

# Using Exponential Lévy Models to Study Implied Volatility patterns for Electricity Options

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**Abstract**—German electricity European options on futures using Lévy processes for the underlying asset are examined. Implied volatility evolution, under each of the considered models, is discussed after calibrating for the Merton jump diffusion (MJD), variance gamma (VG), normal inverse Gaussian (NIG), Carr, Geman, Madan and Yor (CGMY) and the Black and Scholes (B&S) model. Implied volatility is examined for the entire sample period, revealing some curious features about market evolution, where data fitting performances of the five models are compared. It is shown that variance gamma processes provide relatively better results and that implied volatility shows significant differences through time, having increasingly evolved. Volatility changes for changed uncertainty, or else, increasing futures prices and there is evidence for the need to account for seasonality when modelling both electricity spot/futures prices and volatility.

**Keywords**—Calibration, Electricity Markets, Implied Volatility, Lévy Models, Options on Futures, Pricing

## I. INTRODUCTION

GIVEN electricity market characteristics [1], we should not expect a model like B&S to offer reasonable fit to derivatives data. It assumes a constant volatility across moneyness and maturity despite empirical evidence showing that variance of returns is not stationary. Also, the relation between volatility and the commodity changes over time. Several models have been proposed as replacements or extensions of the Black-Scholes model for modeling stock prices, but many of these models or variations of them can also be used for pricing commodities. Lévy market models are now actively used in finance and many models have been developed recently, being mainly calibrated on index options [2], currency options [3], [4] and on European options on futures for the crude oil market [5]. Lot less attention has been given for Lévy market models in electricity markets mainly due to data scarcity problems. Implied volatility in commodities markets lack of attention is due to the fact that exchange-traded commodity options are not always very liquid and implied volatilities from their prices can be unreliable (see [6] for oil options on futures markets using semi-parametric and non-parametric approaches to infer about implied volatility dynamics). But electricity options markets are now more liquid than they were when introduced in 2004.

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We do not attempt to describe the entire class of Lévy models or explain their mathematical properties, but yes to apply four known of these models to electricity options on futures data. As such, a jump diffusion model, two infinite activity models (normal inverse Gaussian and variance gamma) and one tempered stable Lévy process (CGMY) have been estimated and compared in terms of fitting with the B&S model. Option price calculations were obtained by the Fourier transform given that characteristic functions of log prices in the risk neutral measure are known analytically. Calibration to market option prices is parametric and performed through minimizing an objective function, which is a weighted sum of squared deviations of the model and market option prices.

Lévy processes are seen as a random walk in continuous time with jumps occurring at random times. The probability distributions associated with Lévy processes are infinitely divisible and offer more flexibility for fitting financial data, and can have skewed shapes and slow decaying tails (characteristics of electricity markets). Moreover, the model's implied volatility tends to vary both in relation to the state and time, exhibiting a smile or a smirk.

The dynamics of the electricity price may exhibit jumps of different sizes, with small jumps occurring more often than large jumps, leading both to asymmetries and fat tails in electricity returns, contradicting thus the Brownian motion assumption underlying the [7] model. Consequently, extracting densities from option prices allows one to analyze the role of electricity as an asset, besides its role as a commodity, and improves electricity market modeling.

We were thus able to raise the following questions: Among the Lévy models selected, what model best describes a market with special characteristics such as the European Energy Exchange (EEX) electricity market in Germany, and which is better suited for pricing electricity futures options? Given that the level of implied volatilities changes with time, deforming the shape of the implied volatility surface [8], we also try to answer: How does implied volatility behaves for electricity options on futures over time? As such, we aim at analyzing the patterns of implied volatility in electricity markets, similar to [5] that compares two implied volatility reconstruction methodologies for European type options on commodity futures (WTI crude oil). Options allow flexibility in dealing with price risk providing profit from favorable market moves, being useful for hedging. Given that implied volatility is useful to infer information about market's expectations of future price movements; they are a relevant indicator of the uncertainty inherent to the electricity market.

No special attention has been given to the patterns of implied volatility through time in electricity markets, although

it is highly investigated in other financial and commodities markets. Reference [8] study the joint dynamics of all implied volatilities quoted on the S&P500 index options market, while [9] question whether the evolution of implied volatility can be forecasted by studying a number of European and US implied volatility indices. Previously, [10] had explored the ability of alternative continuous-time diffusion and jump-diffusion processes to capture the dynamics of implied volatility indices over time for European and American implied volatility indices and CBOE volatility futures market. They found that the addition of jumps is necessary to capture the evolution of implied volatility indices.

Many models have been tested in the context of the special electricity markets. However, none of these have performed an empirical comparison between Jump diffusion, CGMY and the broader class Lévy market models, in the European Energy Exchange (EEX) market. This study is primarily an empirical comparison of these models and their applicability in the context of options on futures electricity markets. So, we aim to extend the option pricing literature through an empirical comparison of the stated models. Our findings will provide insight into the comparative strengths and weaknesses of alternative model structures, given that existing literature has shown that a model with smaller pricing errors is the best model.

The rest of the work develops as follows. Section 2 exposes the data to be used on the paper, while in section 3 the processes to be used are presented. Section 4 discusses the risk neutral parameter estimation methodology. In section 5 we present and discuss the results attained using the processes of section 3, while section 6 concludes the work, also providing directions for future research.

## II. DATA

This work uses data on EEX European option prices over futures, from one up to the next 5 months, having we randomly selected to use 4 months to maturity call and put options. Maturity is specified as the time at which the respective option concerned can be traded for the last time and exercised. The data covers the period 8 November 2004 until 27 March 2008. Our sample of observations to implied volatility computations correspond to a total of 853 days. We exclude observations of option premiums of 0.001 (the minimum tick value allowed) because these values might be a noisy estimate of the true value of the option. Also, since most of the observations excluded correspond to short term out-of-the-money options, we end up reducing liquidity-related biases.

At the beginning of the sample period, given market immaturity, we had much lower observations and therefore more observations were excluded there. However, as the market evolved liquidity increased and more option contracts end up being traded. Like [11], the joint data set of call and put options is used for implicit parameter estimation. In other implicit parameter estimation studies, [12], only call options are used to simplify estimation. Good models should capture

the behavior of the series over time and using more information should lead to better results.

We have used the entire set of call and put options for parameters estimation. With these parameters we have obtained call prices, those that will be used in the implied volatility estimations through time for each of the processes considered. For the options used on the implied volatility computations we have assumed a moneyness interval of 0.6 to 1.3, treating these as the at-the-money (ATM) options of our sample.

An implied volatility corresponding to an option maturing in 4 months is a forecast of the average volatility over the next 4 month period, whereas the shape of the implied volatility surface can be used to assess the adequacy of an option pricing model: if an option pricing model is correct, then there should be no shape to the implied volatility surface.

## III. MODELS

How to model power prices? This is an issue investigated extensively in the literature. With respect to energy markets, [13] propose a class of stochastic mean reverting models for electricity prices with Lévy process driven Ornstein-Uhlenbeck (O-U) processes being the building blocks. Reference [14] model spot prices in oil and natural gas markets with exponential non-Gaussian O-U processes introducing Lévy processes as the driving noise rather than Brownian Motion (BM), and imposing the normal inverse Gaussian distribution for the Lévy increments they obtain a superior fit (NIGOU model).

Here, it is employed Lévy processes as an underlying asset to calculate option prices and implied volatility surfaces. The Lévy process,  $(L_t)_{t \geq 0}$  characterized by its characteristic triplet  $(\mu, \sigma, \nu)$  can recreate features of option prices such as the smile or sneer behavior in the implied volatility surface.

All the models to be discussed belong to a family of Lévy Processes called "exponential Lévy processes". In this class of Lévy processes, the risk-neutral dynamics of the underlying asset is given by

$$S_t = S_0 \exp(L_t) \quad (1)$$

where  $L_t$  is a Lévy process under the equivalent martingale measure  $Q$  with characteristic triplet  $(\mu, \sigma, \nu)$ . The log returns  $\log(S_{t+s}/S_t)$  of such a model follow the distribution of increments of length  $s$  of the Lévy process  $L_t$ . Our spot price here will be with respect to futures market quotes ( $F_t=S_t$ ).

The absence of arbitrage then imposes that  $\hat{S}_t = S_t e^{-rt} = e^{-rt} e^{L_t}$  be a martingale, and  $r$  is the interest rate. Under the condition that  $S_t e^{-rt}$  satisfies a martingale, Exponential-Lévy models allow us to use Fourier transform methods for option pricing because of the availability of closed-form expressions for characteristic functions of Lévy processes calculated by the Lévy-Khinchin representation. With this we are able to derive the risk-neutral characteristic function for the discounted Lévy process. Different exponential Lévy models proposed in the financial modeling

literature simply correspond to different choices for the Lévy measure  $\nu$  (and for the Gaussian part  $\sigma$  if present).

Before going ahead, we need to define a Lévy process called subordinator which has an important role in the construction of other Lévy processes.

*Definition 1: A subordinator is an increasing (in  $t$ ) Lévy process. Equivalently, for  $S$  to be a subordinator, the triplet must satisfy  $\nu(-\infty, 0) = 0$ ,  $c = 0$ ,  $\int_{(0,1)} \nu(dl) < \infty$  and  $\mu = b + \int_{(0,1)} \nu(dl) > 0$ .*

These are called subordinators because they can be used as time changes for other Lévy processes. The Poisson, the Gamma and the inverse Gaussian process are examples of subordinators.

*Theorem [Lévy-Kinchin representation]:* Let  $(L_t)_{t \leq 0}$  be a Lévy process on  $\mathbb{R}^d$  with Lévy triplet  $(A, \nu, \gamma)$ . Then its characteristic function  $\phi_L$  and characteristic exponent  $\varphi_L$  are given by:

$$\begin{aligned} \phi_L &= E[e^{iz'L_t}] = e^{t\varphi_L(z)} \\ \varphi_L(z) &= -\frac{1}{2}z'Az + i\gamma'z + \int_{\mathbb{R}^d} (e^{iz'l} - 1 - iz'l1_{|l| \leq 1}) \nu(dl) \end{aligned} \quad (2)$$

$(L_t)_{t \leq 0}$  is a Lévy process of finite variation if and only if its Lévy triplet is given by  $(0, \sigma, \nu)$  with  $\int_{\mathbb{R}^d} |l| \nu(dl) < \infty$ .

Here we consider Lévy processes, widely acknowledged in financial literature, which include the two-categories: jump-diffusion models and infinite activity models. The jump diffusion model included is the Merton model and infinite activity models considered here are the normal inverse Gaussian and variance gamma. Moreover, we also apply the tempered stable Lévy process, CGMY. Before presenting them, it is described the Fourier transform method for option pricing.

#### A. Fast Fourier Transform (FFT)

The pricing method by Fourier transforms (due to [15]) is a widely used method to price options in financial models when the risk neutral density of the underlying asset is not given in an analytically tractable form, however the characteristic function, which also fully describes the probabilistic behavior of the underlying, can be evaluated easily. By now, there is a large variety of Fourier-based pricing algorithms [16], yet we restrict the discussion to one of the most common versions.

A method based on Monte Carlo simulation is inefficient, because of slow convergence due to the large magnitude of the jumps, and to inherent difficulties in identifying the optimal exercise policy. Knowing the characteristic function of the Lévy process paves the way for a Fourier approach for pricing options on the spot. We refer to [15] for complete descriptions of the fast Fourier transform technique for calculating option prices when the characteristic function of the log-price is known.

To price derivatives written on electricity futures one needs to take into account the risk preferences of the investors. This is traditionally described by a market price of risk charged for issuing the derivative, which turns out to be an additional

parameter coming from the equivalent martingale measure. Options on futures cannot be calculated explicitly, unless we know the characteristic function of the logarithmic spot prices given the Lévy process and the market price of risk. This is the necessary input for a numerical approach for pricing based on the fast Fourier transform (FFT). Reference [15] invoked the fast Fourier transform for its speed in pricing options. Though adequate for near money options the method is known to break down for deep out the money options where it often gives rise to negative prices.

For this presentation we begin by working with log-price  $s_T = \ln(S_T)$ . This is due to the fact that as we have  $S_0 = 1$ , then  $S_T = \exp(L_T) \Leftrightarrow s_T = \ln(\exp(L_T)) = L_T$ .

The log-price process is then totally described by the risk-neutral Lévy process we use, whose characteristic function is thus the risk-neutral characteristic function of the Lévy process, being

$$\phi_T(u) = E[\exp(ius_T)] = E[\exp(iuL_T)] \equiv \int_{-\infty}^{+\infty} e^{ius} q_T(s) ds \quad (3)$$

Therefore, even though any risk-neutral Lévy process can take on negative values, the exponential of this process is always positive, which is a requisite for a process describing the path of a stock price.

With  $s_T = \ln(S_T)$ ,  $S_T = F_T$ , and considering  $k = \ln(K)$ , being  $K$  the strike price of the option, the value of a European call option, related to the risk neutral density  $q_T$ , with maturity  $T$  as a function of  $k$  is given by

$$C_T(k) = e^{-rT} E[(S_T - K)^+] = \int_k^{\infty} e^{-rT} (e^s - e^k) q_T(s) ds \quad (4)$$

being  $q_T(s)$  the risk-neutral density function of  $s$ . If it is possible to compute the Fourier transform (or characteristic function) of the last equation, then we can derive the price of the call option through the inverse transformation.

Reference [15] define a modified call price function (square integrable for a range of  $\alpha$  values and  $\forall k$ ) given by

$$c_T(k) = e^{-\alpha k} C_T(k), \alpha > 0 \quad (5)$$

given that  $C_T(k)$  is not square integrable (it tends to  $S_0$ , a positive constant, as  $k$  tends to  $-\infty$ ), and so we can't compute its Fourier transform. Now, the modified call price  $c_T(k)$  is square integrable in  $k$  over the whole range for proper positive values of  $\alpha$ , allowing us to consider the Fourier transform of  $c_T(k)$ . If in applications all processes have finite variances, setting  $\alpha=1$  seems to be a valid choice. Though the choice appears arbitrary to some extent, we may not find any improvement in results when changing  $\alpha$  to other values.

The parameter  $\alpha$  is the dampening factor, being the Fourier transform and inverse Fourier transform of  $c_T(k)$  given by:

$$F_{c_T}(v) = \varphi_T(v) = \int_{-\infty}^{\infty} e^{ivk} c_T(k) dk \quad (6)$$

$$c_T(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ivk} F_{c_T}(v) dv = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ivk} \varphi_T(v) dv$$

Thus,

$$C_T(k) = e^{-\alpha k} c_T(k) = e^{-\alpha k} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ivk} \varphi_T(v) dv \quad (7)$$

Given that we can write

$$\begin{aligned} \int_{-\infty}^{\infty} e^{-ivk} \varphi_T(v) &= \int_0^{\infty} e^{-ivk} \varphi_T(v) + \int_{-\infty}^0 e^{-ivk} \varphi_T(v) \\ &= \int_0^{\infty} e^{-ivk} \varphi_T(v) + \int_0^{\infty} e^{-ivk} \varphi_T(-v) \\ &= 2 \operatorname{Re} \int_0^{\infty} e^{-ivk} \varphi_T(v) \end{aligned} \quad (8)$$

being the last equality given by the fact that when  $\varphi_T(v)$  is symmetric we can write  $\varphi_T(v) = \varphi_T(-v)$ , and  $\operatorname{Re}(\cdot)$  stands for the real part. Since  $C_T(k)$  is real, we thus have  $C_T(k) = \frac{e^{-\alpha k}}{\pi} \int_0^{\infty} e^{-ivk} \varphi_T(v)$ .

Now we just need to retrieve an analytic form for  $\varphi_T(v)$  to put it in  $C_T(k) = \frac{e^{-\alpha k}}{\pi} \int_0^{\infty} e^{-ivk} \varphi_T(v)$ , in order to get back the price of the option.

Reference [15] have derived the following form for  $\varphi_T(v)$

$$\varphi_T(v) = \frac{e^{-rT} \phi_T(v - (\alpha + 1)i)}{\alpha^2 + \alpha - v^2 + i(2\alpha + 1)v} \quad (9)$$

where  $\phi_T(v) = E[e^{ivS_T}]$  is the characteristic function of the log price  $S_T = \ln(S_T/S_0) = \ln(S_T)$  given that we define  $S_0 = 1$ .

Developed by [17], the FFT algorithm consists in approximating the continuous Fourier transform (CFT) with its discrete counterpart (DFT)

$$\int_0^{\infty} e^{-ixu} h(u) du \approx \sum_{j=0}^{N-1} e^{-i \frac{2\pi}{N} k_j} h_j, \quad k = 0, \dots, N-1 \quad (10)$$

Using an integration rule as the trapezoidal rule, we can rewrite the integral  $\int_0^{\infty} e^{-ivk} \varphi_T(v)$  for option prices as

$$\int_0^{\infty} e^{-ivk} \varphi_T(v) dv \approx \sum_{j=0}^{N-1} e^{-iv_j k} \varphi'(v_j) \eta \quad (11)$$

where  $\eta$  is the discretization step or grid spacing (corresponding to  $\Delta x$  in the definition of the trapezoidal rule), the points  $v_j$  are chosen to be equidistant with grid spacing  $\eta$ , that is  $v_j = \eta j$ .  $\varphi'$  is just  $\varphi$  weighted by the integration rule. The upper limit of the integration is thus  $a = \eta N$  so the value of  $\eta$  has to be small enough to allow a good approximation but not too small to guarantee that the characteristic function is equal to zero for any point  $a > a$ .

### B. Black and Scholes (B&S)

The [7] model remains the paradigm of option pricing and the benchmark against which all other models and extensions are compared. The drawbacks in B&S have guided some of its extensions. For example, [12] put forward that B&S ignores possible price jumps of the underlying and assumes constant

volatility across moneyness and maturity. With the aim to increase accuracy many models followed after.

The [7] model assumes that the risk-neutral price process for the underlying stock follows a geometric Brownian motion:

$$S_t = S_0 e^{(r-q-\sigma^2/2)t + \sigma W_t} \quad (12)$$

where  $t \geq 0$ , is the time remaining until maturity,  $r \geq 0$  is the risk-free interest rate,  $q \geq 0$  is the dividend yield,  $\sigma > 0$  is the volatility parameter and  $W_t$  is a standard Brownian motion. The underlying  $S_t$  satisfies a log-normal distribution. By introducing the auxiliary stochastic variable,  $X_t = \log\left(\frac{S_t}{S_0}\right) - (r-q)t$ , we get as characteristic

function  $\phi_X(u) = E[e^{iuX}] = \exp(-\frac{1}{2}u(u+i)\sigma^2 t)$ . Moreover, the simulation of the underlying is straightforward from the discrete-time version of the above expression:  $S_{t+\Delta t} = S_t e^{(r-q-\sigma^2/2)\Delta t + \sigma W_{\Delta t}}$ , where  $W_{\Delta t}$  is a draw from a normal distribution with mean zero and standard deviation  $\sqrt{\Delta t}$ .

When the B&S model is used to calculate implied volatilities one often obtain different numbers for different values of  $K$  and  $T$ . In particular, a "smile" or "smirk" shape is often observed in the plot of implied volatility versus strike price. Implied volatility tends to increase with maturity time, but is often larger for options with very short maturities. This is due to the increase in price that sometimes occurs when options are close to maturity as the price and the payoff converge. Given that prices revealed to be very different and inconsistent, and to not incur in the "bands" problem we end working with options on futures up to 4 months to maturity.

### C. Jump diffusion models (the Merton model)

Assuming that the process  $L = (L_t)_{t \geq 0}$  is a Lévy jump-diffusion, meaning a Brownian motion plus a compensated compound Poisson process, it can be described by

$$L_t = \gamma t + \sigma W_t + \sum_{i=1}^{N_t} Y_i \quad (13)$$

where  $\gamma \in \mathbb{R}$ ,  $\sigma \in \mathbb{R}_+$ ,  $W = (W_t)_{t \geq 0}$  is a standard Brownian Motion,  $N = (N_t)_{t \geq 0}$  is a Poisson process with parameter  $\lambda$  ( $E[N_t] = \lambda t$ ) and  $Y = (Y_t)_{t \geq 1}$  is an i.i.d. sequence of random variables with probability distribution  $F$  and  $E[Y] = k < \infty$ ,  $F$  describes the distribution of jump size. All sources of randomness are mutually independent.

Reference [18] was the first to use a discontinuous price process to model asset returns. In the Merton model (MJD), jumps in the log-price  $X_t$  are assumed to have a Gaussian distribution. The canonical decomposition of the driving process is

$$X_t = \mu t + \sigma W_t + \sum_{i=1}^{N_t} Y_i \quad (14)$$

where  $Y_i \sim N(\alpha, \delta^2)$ . The distribution of the jump size has the density

$$F(x) = \frac{1}{\delta \sqrt{2\pi}} \exp\left[-\frac{(x-\alpha)^2}{2\delta^2}\right] \quad (15)$$

and the Lévy density/measure is given by:

$$v(x) = \lambda F = \frac{\lambda}{\delta\sqrt{2\pi}} \exp\left[-\frac{(x-\alpha)^2}{2\delta^2}\right] \quad (16)$$

The Merton model has thus 4 parameters (excluding the drift  $\mu$ ): the diffusion volatility  $\sigma$ ,  $\lambda$  the jump intensity,  $\alpha$  the mean jump size and  $\delta$  the standard deviation of the jump size:  $\theta = (\sigma, \lambda, \alpha, \delta)$ .

By using the Lévy-Khinchin formula, MJD has the characteristic function:

$$E[e^{iuX_t}] = \phi_{X_t}(u) = \exp\left\{t\left(i\mu u - \frac{1}{2}\sigma^2 u^2 + \lambda\left(e^{i\alpha u - \frac{1}{2}\delta^2 u^2} - 1\right)\right)\right\} \quad (17)$$

being the characteristic exponent  $\phi_X(u)$

$$\phi_X(u) = i\mu u - \frac{1}{2}\sigma^2 u^2 + \lambda\left(e^{i\alpha u - \frac{1}{2}\delta^2 u^2} - 1\right) \quad (18)$$

and the Lévy triplet  $(\mu, \sigma^2, \lambda F)$ .

The density of  $X_t$  is not known in closed form, but it admits a series expansion

$$q_t(x) = e^{-\lambda t} \sum_{k=0}^{\infty} \frac{(\lambda t)^k \exp\left[-\frac{(x-\mu t - \alpha k)^2}{2(\sigma^2 t + \delta^2 k)}\right]}{k! \sqrt{2\pi(\sigma^2 t + \delta^2 k)}} \quad (19)$$

and the first two moments are

$$E[X_t] = t(\mu + \lambda\delta), \text{Var}[X_t] = t(\sigma^2 + \lambda\alpha^2 + \lambda\delta^2) \quad (20)$$

#### D. Infinite Activity Models (variance gamma and normal inverse Gaussian models)

In this class of models we have the Variance Gamma (VG) model and the Normal Inverse Gaussian (NIG) model. The VG model was introduced by [19] as a model for stock returns considering the symmetric case with  $\theta=0$ . Reference [20] presents the general case. The NIG distribution was first introduced by [21].

Both models are famous for their easy simulation through Brownian subordination. The subordinating processes under VG and NIG are respectively the Gamma and Inverse Gaussian processes.

Let  $T_t$  be a subordinator, meaning its trajectories are almost surely non-decreasing with Laplace exponent  $l(u) = \int_0^{\infty} (e^{ux} - 1)\rho(dx)$  where  $E[e^{uT_t}] = e^{t l(u)}$ . An infinitely activity Lévy process can be obtained by subordinating Brownian motion by subordinator  $T_t$  as follows:

$$X_t = \mu T_t + \sigma W(T_t) \quad (21)$$

Then the characteristic exponent of  $X_t$  is given by  $\phi_X(u) = l(-u^2\sigma^2/2 + i\mu u)$  and the Lévy triplet  $(A_X, v_X, \mu_X)$  of  $X_t$  is given by

$$A_X = 0, v_X = \int_0^{\infty} p_s^T(x)\rho(ds), \mu_X = \int_0^{\infty} \rho(ds) \int_{|x|\leq 1} xp_s^T(dx) \quad (22)$$

where  $p_s^T$  is the probability distribution of the subordinator  $T_t$ . This process is a Brownian motion observed on a new time scale, say business time.

Usually, subordinators are  $\alpha$ -stable processes with  $\alpha \in [0, 1)$ . Subordinators have no diffusion component, only positive jumps of finite variation and positive drift, the Lévy measure

of a real valued  $\alpha$ -stable process is of the form  $\frac{ce^{-\lambda x}}{x^{\alpha+1}} 1_{x>0}$  and

by tempering this Lévy measure, we obtain a tempered stable subordinator, which is a three parameter process with Lévy measure  $\rho(x) = \frac{ce^{-\lambda x}}{x^{\alpha+1}} 1_{x>0}$  where  $c>0$  alters the intensity of

jumps of all sizes simultaneously (time scale of the process),  $\lambda>0$  fixes the decay rate of big jumps, and  $1>\alpha\geq 0$  determines the relative importance of small jumps in the path of the process. The probability density of tempered stable subordinator is only known in explicit form for  $\alpha=0$  (variance gamma) and  $\alpha=1/2$  (normal inverse Gaussian), so the corresponding subordinated processes have been widely used because they are easier to simulate and more mathematically tractable [16].

The variance gamma process is a finite variation process with infinite but relatively low activity of small jumps obtained by evaluating Brownian motion with drift  $\mu$  and volatility  $\sigma$  at an independent gamma time,  $X_t = \mu T_t + \sigma W(T_t)$ , where  $T_t$  is a gamma process with mean rate  $t$  and variance rate  $\kappa t$ . The Gamma distribution has density function of the gamma time change  $g$  over a finite interval  $t$ , with parameters  $a=(t/\kappa)$ ,  $b=(1/\kappa)>0$ , given by

$$f_g(g) = p_t^T(g) = \frac{b^a}{\Gamma(a)} g^{a-1} e^{-gb}, g > 0 \quad (23)$$

Being the characteristic function given by

$$\phi_{Gamma}(u; a, b) = \left(1 - \frac{iu}{b}\right)^{-a},$$

having the Gamma distribution a mean  $a/b$  and variance  $a/b^2$ . The Lévy measure of  $T_t$ , the rate of arrival as a function of the jump size  $x$ , is given

$$\text{by } \rho(x) = \frac{1}{\kappa} \frac{e^{-x/\kappa}}{x} 1_{x>0}, \quad \text{with Laplace exponent}$$

$$l(u) = -\frac{1}{\kappa} \ln(1 - \kappa u).$$

There are 3 parameters in the gamma process that should be considered:  $\theta = (\mu, \sigma, \kappa)$ , where  $\mu$  is the diffusion drift,  $\sigma$  the diffusion volatility and  $\kappa$  the variance of the subordinator.

The characteristic function of the VG process is given by

$$E[e^{iuX_t^{VG}}] = \phi_{VG}(u; \sigma\sqrt{t}, \mu/t, t\kappa) = (\phi_{VG}(u; \sigma, \mu, \kappa))^t = \left(1 - iu\kappa\mu + \frac{1}{2}\sigma^2\mu u^2\right)^{-t/\mu},$$

$$\text{being its first two moments:} \quad E[X_t] = \kappa t, \text{Var}[X_t] = \sigma^2 t + \kappa^2 \mu t \quad (24)$$

whose characteristic exponent  $\phi_X(u)$  is:

$$\phi_X(u) = l(-u^2\sigma^2/2 + i\mu u) = -\frac{1}{\kappa} \ln\left(1 - i\mu\kappa u + \frac{1}{2}\sigma^2\kappa u^2\right) \quad (25)$$

and the characteristic function  $\phi_{X_t}(u)$  of  $X_t$  is therefore

$$\phi_{X_t}(u) = E[\exp iuX_t] = e^{i\varphi_X(u)} = e^{-a \ln\left(1 - i\mu\kappa u + \frac{1}{2}\sigma^2\kappa u^2\right)} \quad (26)$$

The normal inverse Gaussian process is an infinite variation process with stable like behavior of small jumps constructed from Brownian subordination at an independent inverse Gaussian time,  $X_t = \mu T_t + \sigma W(T_t)$ , where  $T_t$  is an independent inverse Gaussian process with mean  $t$  and variance  $\kappa t$ .

There are 3 parameters in the normal inverse Gaussian (NIG) process:  $\theta = (\mu, \sigma, \kappa)$ , where  $\mu$  is the diffusion drift,  $\sigma$  the diffusion volatility and  $\kappa$  the variance of the subordinator.

It's characteristic exponent  $\varphi_X(u)$  is

$$\varphi_X(u) = l\left(-u^2\sigma^2/2 + i\mu u\right) = e^{i\left(b - b\sqrt{1 - 2i\mu\kappa u + \sigma^2\kappa u^2}\right)} \quad (27)$$

and being the characteristic function  $\phi_{X_t}(u)$ :

$$\phi_{X_t}(u) = b - b\sqrt{1 - 2i\mu\kappa u + \sigma^2\kappa u^2} \quad (28)$$

This characteristic function is infinitely divisible, so we can define the NIG process  $X^{\text{NIG}} = \{X_t^{\text{NIG}}, t \geq 0\}$  with three parameters  $\mu$ :  $\mu > 0$ ,  $\sigma$ :  $-\mu < \sigma < \mu$  and  $\kappa$ :  $\kappa > 0$ , with  $X_0^{\text{NIG}} = 0$  stationary and independent NIG distributed increments.

The first two moments of the NIG process are given by

$$E[X_t] = \kappa\sigma\sqrt{\mu^2 - \sigma^2}; \text{Var}[X_t] = \mu^2\kappa(\mu^2 - \sigma^2)^{-3/2} \quad (29)$$

#### E. Tempered stable Lévy processes CGMY process

These are Lévy processes  $\mathfrak{R}$  with no Gaussian component, constructed by directly specifying a Lévy measure of the form

$$\nu(x) = \frac{c_-}{(-x)^{1+\alpha_-}} e^{-\lambda_- x} 1_{x < 0} + \frac{c_+}{x^{1+\alpha_+}} e^{-\lambda_+ x} 1_{x > 0} \quad (30)$$

where  $c_{\pm}, \lambda_{\pm} > 0$  and  $\alpha_{\pm} < 2$ . We can represent these as a time changed Brownian motion (with drift) if and only if  $c_- = c_+$  and  $\alpha_- = \alpha_+ = \alpha \geq -1$ . This enforcement implies that small jumps must be symmetric whereas decay rates for big jumps may be different. As discussed previously, the main impact on option prices are large jumps. The CGMY subclass of tempered stable Lévy processes is flexible allowing for asymmetry of small jumps. For more details on the CGMY process, please see [22].

CGMY process is of finite variation if  $0 \leq \alpha < 1$  and of infinite variation if  $\alpha \geq 1$ . In this class of processes 4 parameters should be taken into account:  $\theta = (c, \alpha, \lambda_-, \lambda_+)$ , where  $c$  determines the overall and relative frequency of jumps;  $\alpha$  determines the local behavior of the process (how the price evolves between big jumps);  $\lambda_-, \lambda_+$  determine the tail behavior of the Lévy measure.

CGMY process  $X_t$  characteristic function  $\phi_{X_t}(u)$  is given by

$$\phi_{X_t}(u) = E[\exp iuX_t] = e^{\{c\Gamma(-\alpha)\left[(\lambda_- - iu)^\alpha - \lambda_-^\alpha + (\lambda_+ + iu)^\alpha - \lambda_+^\alpha\right]\}} \quad (31)$$

being the characteristic exponent  $\varphi_X(u)$

$$\varphi_X(u) = c\Gamma(-\alpha)\left[(\lambda_+ - iu)^\alpha - \lambda_+^\alpha + (\lambda_- + iu)^\alpha - \lambda_-^\alpha\right] \quad (32)$$

where  $0 < \alpha < 1$  or  $\alpha > 1$ .

#### IV. RISK NEUTRAL ESTIMATION OF PARAMETERS

Now that we have presented the processes and FFT method to price options when the characteristic function of those processes are known in closed form, we can use a number of different objectives to calibrate option pricing models. Among them, the minimization of the absolute pricing error gives more weight to in the money, long term options, while the minimization of the relative error gives more weight to out of the money, short term options. Here we settle this problem by using the Maximum Likelihood Estimation (MLE) method of [20]. Let  $c_i^{\text{Market}}$  be the observed market price on the  $i$ -th option and  $c_i^{\text{Model}}$  be the price according to the model under use; additionally, we assume that  $c_i^{\text{Market}}$  and  $c_i^{\text{Model}}$  satisfy the following equation,

$$c_i^{\text{Market}} = c_i^{\text{Model}} \exp(\theta \varepsilon_i - \theta^2 / 2) \quad (33)$$

Where  $\varepsilon_i \sim N(0,1)$ . The corresponding log likelihood function for  $c_i^{\text{Market}}$  is,

$$\log(f(\theta; c_i^{\text{Market}})) = -\frac{1}{2} \sum_{i=1}^M \left( \frac{\log(w_i) - \log(c_i^{\text{Model}}) + \theta}{\theta} \right)^2 \quad (34)$$

where  $\theta$  is the parameter vector. Reference [20] show that maximum likelihood estimation is equivalent to the minimization of

$$k = \sqrt{\frac{1}{M} \sum_{i=1}^M (\log(c_i^{\text{Model}}) - \log(c_i^{\text{Market}}))^2} \quad (35)$$

This is a non-linear minimization problem. We apply the standard Levenberg-Marquardt method to achieve the minimum point in the parameter space. To avoid local minima, we start with more than a hundred initial parameters guesses for each model. The interest rate considered was of 5%. The parameter set  $\theta$  is then determined minimizing the following expression

$$\hat{\theta} = \arg \min_{\theta} \frac{1}{N} \sum_{i=1}^N (c_i^{\text{Model}} - c_i^{\text{Market}})^2 \quad (36)$$

So, the objective function consists of finding the minimum value of a averaged sum of  $N$  squared residuals (difference between model prices and market prices) with respect to a set of  $n$  parameters of a model. Given that there are huge variations in the underlying prices (futures prices) the average squared errors are not comparable through time, and for the parameters estimates and implied volatility plot over time, we had to take into account the average relative squared errors:

$$\frac{1}{N} \sum_{i=1}^N \frac{(c_i^{\text{Model}} - c_i^{\text{Market}})^2}{c_i^{\text{Model}}} \quad (37)$$

Usually the error produced by Fourier approximations is very little, except when maturity tends to zero (with very short maturities the Fourier method cannot provide satisfactory prices; one of the reasons to justify the choice of 4 months to maturity options on futures) or for deep-in-the-money or deep-out-of-the-money options. As such, there was two steps on the estimation: Initially parameters for each model are estimated

using both the set of call and put prices. After the model calibration and parameters estimations we used the iterative search procedure Newton-Raphson method, to find implied volatilities for every option price and plot the implied volatility; Second we use parameter values estimated to price call options. In the next section we will discuss and present volatility surface plots considering the five processes discussed above for the last day of our sample. For implied volatility patterns discussion, implied volatility surfaces were computed daily, throughout the entire sample available.

#### V. EMPIRICAL RESULTS

Reference [18] proposed a jump diffusion model (MJD), under which the log return has both a diffusion component and a jump component (assumed to be a compound Poisson jump process). But, Poisson jumps happen at a very "slow" rate, that is, imply finite activity. Right after that [19] and [20], discussed the so called variance gamma model (VG) under which the jump component of the log return is an infinite activity process. This model was generalized in [22], CGMY, allowing for diffusions and jumps of both finite and infinite activity. Importantly, [23] and [24] argued that in infinite activity jump models, the diffusion component of the log return is redundant. Therefore, all these models can be simplified to pure jump models. Jump processes were also modeled by [21] and [25]. Complementary to the previous papers, there is also a large empirical literature that tests the performance of the models, but these use mostly index or currency options. Reference [26] studies the behavior of Merton jump diffusion (MJD) before the 1987 crash, and shows that the S&P500 option market experienced crash fears before October, 1987, while [27] test the performance of VG in the Hang Seng index options market and conclude that VG marginally outperforms B&S. Also [23] test for the empirical performance of Lévy option pricing models. But, most of them investigate the performance on the call side, concluding that Lévy option pricing models exhibit better calibration performance. However, basing tests only on the call side is insufficient to draw the conclusion that more general Lévy models can explain market prices better than the traditional pure diffusion models. In this essay, we investigate model performance using both call and puts for each pricing model. Reference [22], using CGMY, also show that the index dynamics are devoid of a diffusion component. After the development of time changed Lévy processes, [22], [25] and [24] investigate their empirical performance in the S&P500 index options market. Moreover, [23] compare the performance of B&S, MJD and VG in the Deutsche mark foreign currency options market, finding that VG has better out of sample performance and less entropy between the statistical measure and the risk neutral measure. More recently, [3] present evidence of stochastic skewness in over-the-counter currency option markets, and develop a class of models that capture the stochastic skewness, where results favored these. Overall the extensions to B&S show strong empirical performance, especially the time changed Lévy processes, at least for financial markets.

The rational for having chosen Lévy models is that exponential distributions are known to have a good entropy.

As such, it contains most information possible, capturing all available data, and also turns option price computation easier given that we may use FFT. Therefore, [5] calibration to market options is parametric, performed by minimizing an objective function, which in our case is the weighted sum of squared deviations, adjusted after for weighted relative sum of squared deviations, between model and market options. Reference [5] was the first to compare different methods to calculate implied volatilities for options on non-tradable spots, using NYMEX data on WTI European-type oil options on futures as example. He fits an exponential mean-reverting jump-diffusion model to market prices and a forward curve, and uses a stochastic volatility inspired (SVI) parameterization. We will start this section by presenting the implied volatility plots for each of the considered processes, in Fig. 1, considering electricity options on futures quoted on 27 March 2008. Fig. 1 shows the volatility surface generated with 5 models. Next we will show that the B&S model fails in modeling option prices on futures while Lévy processes can reproduce conveniently option prices and the volatility surface. Using model implied volatilities is also useful due to limited liquidity of market exchange-traded options in power. Therefore relying solely on market traded option prices to assess market fit is a disadvantage. Model implied volatility analysis have received attention on equity markets [28]-[30] and more recently in electricity markets by [31], although the latter's only consider a spike mean-reverting model applied to the Nord Pool market. Reference [32] proposes to update implied volatility models on a day-by-day basis, whereby increasing the accuracy of the estimation, concluding that IVS evolves dynamically though time. As such, implied volatility modeling is sparse in commodity markets. Implied volatility plots across strike prices and time to expiration (implied volatility surface or IVS) is a convenient tool to illustrate discrepancies between market reality and theory. However, it should be noticed that implied volatility as a function of strike does not adequately capture volatility market movements, but the implied volatility as a function of moneyness parameter does. Therefore, we use as a measure of moneyness the ratio of the strike over the futures price ( $K/F$ ). Implied volatility for each model is plotted on time to maturity ( $T$ ) and moneyness ( $K/S = K/F$ ).

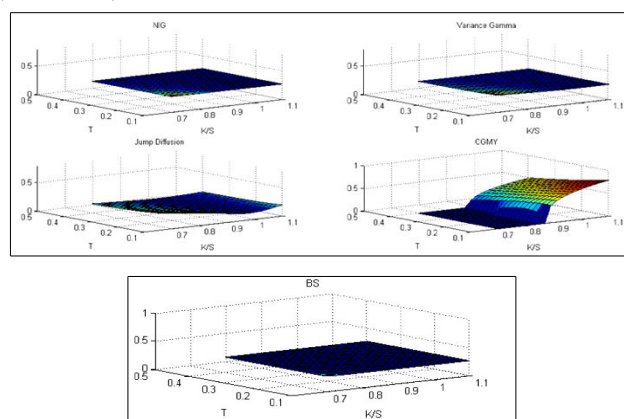


Fig. 1 Volatility surfaces generated with the MJD, VG, NIG, CGMY and B&S models on 27 March 2008

First, noticed that we have calibrated the models for the entire sample, having obtained a volatility surface for each of the days in the sample. From all the computations it was clearly visible that the CGMY model provides the worst performance. VG being a special case of the CGMY model provides the best fit despite the day in the sample.

Parameter values estimates obtained on that specific day for the MJD were 0.7402 for the diffusion volatility ( $\sigma$ ), 0.1968 for the jump intensity ( $\lambda$ ), 0.0199 for the mean jump size ( $\alpha$ ) and 0.01 for the standard deviation of the jump size ( $\delta$ ). The diffusion drift ( $\mu$ ) was 0.0295 and 11.4459 for the VG and NIG models, respectively, while diffusion volatility values ( $\sigma$ ) were 0.2384 for the VG process and -3.7489 for NIG. Finally, variance of the subordinator ( $\kappa$ ) was -0.1516 and 0.1194 for VG and NIG, respectively.

For the B&S model that uses a single volatility parameter ( $\sigma$ ), the fitting wasn't better than for exponential Lévy models, being the parameter obtained of 0.2403.

The CGMY model parameter values were 0.01, 0.4102, 7.572 ( $\lambda_-$ ) and 1 ( $\lambda_+$ ) for the relative frequency of jumps ( $c$ ), the local behavior of the process ( $\alpha$ ) which shows how the price evolves between big jumps, and for the tail behavior of the jumps ( $\lambda$ ), respectively.

In Fig. 2 to 6 we plot the relative quadratic mean errors and histograms obtained with the parameters found after calibration for all the models. Relative quadratic mean errors are represented on the vertical axis against the sample N on the horizontal axis.

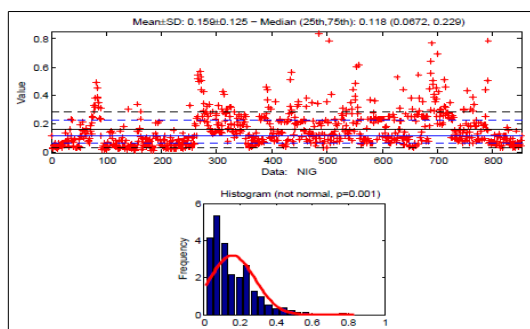


Fig. 2 Relative quadratic mean error and histogram plots using calls and puts testing under NIG

Fig. 2 shows a mean relative error per option of 0.159 for the NIG case with standard deviation of 0.125. However, for the VG process, the mean relative error per option was 8.44%, while 11.53% for standard deviation. Visually (Fig. 3) we can also see that this was the model for which model prices mostly fit market prices. Notices that, outliers can either indicate measurement errors or that the population has a heavy tailed distribution, and that we are dealing with options on futures. Initially, we should expect futures prices to be much more stable than spot prices in electricity markets. But, as futures depend on the evolution of the spot these should be modeled more accurately if we consider heavy-tailed distributions, like the ones we are applying.

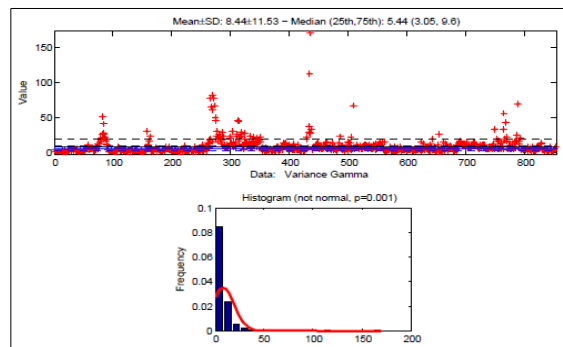


Fig. 3 Relative quadratic mean error and histogram plots using calls and puts testing under VG

The MJD model identified previously in the literature as being a good model for stock and commodities markets has mean relative error of 0.174 and standard deviation of 0.124, where relative quadratic mean errors plot show an higher dispersion of values inside the entire sample.

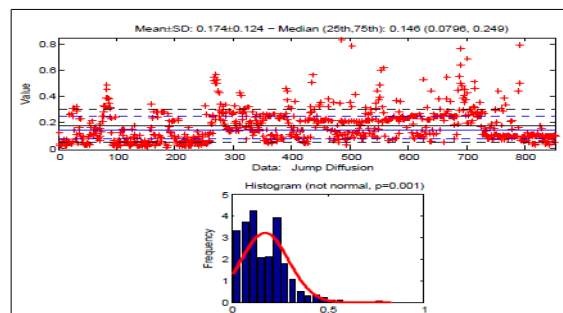


Fig. 4 Relative quadratic mean error and histogram plots using calls and puts testing under MJD

The second lowest mean relative error (0.142) was obtained for the CGMY model and this revealed to be the model for which we have obtained the most unstable results. Plots for the CGMY process are presented in Fig. 5.

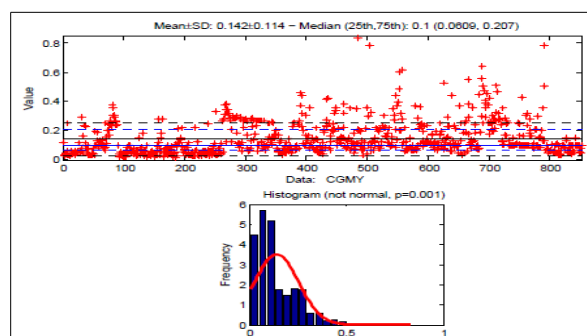


Fig. 5 Relative quadratic mean error and histogram plots using calls and puts testing under CGMY

Standard deviation for the CGMY process was 0.114, lower than that of the B&S model (0.124). The diffusion volatility parameter ( $\sigma$ ) is 0.2403 for the B&S model while being 0.7402 in the MJD process. This result was surprising in the sense that volatility of MJD comes from both the diffusion and jump components, and we should thus expect a lower value.



Maybe outlier's presence could explain this result, and thus conclusions to be drawn deserve a more careful treatment.

Moreover, the mean relative error per option was 0.155 for the B&S model, higher than that of the CGMY and VG processes. For all the processes used, histogram of relative quadratic mean errors using both calls and puts reinforce the idea on non-normality presented in electricity derivatives data. Given that a model with smaller pricing errors is the one that offers the best fit, in our analysis this role is attributed to the VG process.

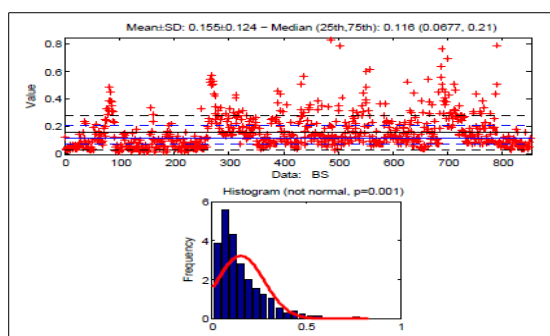


Fig. 6 Relative quadratic mean error and histogram plots using calls and puts testing under B&S

Implied volatility surface plots are not new. An implied volatility is the volatility implied by the market price of an option based on the option pricing model. With this we mean the volatility that when used in the pricing model, yields a theoretical value for the option equal to the current market price of that option. We will present graphically the results obtained for both NIG and VG processes, thus considering the joint patterns of all implied volatilities quoted on the EEX electricity market for "ATM" options on futures of a 4 month maturity.

Volatility surfaces need to be updated when forward prices changes or the implied volatility levels experience variations. Changes in the shape of implied volatilities for different points in the surface are usually highly correlated across strikes and maturities due to various arbitrage relationships. Therefore, if we have updated information on the implied volatilities for the most liquid options traded in the market, we can treat those as a volatility shock.

For many commodities where the price is supported by non-market effects (government intervention or production costs) it is common to see a flat or negative skew for puts and a pronounced positive skew for calls, due to the risk of price spikes.

The shape of the volatility smile can also provide information about the correlation of price movements and changes in the implied volatility. For certain commodities, it is reasonable to expect that price increases are more likely to be associated with higher implied volatilities than price decreases. For example, in the oil market, when price goes up it is usually due to increased geopolitical instability, and this is commonly associated with higher levels of volatility. In the case of natural gas and electricity markets, large price increases are usually the result of supply or demand shocks that tend to be followed by higher implied volatilities for traded options.

In order to price a European call or put option with the same strike and maturity with the B&S model, the same volatility should be used. This is true for European options when put-call parity holds, and it does not depend on the future probability distribution of the underlying, due to the fact that it is based on simple arbitrage arguments. Therefore, when we are talking about implied volatilities, it does not matter whether we are referring to calls or puts, because they should be the same. However, in electricity put-call parity does not hold due to the asset non-storability, and we decided to price only call options.

We have obtained implied volatility surfaces for each of the available daily quotes and retained implied volatility, average squared errors and relative average squared errors for each of the processes. Fig. 7 presents the implied volatility daily plot for the entire observation period. The blue line is for the NIG process while the red line is for the VG process.

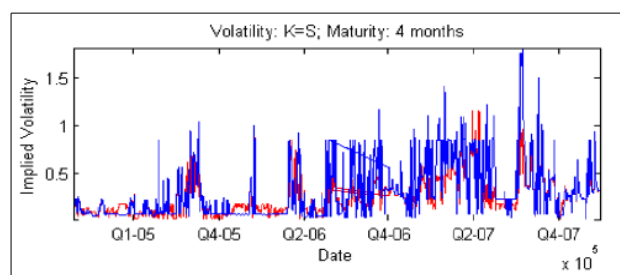


Fig. 7 Implied volatility daily plot using at-the-money options for the entire sample period

It was observed that the level of implied volatilities change with time, deforming the shape of the implied volatility surface. Therefore, the evolution in time of this surface captures the evolution of prices in the options market. The implied volatility surface also changes dynamically over time in a way that is not taken into account by current modeling approaches, but that should.

In fact, if we look at Fig. 7 we see that implied volatility was relatively lower at the beginning of the sample period and increased substantially as time passed by. This increase was even higher during 2007 with implied volatility values of more than 1.5 for that period under the NIG model, and of around 1 for the VG process. Market changing rules, like CO<sub>2</sub> contracts introduction and development, or even fuel prices evolution, increased liquidity in the options on futures market, extreme weather conditions, or even the financial crisis could explain this implied volatility movement on the market. However, the causes underlying the dynamics of implied volatility over time are left for future works, where we intend to apply semi-parametric or even non-parametric methods in order to be able to explain this dynamic.

To reinforce our results we have also computed mean and variance values obtained from implicit volatility and relative quadratic mean errors for the data gathered. We present results on Tables I and II for each of the processes used. In fact, mean and variance implied volatility values increased suddenly from 2007 onwards, independently of the process under analysis, to decrease after in 2008, and explaining this behavior will be a challenging task. For now we can only say that this increase

is due to also higher prices verified in the market in that period. Implied volatility values have varied from as low as 0 for CGMY, to as high as 1.79 for the NIG model.

TABLE I  
 MEAN AND VARIANCE VALUES OF IMPLIED VOLATILITY BY MONTH FOR THE ENTIRE SAMPLE PERIOD, FOR PROCESSES VG, NIG, MJD, CGMY AND B&S

		Mean values											
Model	Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
VG	2004											0.162	0.129
	2005	0.147	0.141	0.176	0.122	0.121	0.149	0.217	0.424	0.162	0.090	0.159	0.223
	2006	0.109	0.126	0.216	0.297	0.137	0.172	0.291	0.260	0.220	0.205	0.229	0.201
	2007	0.277	0.267	0.280	0.301	0.624	0.469	0.266	0.190	0.446	0.204	0.425	0.274
	2008	0.296	0.204	0.291									
NIG	2004											0.101	0.105
	2005	0.129	0.097	0.129	0.120	0.161	0.140	0.227	0.575	0.172	0.121	0.112	0.166
	2006	0.120	0.097	0.209	0.259	0.115	0.240	0.225	0.446	0.225	0.515	0.255	0.212
	2007	0.299	0.561	0.716	0.620	0.695	0.506	0.606	0.227	0.795	0.454	0.410	0.266
	2008	0.296	0.227	0.425									
MJD	2004											0.162	0.129
	2005	0.122	0.107	0.122	0.112	0.124	0.126	0.117	0.229	0.150	0.020	0.121	0.161
	2006	0.104	0.090	0.147	0.141	0.106	0.110	0.227	0.125	0.171	0.221	0.295	0.245
	2007	0.169	0.171	0.181	0.204	0.266	0.175	0.121	0.141	0.216	0.260	0.297	0.214
	2008	0.260	0.277	0.207									
CGMY	2004											0.202	0.159
	2005	0.202	0.220	0.282	0.260	0.167	0.225	0.222	0.420	0.220	0.159	0.201	0.266
	2006	0.207	0.266	0.246	0.220	0.032	0.126	0.292	0.249	0.220	0.412	0.209	0.226
	2007	0.204	0.266	0.512	0.602	0.604	0.475	0.202	0.075	0.190	0.200	0.427	0.242
	2008	0.263	0.267	0.220									
B&S	2004											0.160	0.121
	2005	0.127	0.154	0.176	0.124	0.122	0.147	0.154	0.272	0.162	0.100	0.160	0.220
	2006	0.201	0.145	0.175	0.171	0.161	0.172	0.292	0.275	0.224	0.272	0.221	0.202
	2007	0.274	0.279	0.292	0.492	0.460	0.292	0.274	0.200	0.257	0.226	0.209	0.264
	2008	0.292	0.221	0.246									

		Variance values											
Model	Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
VG	2004											0.003	0.001
	2005	0.003	0.001	0.004	0.005	0.005	0.008	0.032	0.039	0.011	0.005	0.003	0.011
	2006	0.034	0.002	0.035	0.056	0.003	0.009	0.017	0.013	0.007	0.014	0.002	0.002
	2007	0.018	0.011	0.014	0.016	0.067	0.067	0.007	0.001	0.102	0.033	0.046	0.020
	2008	0.003	0.011	0.013									
NIG	2004											0.005	0.000
	2005	0.003	0.000	0.004	0.004	0.028	0.011	0.025	0.078	0.010	0.003	0.002	0.022
	2006	0.037	0.000	0.063	0.080	0.006	0.081	0.075	0.096	0.069	0.087	0.007	0.012
	2007	0.086	0.064	0.110	0.049	0.085	0.063	0.118	0.004	0.339	0.083	0.117	0.047
	2008	0.005	0.031	0.036									
MJD	2004											0.003	0.009
	2005	0.005	0.000	0.004	0.006	0.006	0.008	0.007	0.008	0.011	0.006	0.002	0.016
	2006	0.000	0.000	0.012	0.002	0.002	0.004	0.026	0.004	0.009	0.011	0.010	0.011
	2007	0.010	0.010	0.017	0.047	0.040	0.018	0.003	0.004	0.126	0.037	0.072	0.017
	2008	0.013	0.015	0.012									
CGMY	2004											0.008	0.011
	2005	0.014	0.004	0.010	0.036	0.016	0.023	0.044	0.047	0.009	0.009	0.009	0.008
	2006	0.019	0.009	0.012	0.030	0.006	0.050	0.021	0.103	0.067	0.090	0.034	0.026
	2007	0.072	0.073	0.126	0.045	0.084	0.065	0.111	0.034	0.046	0.023	0.082	0.028
	2008	0.004	0.007	0.019									
B&S	2004											0.003	0.001
	2005	0.005	0.002	0.004	0.005	0.005	0.009	0.033	0.008	0.010	0.005	0.002	0.010
	2006	0.033	0.002	0.004	0.004	0.004	0.010	0.018	0.012	0.006	0.007	0.002	0.002
	2007	0.013	0.012	0.014	0.012	0.017	0.016	0.005	0.010	0.005	0.016	0.020	0.011
	2008	0.002	0.004	0.001									

of 0.88 for all the processes analyzed. In general all models have showed a high instability when estimated through time.

The evolution of calls implied volatility is also different throughout months for 4 months options on futures, and this could also be different for different options. It would be useful to analyze implied volatility surface evolution from day-to-day, which can be modelled in the spirit of yield and forward curve modelling by applying principal component analysis (PCA) methods. On Fig. 8 it is plotted the implied volatility values obtained for each of the fitted processes, which generated those values reported on Table I.

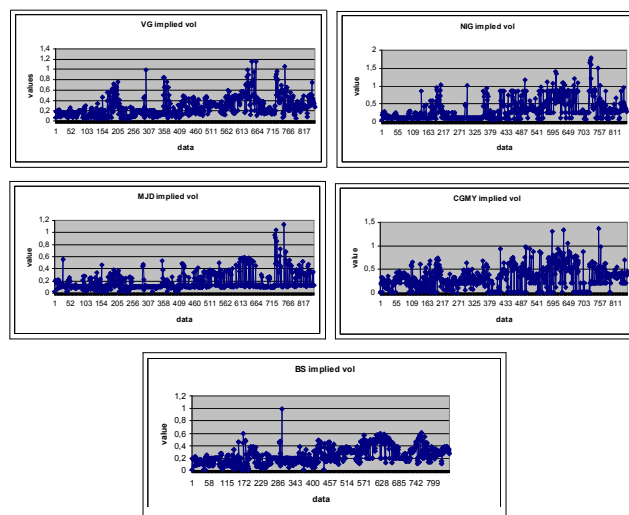


Fig. 8 Implied volatility patterns for VG, NIG, MJD, CGMY and B&S processes

Greater sensitivity to economic, financial or political information which impact demand and supply and therefore commodities markets, can be added to account for uncertainty which turns difficult to derive a general profile for implied volatilities. The variance gamma process implied volatility highest value occurred in 14 June 2007, being 1.159. As for NIG the highest value turns to be on 21 September 2007 (1.790), while for MJD and CGMY the highest implied volatility value occurred in 29 October 2007, which have been 1.134 and 1.367, respectively. The B&S model implied volatility computed values presents the highest value on January 6, 2006, being only 0.99. We may infer from here the underestimation of implied volatility obtained using B&S as it has already been reported previously in the literature [13], [10] and [5]. We could model implied volatility behavior according to moneyness and option maturity using IVFs in the spirit of [32] OLS regressions. Moreover, generalizations to incorporate additional state variables inherent to electricity markets like volatile fuel prices, CO<sub>2</sub> allowances prices, and a dummy to account for market change rules impact, are among those possibilities that offer promising areas for future research, allowing for a deeper understanding of electricity implied volatilities.

Relative mean squared errors in our implied volatility estimates ranged from 0.009 for almost every process to a high

TABLE II

MEAN AND VARIANCE VALUES FOR RELATIVE MEAN SQUARED ERRORS BY MONTH FOR THE ENTIRE SAMPLE PERIOD AND FOR PROCESSES VG, NIG, MJD, CGMY AND B&S

Mean values													
Model	Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
VG	2004											0.047	0.091
	2005	0.082	0.168	0.2027	0.067	0.061	0.093	0.057	0.101	0.052	0.066	0.243	0.258
	2006	0.222	0.207	0.173	0.077	0.200	0.136	0.174	0.181	0.140	0.195	0.171	0.202
	2007	0.250	0.119	0.166	0.146	0.171	0.163	0.234	0.345	0.260	0.137	0.183	0.241
	2008	0.074	0.083	0.064									
NIG	2004											0.048	0.091
	2005	0.082	0.168	0.203	0.068	0.071	0.094	0.057	0.100	0.052	0.067	0.243	0.258
	2006	0.222	0.207	0.181	0.073	0.200	0.156	0.191	0.200	0.147	0.189	0.171	0.202
	2007	0.251	0.155	0.200	0.166	0.167	0.180	0.273	0.345	0.244	0.134	0.183	0.229
	2008	0.074	0.084	0.064									
MJD	2004											0.047	0.165
	2005	0.082	0.168	0.213	0.067	0.061	0.093	0.147	0.130	0.052	0.066	0.252	0.258
	2006	0.223	0.207	0.176	0.151	0.208	0.136	0.192	0.225	0.160	0.205	0.183	0.207
	2007	0.279	0.212	0.224	0.202	0.201	0.262	0.301	0.345	0.252	0.132	0.155	0.203
	2008	0.080	0.087	0.079									
CGMY	2004											0.055	0.092
	2005	0.075	0.147	0.164	0.054	0.056	0.076	0.053	0.093	0.051	0.063	0.179	0.235
	2006	0.187	0.154	0.149	0.071	0.200	0.135	0.150	0.180	0.139	0.189	0.166	0.186
	2007	0.248	0.112	0.162	0.143	0.160	0.170	0.267	0.322	0.239	0.120	0.162	0.219
	2008	0.068	0.078	0.082									
B&S	2004											0.047	0.091
	2005	0.082	0.168	0.203	0.067	0.061	0.093	0.061	0.103	0.052	0.066	0.243	0.258
	2006	0.222	0.207	0.174	0.079	0.200	0.136	0.174	0.181	0.140	0.196	0.171	0.202
	2007	0.251	0.120	0.166	0.146	0.173	0.164	0.234	0.345	0.260	0.138	0.183	0.241
	2008	0.074	0.083	0.064									

Variance values													
Model	Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
VG	2004											0.001	0.003
	2005	0.003	0.007	0.029	0.002	0.002	0.009	0.001	0.003	0.001	0.001	0.038	0.007
	2006	0.008	0.006	0.004	0.001	0.016	0.006	0.024	0.006	0.009	0.032	0.025	0.007
	2007	0.030	0.006	0.004	0.011	0.010	0.021	0.040	0.015	0.006	0.004	0.012	0.033
	2008	0.001	0.002	0.001									
NIG	2004											0.001	0.003
	2005	0.003	0.007	0.027	0.002	0.005	0.008	0.001	0.004	0.001	0.001	0.038	0.007
	2006	0.008	0.006	0.004	0.001	0.016	0.006	0.023	0.005	0.009	0.030	0.025	0.007
	2007	0.030	0.008	0.005	0.014	0.010	0.022	0.034	0.015	0.008	0.004	0.012	0.034
	2008	0.001	0.002	0.001									
MJD	2004											0.001	0.011
	2005	0.003	0.007	0.027	0.002	0.002	0.009	0.014	0.006	0.001	0.001	0.036	0.007
	2006	0.008	0.006	0.004	0.009	0.015	0.006	0.022	0.003	0.009	0.029	0.024	0.006
	2007	0.023	0.005	0.003	0.012	0.009	0.010	0.027	0.015	0.008	0.004	0.013	0.038
	2008	0.001	0.001	0.001									
CGMY	2004											0.003	0.006
	2005	0.003	0.004	0.017	0.001	0.002	0.006	0.001	0.003	0.002	0.002	0.019	0.006
	2006	0.008	0.008	0.004	0.001	0.016	0.006	0.014	0.006	0.009	0.032	0.023	0.006
	2007	0.030	0.007	0.005	0.011	0.009	0.020	0.026	0.010	0.009	0.002	0.009	0.034
	2008	0.001	0.001	0.001									
B&S	2004											0.001	0.003
	2005	0.003	0.007	0.029	0.002	0.002	0.009	0.001	0.003	0.001	0.001	0.038	0.007
	2006	0.008	0.006	0.004	0.001	0.016	0.006	0.024	0.006	0.009	0.032	0.025	0.007
	2007	0.030	0.006	0.004	0.011	0.010	0.021	0.040	0.015	0.006	0.004	0.012	0.033
	2008	0.001	0.002	0.001									

Relative mean squared errors mean and variance values over months, show that the pattern of increasing values is mostly felt in 2007, especially during summer (June, July) and winter (December, January) due to the commodity seasonality cooling and heating needs, respectively, to start decreasing from 2008 onwards. In fact, more options started to be traded from 2007 onwards and market learning can be one of the justifications. But price increases verified could also be explained by other causes. The VG process shows mean and variance values lower than those of NIG and MJD processes, for most of the months, but always monthly lower than CGMY, at least until 2007. From that moment onwards results change a lot.

TABLE III

MEAN AND VARIANCE VALUES FOR IMPLICIT VOLATILITY AND RELATIVE MEAN SQUARED ERRORS FOR THE ENTIRE SAMPLE PERIOD AND FOR PROCESSES VG, NIG, MJD, CGMY AND B&S

Implied Volatility									
VG		NIG		MJD		CGMY		B&S	
Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
0.2666	0.0318	0.3232	0.0845	0.1769	0.0183	0.3050	0.0510	0.2433	0.0176

Relative Mean Squared Errors									
VG		NIG		MJD		CGMY		B&S	
Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
0.1546	0.0155	0.1594	0.0156	0.1744	0.0154	0.1419	0.0129	0.1550	0.0154

Since we have implied volatility values and relative mean squared errors computed for each of the days in the sample, in Table III we report implicit volatility and relative mean squared errors, for the entire sample period, based on mean and variance values. We observe that CGMY has the largest implicit volatility mean, while NIG has the highest variance. Once again we see that the model with the smaller mean relative pricing error is the VG model, given that CGMY should be excluded. The CGMY model was the one that revealed the worst adjustment independently on the day, in terms of very strange, unstable and hard values to interpret. Hence we can get improvements if we employ more general Lévy processes than Brownian motion in option pricing. Moreover, the B&S model has a mean relative mean squared error lower than that of the MJD model indicating that jumps are necessary to include in option pricing. The bad performance of the B&S model also suggests that we need additional parameters to capture skewness and kurtosis of futures contracts (the underlying) which also implies increased computational time and hardness. However, a huge array of possibilities emerges with respect to the "cause-effect" standards, and studying these deeply would be our next step. The main conclusion therefore remains: Heavy tailed modeling in electricity options on futures is extremely important.

## VI. CONCLUSION

This paper addressed option pricing models from the perspective of Lévy processes, which offer better tools for analyzing skewness, fat tails, and stochastic volatility in financial data than the classical diffusions or jump-diffusion models. We estimate relative pricing errors for the Affine Jump Diffusion model (MJD) and Lévy models presented, using the B&S model as a benchmark, on electricity futures options. The stability of the implied volatility estimates over time is also examined, with an increasing pattern identified in 2007. Using these Lévy models, we clearly observe a significant improvement with respect to the B&S model. We can conclude that the more flexible Lévy processes are more suitable than the normal distribution. Volatility surface plots on 27 March 2008 show that the CGMY model provides the worst performance compared to VG. Therefore, allowing for diffusions and jumps of both finite and infinite activity (CGMY model) seems redundant in electricity. In fact, from all the processes analyzed, CGMY model gave the worst and incorrect results. Implied volatility surface changed

dynamically over time in a way that is not taken into account by current modeling approaches in electricity options on futures, with visible patterns. The increased pattern verified on implied volatilities from 2007 onwards is due to larger prices verified in the market in that period. In the present setting, and considering that modeling adjustment is measured by that model with the smallest relative pricing error, the Variance Gamma (VG) process should be the one selected for electricity options on futures. The analysis undertaken over implied volatility indicates that an increase in the underlying price will increase the implied volatility, but will also increase relative mean squared errors. Moreover, differences in changes of implied volatility found between different models revealed to be seasonal, where in general mean and variance values changed in the same way for every model from one month to another, especially during winter and summer. As such, volatility changes for changed uncertainty, or else, increasing futures prices. This also induces the need to account for seasonality when modeling electricity spot/futures price volatility, and not only to account for this seasonality for electricity prices (as it has been extensively noticed in the literature). However, we should bear in mind that the higher the volatility, the higher the premium to be demanded and the more difficult it will be to pay for the option. Despite the future work tips provided during the exposition, we should also consider some other possibilities. In the future, it would be interesting to calibrate with more option data as it becomes available. We also should take into account non-constant interest rates and Lévy processes with stochastic volatility. More research is needed also on how to link the implied volatility dynamics to return factors, market specificities as well as discontinuous movements in the price and implied volatility dynamics, and still retaining simplicity and tractability. Moreover, computation of the Greeks under infinite activity models would be interesting for risk management purposes.

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