_5 & \gamma_6 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3
\end{bmatrix} \tag{5}$$

where

$$\begin{aligned}\boldsymbol{\alpha}=[&0.33838609728386, 0.53083618701374,\\ &0.72328627674361, 0.23896417190576,\\ &0.04651408217589, -0.14593600755399]\\ \boldsymbol{\beta}=[&-0.11737701613483, 0.54433105395181,\\ &-0.01870574735313, -0.69911956479289,\\ &-0.13608276348796, 0.42695403781698]\\ \boldsymbol{\gamma}=[&0.40363686892892, -0.62853936105471,\\ &0.46060475252131, -0.40363686892892,\\ &-0.07856742013185, 0.24650202866523]\end{aligned} \quad (6)$$

It's easy to verify that

$$\sum_{i=1}^{6} \alpha_i = \sqrt{3} \quad (7)$$

$$\sum_{i=1}^{6} \beta_i = \sum_{i=1}^{6} \gamma_i = 0 \quad (8)$$

$$\|\boldsymbol{\alpha}\| = \|\boldsymbol{\beta}\| = \|\boldsymbol{\gamma}\| = 1 \quad (9)$$

$$\boldsymbol{\alpha}\cdot\boldsymbol{\beta} = \boldsymbol{\beta}\cdot\boldsymbol{\gamma} = \boldsymbol{\gamma}\cdot\boldsymbol{\alpha} = 0 \quad (10)$$

This wavelet matrix can be extended to any size of $3^n k \times 3^n k$, where $n, k \in N$.

Fig. 2 (a) Original image (b) DMWT of the image

B. Discrete Cosine Transform (DCT)

A discrete cosine transform (DCT) expresses a sequence of data points in terms of cosine functions oscillating at different frequencies. DCTs are important to numerous applications in science and engineering, such as lossy compression of audio and images, where small high-frequency components can be discarded. Based on the following formula, we can get the 2-D DCT of an $n \times m$ image. Define

$$c_u = \begin{cases} \sqrt{1/n} & \text{if } u = 0 \\ \sqrt{2/n} & \text{if } u = 1, \ldots, n-1 \end{cases} \quad (11)$$

$$d_v = \begin{cases} \sqrt{1/m} & \text{if } v = 0 \\ \sqrt{2/m} & \text{if } v = 1, \ldots, m-1. \end{cases} \quad (12)$$

Then the DCT of image I is given by

$$H_{uv} = c_u d_v \sum_{x=0}^{n-1} \cos\frac{\pi(2x+1)}{2n} \sum_{y=0}^{m-1} I_{xy} \cos\frac{\pi(2y+1)}{2m} \quad (13)$$

where $u = 0,1 \cdots n-1; v = 0,1 \cdots m-1$.

The inverse of DCT is then given by:

$$I_{xy} = \sum_{u=0}^{n-1}\sum_{v=0}^{m-1} c_u d_v H_{uv} \cos\frac{\pi(2x+1)u}{2n} \cos\frac{\pi(2y+1)v}{2m} \quad (14)$$

where $x = 0,1 \cdots n-1; y = 0,1 \cdots m-1$.

C. Random Number Generator (RNG)

To enhance the security of watermark further, we apply a random number generator to randomize watermark image. But it's impossible to generate real random numbers; so we use pseudo-RNG instead, which is enough for our research. Suppose the watermark matrix $J = [J_{ij}]$ is $n \times n$. Define

$$f(J_{ij}) = B_{ij} = \begin{cases} 2J_{ij} & \text{if } J_{ij} \in [0,127] \\ J_{ij} & \text{if } J_{ij} \in [128,255]. \end{cases} \quad (15)$$

At each position (i, j), we apply RNG to the interval (0, 1) to generate r_{ij}. Define

$$g: (0,1) \to \{ n/255 \mid n \in N, 8 \le n \le 255 \} \quad (16)$$

by

$$g(r_{ij}) = R_{ij} = fix(8 + 248 r_{ij})/256 \quad (17)$$

where

$$fix(x) = \begin{cases} \lfloor x \rfloor & \text{if } \{x\} \in [0, 0.5) \\ \lceil x \rceil & \text{if } \{x\} \in [0.5, 1]. \end{cases} \quad (18)$$

The code book is then created with $R = [R_{ij}]$. Finally we compute the randomized watermark $K = [K_{ij}]$, where

$$K_{ij} = B_{ij} R_{ij} \quad (19)$$

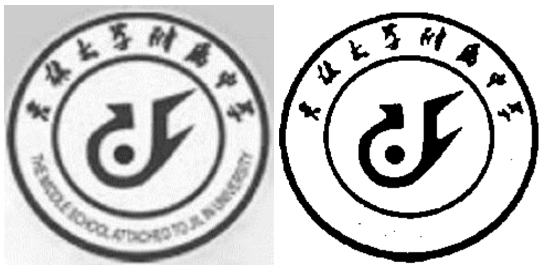
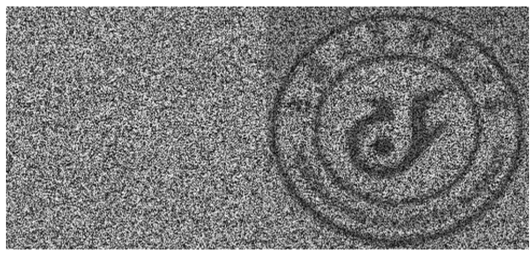


Fig. 3 (a) Original watermark (b) Binary watermark



(c) Code book (d) Randomized watermark

D. Principal Components Analysis (PCA)

Principal component analysis (PCA) is a mathematical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of de-correlated variables called principal components. The number of principal components is less than or equal to the number of original variables. This transformation is defined in such a way that the first principal component has a variance as high as possible (maximum), and each following principal component has the highest variance possible comparing with the rest of the components, and is orthogonal to all the components proceeding to it. PCA is sensitive to the relative scaling of the original variables.

In this research, we apply PCA in our watermarking scheme which combines two transformations: DMWT and DCT. In our algorithm, the greatest variation data happens to the first principal component, the second greatest variation to the second principal component, and so on. This transformation orthogonalizes the components of the input data vectors so that they are completely de-correlated, and most of the energy is concentrated into the first several principal components. Due to the excellent energy compaction property, components that contribute the least variation in the data set are eliminated without much loss of information. The PCA does not have a fixed set of basis functions, instead its basis functions depend on the data set only.

III. Watermark Embedding Procedure

To embed watermark, we carry out the following steps:

Step 1**:** Apply DMWT to the three colored image matrices F_k and denote the DMWT of F_k by G_k and approximation sub-band matrix of G_k by A_k , $k = 1,2,3$.

Step 2**:** Apply DCT to A_k to obtain H_k.

Step3: Apply RNG to the watermark image described in previous section to obtain randomized watermark matrix K.

Step 4: Decompose H_k into $l \times l$ sub-matrices H_{mn}. Compute

$$H'_{mn,ij} = H_{mn,ij} - \frac{\sum_{k=1}^{l} H_{mn,kj}}{l}, i\ , j\ =\ 1, \dots, l \qquad (20)$$

so that the sum of each column of H'_{mn} is 0.

Step 5: Compute

$$X_{mn} = H_{mn}' \times H_{mn}'^{T} \qquad (21)$$

Step 6: Compute the eigenvectors and eigenvalues of X_{mn}, and then sort the eigenvalues in descending order. Also sort the eigenvectors in the same order as eigenvalues; the matrix made up of eigenvectors is denoted by R_{mn}. Compute

$$P_{mn} = R_{mn}^{T} H'_{mn} \qquad (22)$$

then P_{mn} is the principal component corresponding to H'_{mn}, and it's used for embedding watermark.

Step 7: Decompose the watermark matrix K into $l \times\ l$sub-matrices K_{mn}. Let e be the strength of the watermark, then compute

$$P'_{mn} = P_{mn} + e\ K_{mn} \qquad (23)$$

Step 8: Apply IPCA to P'_{mn} to get

$$H_{mn}'' = R_{mn} P'_{mn} \qquad (24)$$

and then combine all sub-matrices H_{mn}'' to create the modified cosine transform matrix H'_k.

Step 9: Apply IDCT to H'_k to get $A_k{}'$.

Step 10: Replace the approximation sub-band of G_k by $A_k{}'$ without changing the detail sub-matrices, and call the new matrix $G_k{}'$, and then apply IDWT to obtain

$$F'_k = W^T G'{}_k{}' W \qquad (25)$$

Combine $F'_1, F'_2,$ and F'_3 to obtain watermarked image F'.

IV. Experimental Results

The randomized logo of our school is embedded in the original image, "the cloud and sky", as the watermark. To our surprise, the watermark is much more robust than we expected. We thought the extracted watermark would be beyond recognition, but it didn't.

Fig. 4 (a) Original image (b) Watermarked image

The purpose of the watermark is to secure the authentication of a multimedia data or visual art work. There are always some unlawful people trying to attack or destroy the watermark by all means possible, for instance, rotating, cutting, distorting, even Gaussian noise, etc. So it's important to protect the products and art works from being copied or stolen. The most important indicators of watermark are the security and robustness. A watermark with bad security and robustness can be attacked or destroyed easily, and this makes it meaningless.

The security of our algorithm is excellent as explained in previous sections. To check the robustness of the watermark, we have to attack the watermarked image, and then check if we can extract the watermark, and whether it's effective. The followings are the attacked images and statistics results. The examples of attacked images are based on rotating, cutting, and Gaussian noise attacking.

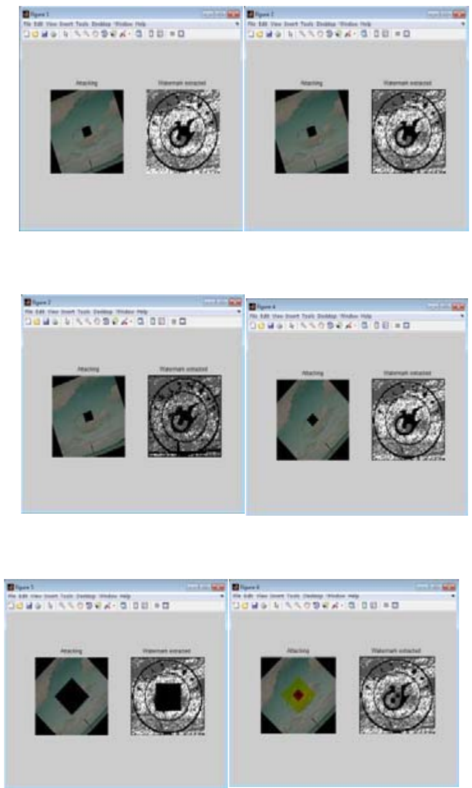


Fig. 5 Attacked images by rotating, cutting, and Gaussian noise

TABLE I
ATTACKING EXPERIMENT RESULTS

	h	*D*	Cutting size	*CH*
1	0.2	20	1/9*1/9	0.7836
2	0.4	20	1/9*1/9	0.7535
3	0.6	20	1/9*1/9	0.5397
4	0.4	45	1/9*1/9	0.7535
5	0.4	45	1/3*1/3	0.7476
6	0.4	45	*complex*	0.7366

In the TABLE I above, *h* is the rating of Gaussian noise attacking; *D* is the rotating angle; *CH* is the similarity of the extracted watermark and the original watermark: 1 means completely the same and 0 means totally different. *Complex* means the cutting sizes are different in RGB matrices.

Our experiment shows that the rotating angle doesn't influence the effective of watermark; light attacking influences the watermark quality slightly (the Gaussian noise level from 0.2 to 0.6). This means the robustness of the watermark has achieved our expectation.

V. CONCLUSION

In this paper, we present a new approach based on MWT, DCT, and RNG, incorporating PCA to digital image watermark. We have shown the efficiency in applying our method for performing watermark embedding and verification. Since we have used two transforms—MWT and DCT, this method greatly increases difficulties for attackers to destroy a watermark. As a result, the watermarked image with such a well-chosen embedding domain is more robust against attacking than watermarks embedded in the MWT or DCT domain only.

In order to enhance the security further, we use 3 code books in the randomized watermark, so that if one code book is stolen by someone, they still can't remove the watermark. Comparing with existing watermark algorithms, the attacking experiment shows that our methods are much safer.

This algorithm may also be used for multimedia files encryption, security information transmission, etc.

ACKNOWLEDGMENT

We would like to thank Affiliated High School to Jilin University, Jilin University, and Western Connecticut State University for their exceptional supports. We realize that without their constant encouragement and necessary funding to complete this research is not possible.

REFERENCES

[1] K. R. Rao and P. Yip, *Discrete Cosine Transform: Algorithms, Advantages, Applications*. San Diego, CA: *Academic Press*, 1990.

[2] J.S. Walker, *A primer on wavelets and their scientific applications*. Boca Raton, FL: Chapman & Hall/CRC, 2008.

[3] P. Steffen, P.N. Heller, R.A. Gopinath, and C.S Burrus, "Theory of regular m-band wavelet bases," *IEEE Trans. Signal Processing*, Vol. 41, pp. 3497-3511, Dec. 1993.

[4] A.K. Mostafa, A.S. Tolba,,F.M. Abdelkader, and H.M. Elhind, "Video Watermarking Scheme Based on Principal Component Analysis and Wavelet Transform," *Int. J. of Computer Science and Network Security,* Vol.9, pp. 45-52, Aug. 2009.

[5] X. Kang, W. Zeng, and J. Huang, "A Multi-band Wavelet Watermarking Scheme," *Int. J. of Network Security,* Vol.6, pp. 121-126, Mar. 2008.

[6] D. Cotting, T. Weyrich, M. Pauly, and M. Gross, "Robust watermarking of point-sampled geometry," in *Proc. SMI Conf.,* pp. 233-242, June 2004.

[7] I.J. Cox, J. Kilian, F. T. Leigton, and T. Shamoon, " Secure spectrum watermarking for multimedia," *IEEE Trans .Image Processing,*Vol.6, pp. 1673-1687, Dec. 1997.

[8] W. Zhu, Z. Xiong, and Y. Zhang, " Multiresolution watermarking for image and video," *IEEE Trans. Circuits and Systems for Video Technology,* Vol.9, pp. 545-550, June 1999.

[9] H. Nikmehr, S.T. Hashemy, "A new approach to audio watermarking using discrete wavelet and cosine transforms," In *Proc. 1st Int. Conf. on Communications Engineering*, pp. 1-10, Dec. 2010.

[10] A. Singh, and A. Tayal, " Performance analysis of digital image watermarking using discrete wavelet transform, discrete cosine transform and singular value decomposition based on PSNR and NC," in 2012 *Proc. Int. Conf. on Computing and Control Engineering*, ICCCECS730, April 2012

[11] N.I. Yassin, N. M. Salem, and M. I. El Adawy, "Block based video watermarking scheme using wavelet transform and principle component analysis," *Int. J. of Computer Science*, Vol. 9, Issue 1, pp. 296-301, Jan. 2012.

[12] M.Luo, and A Bors, "Principal component analysis of spectral coefficients for mesh watermarking," in *Proc15th IEEE Int. Conf. on Image Processing,* pp. 441-444, Oct. 2008.